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Performance degradation due to transfer function errors in acoustic brightness and contrast control: sensitivity analysis

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ABSTRACT

Acoustic brightness/contrast control is a method to generate acoustically bright zone (loud region) or acoustically bright zone and dark zone (quiet region) at the same time using several sound sources. For example, in implementing private audio system, it has been demonstrated that acoustic contrast control is one of effective means to maximize the acoustic energy density ratio between acoustically bright zone (listener region) and dark zone (elsewhere). In acoustic brightness /contrast control, measured transfer functions, which shows the relation between the input signal of a sound source and the output signal of a microphone, are normally used because there are several components in the system, i.e. microphone array, loudspeaker array, and measurement/playback devices. If there are errors in measuring transfer function due to noises, system nonlinearity, or any kinds of disturbances, the desired performance might be distorted by the errors. These errors degrade the system performance whose measure is brightness, contrast and spatial mean-squared-error of the control zone. In this paper, the errors are expressed in terms of magnitudes and phases in general, and we have formulated the performance variation of acoustic brightness/contrast control due to the errors in measuring transfer functions mathematically and evaluated its validity through the simulation results.

MOTIVE AND OBJECTIVE

Using acoustic brightness/contrast control[1], we can obtain the optimal filter as the input of multiple loudspeakers, which maximize acoustic energy in a certain region for given control effort or acoustic energy ratio between two regions.

Figure 1 shows the details of acoustic brightness/contrast control and we can observe how to get the optimal filter becomes simply the maximum eigenvalue problem of spatial correlation matrix ($\mathbf{R}_b$ and $\mathbf{R}_{d\,\mathrm{or}\,t}^{-1}\mathbf{R}_b$). A spatial correlation matrix consists of transfer functions $h(\vec{r}_m \mid \vec{r}_{s,l}; f)$ which show the relation between the input signal of a sound source and the output signal of a microphone. Therefore, if there are errors in transfer functions, they cause the error of spatial correlation matrix which finally induces the distortion of control results. In this paper, we try to investigate the performance degradation of acoustic brightness and contrast control due to transfer function error mathematically and to evaluate the validity through a numerical simulation.

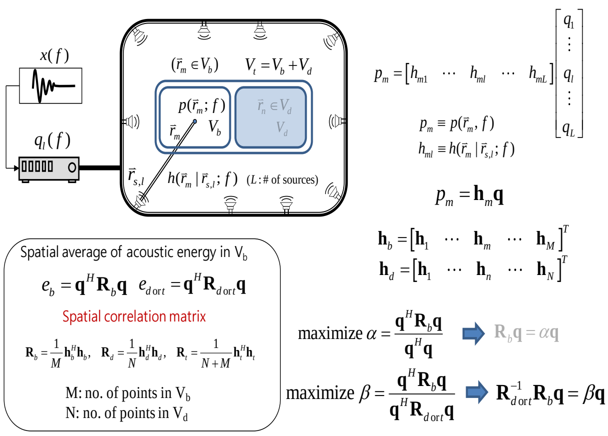

Figure 1. Acoustic brightness/contrast control (V_b : bright zone, V_d : dark zone, m: the index of measurement points in V_b, M : the total number of points in V_b, n : the index of measurement points in V_d , N : the total number of points in V_d , h_{ml} : transfer function between $\vec{r}_m$ and $\vec{r}_{s,l}$, L : the total number of sound source, q_l : source input, $\mathbf{R}_b$: spatial correlation matrix of bright zone, $\mathbf{R}_{d\,\mathrm{or}\,t}$: spatial correlation matrix of dark zone(or total zone of interest), α : acoustic brightness, β : acoustic contrast)

PROBLEM DEFINITION

Definition of measured transfer function

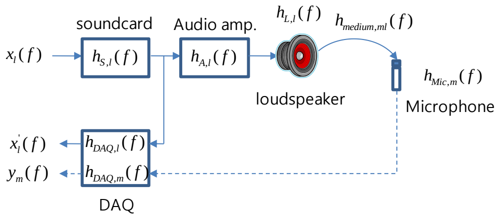

Figure 2. Transfer function in terms of all elements of the measurement system (l : the index of input channel, m : the index of output channel, h : transfer function, x_l : the input signal of the l -th input channel, y_m : the output signal of the m -th output signal)

As depicted in Figure 2, a measured transfer function includes the transfer functions of playback and measurement devices, i.e. soundcard, audio amplifier, loudspeaker, microphone, and data acquisition (DAQ) device. These transfer functions can be classified by the input and the output component of transfer function. That is

$$h_{ml}(f) = h_{input,ml}(f)h_{output,ml}(f) \tag{1}$$

where $h_{input,ml}(f) = \dfrac{h_{L,l}(f)h_{A,l}(f)h_{S,l}(f)}{h_{DAQ,l}(f)}$ and

$$h_{output,ml}(f) = h_{DAQ,m}(f)h_{Mic,m}(f)h_{medium,ml}(f).$$

Measured transfer function including the error

If there are errors in a measured transfer function, the errors can be defined in complex domain because a transfer function is a complex value: the magnitude error and the phase error of a transfer function. The sources of transfer function error can be classified as the error of $h_{input,ml}(f)$ and $h_{output,ml}(f)$ in Eq. (1). However, the possibility of the error of $h_{input,ml}(f)$ may be small because most of playback and DAQ device are guaranteed with the very small error in operation and loudspeakers have the fixed position. In this regard, it is noteworthy to consider the error of $h_{output,ml}(f)$ which may stem from sensor position mismatch, noise and the variation of measurement condition (for example, the instant fluctuation of temperature and humidity, and the variation of furniture arrangement in a room). Therefore, the measured transfer function including errors ($\tilde{h}_{ml}$) can be expressed as

$$\tilde{h}_{ml} = h_{ml}(1+\varepsilon_a(m))e^{j\varepsilon_\phi(m)} \tag{2}$$

where $\varepsilon_a(m)$ and $\varepsilon_\phi(m)$ are the magnitude and the phase error of $h_{output,ml}(f)$ [2]. If we assume that these errors are reasonably small, then this assumption enables Eq. (2) to be expressed in terms of Taylor series. That is

$$\begin{aligned}\tilde{h}_{ml} &= h_{ml}\left(1+\varepsilon_a(m)+j\varepsilon_\phi(m)-\frac{1}{2}\varepsilon_\phi^2(m)+j\frac{1}{2}\varepsilon_a(m)\varepsilon_\phi(m)\right)\\ &= h_{ml}+\delta h_{ml}\end{aligned} \tag{3}$$

where the diacritical mark tilde(~) means a variable consisting of true value and the error ($\tilde{a} = a + \delta a$, a : a variable).

Assumptions of error sources

First, let us consider that there is a sensor position mismatch in transfer function measurement. It implies the magnitude change of measured pressure, therefore it may cause the magnitude error in measured transfer function. The magnitude error estimation of measured transfer function is not possible because the sensor position mismatch is arbitrary in a measurement event. However, if we know the statistical properties of the error, we can analyze the probability of the error in a measurement event. Bias and random error can express the probability of the error. In the same manner, the phase error of measured transfer function can be considered.

Let us consider that the mean and variance of the population $S_{a,\phi}$ are 0 and $\sigma_{a,\phi}^2$. Because the error sources are drawn arbitrarily from the populations, the ensemble average of $\varepsilon_{a,\phi}$ and $\varepsilon_{a,\phi}^2$ at a point are equal to 0 and $\sigma_{a,\phi}^2$. By the same reason, $\varepsilon_{a,\phi}$ at a point are uncorrelated with those at other points. These relations can be written as

$$\begin{aligned}&E\left[\varepsilon_{a,\phi}(m)\right]=0\\ &E\left[\varepsilon_{a,\phi}(m)\varepsilon_{a,\phi}(m')\right]=\sigma_{a,\phi}^2\delta_{mm'}\end{aligned} \tag{4}$$

where E represents ensemble average. In addition, the error sources can be assumed to be statistically independent of one another. That is

$$E\left[f(\varepsilon_a)g(\varepsilon_\phi)\right]=E\left[f(\varepsilon_a)\right]E\left[g(\varepsilon_\phi)\right] \tag{5}$$

where f and g are arbitrary functions.

ACOUSTIC BRIGHTNESS AND CONTRAST CONTROL WITH TRANSFER FUNCTION ERROR

Perturbation of optimal filter

Transfer function error causes the distortion of optimal filter. As shown in Figure 1, acoustic brightness/contrast control is a kind of eigenvalue problem and the optimal filter is the eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_b$ or $\mathbf{R}_{d\text{ or }t}^{-1}\mathbf{R}_b$. We note that the subscript d (or t) denotes the contrast control when a contrast is defined as the spatially averaged acoustic energy ratio of the bright zone to the dark zone(or the total zone of interest). If there is the error in a transfer function, then the distorted optimal filter is obtained through

$$\tilde{\mathbf{R}}_b\tilde{\mathbf{q}} = \tilde{\alpha}\tilde{\mathbf{q}}, \tag{6}$$

$$\tilde{\mathbf{R}}_b\tilde{\mathbf{q}} = \tilde{\beta}\tilde{\mathbf{R}}_{d\text{ or }t}\tilde{\mathbf{q}} \tag{7}$$

The perturbation of the optimal filter is mathematically calculated by the eigenvalue perturbation theory [3] as

$$\begin{aligned}&\delta\mathbf{q}_1=\sum_{j=2}^{L}c_{1j}\mathbf{q}_j\\ &c_{1j}=\begin{cases}c_{b,1j} & \text{for brightness control}\\ c_{d\text{ or }t,1j} & \text{for contrast control}\end{cases}\end{aligned} \tag{8}$$

where $c_{b,1j}=\dfrac{\mathbf{q}_j^H\delta\mathbf{R}_b\mathbf{q}_1}{(\alpha_1-\alpha_j)\mathbf{q}_j^H\mathbf{q}_j}$, $c_{d\text{ or }t,1j}=\dfrac{\mathbf{q}_j^H(\delta\mathbf{R}_b-\beta_1\delta\mathbf{R}_{d\text{ or }t})\mathbf{q}_1}{(\beta_1-\beta_j)\mathbf{q}_j^H\mathbf{R}_{d\text{ or }t}\mathbf{q}_j}$, $\delta\mathbf{R}_b$ and $\delta\mathbf{R}_{d\text{ or }t}$ are the perturbation of $\mathbf{R}_b$ and $\mathbf{R}_{d\text{ or }t}$ due to transfer function error, α_1 (β_1) and α_j (β_j) are the maximum eigenvalue and other eigenvalue, $\mathbf{q}_1$ and $\mathbf{q}_j$ are the optimal filter and the non-optimal filter (see the Appendix A).

Perturbation of acoustic brightness and contrast

In case there is the distortion of the optimal filter, acoustic brightness and contrast might be distorted. The distorted acoustic brightness is expressed as

$$\hat{\alpha}_1 = \frac{\tilde{\mathbf{q}}_1^H \mathbf{R}_b \tilde{\mathbf{q}}_1}{\tilde{\mathbf{q}}_1^H \tilde{\mathbf{q}}_1} \tag{9}$$

Eq. (9) can be rewritten in terms of the perturbation of the optimal filter as

$$\hat{\alpha}_1 = \alpha_1 + \frac{\delta\mathbf{q}_1^H (\mathbf{R}_b - \alpha_1 \mathbf{I})\delta\mathbf{q}_1}{\mathbf{q}_1^H \mathbf{q}_1} \quad \text{(Appendix B)} \tag{10}$$

where $\mathbf{I}$ is an identity matrix. Eq. (10) implies that the distortion of acoustic brightness stems from the distortion of optimal filter and the brightness decreases due to the transfer function error because $\mathbf{R}_b - \alpha_1 \mathbf{I}$ is a negative definite matrix. If we substitute Eq. (8) into Eq. (10), then we can get the expression of distorted acoustic brightness as

$$\begin{aligned}\hat{\alpha}_1 &= \alpha_1 + \sum_{j=2}^{L}\sum_{j'=2}^{L} c_{b,1j}^{*} c_{b,1j'} \frac{\mathbf{q}_j^H (\mathbf{R}_b - \alpha_1 \mathbf{I})\mathbf{q}_{j'}}{\mathbf{q}_j^H \mathbf{q}_j} \\ &= \alpha_1 - \sum_{j=2}^{L} | c_{b,1j} |^2 (\alpha_1 - \alpha_j)\end{aligned} \quad .(11)$$

By the same token, the distorted contrast is given as

$$\hat{\beta}_1 = \frac{\tilde{\mathbf{q}}_1^H \mathbf{R}_b \tilde{\mathbf{q}}_1}{\tilde{\mathbf{q}}_1^H \mathbf{R}_{d\,\mathrm{or}\,t} \tilde{\mathbf{q}}_1} \tag{12}$$

Eq. (12) can be rewritten as

$$\hat{\beta}_1 = \beta_1 + \frac{\delta\mathbf{q}_1^H (\mathbf{R}_b - \beta \mathbf{R}_{d\ \mathrm{or}\ t})\delta\mathbf{q}_1}{\mathbf{q}_1^H \mathbf{R}_{d\ \mathrm{or}\ t} \mathbf{q}_1} \quad \text{(Appendix C)} \tag{13}$$

$$\hat{\beta}_1 = \beta_1 - \sum_{j=2}^{L} | c_{d\,\mathrm{or}\,t,1j} |^2 (\beta_1 - \beta_j) \tag{14}$$

Sensitivity of acoustic brightness/contrast to transfer function error

Eq. (11) and (14) can be rewritten in terms of transfer function errors as

$$\hat{\alpha}_1 = \alpha_1 - \sum_{m=1}^{M}\sum_{m'=1}^{M}\left\{\sum_{j=2}^{L}\frac{4\left(\mathbf{q}_j^H \mathbf{h}_m^H \mathbf{h}_m \mathbf{q}_1\right)^H \mathbf{q}_j^H \mathbf{h}_{m'}^H \mathbf{h}_{m'} \mathbf{q}_1}{M^2(\alpha_1 - \alpha_j)(\mathbf{q}_j^H \mathbf{q}_j)^2}\right\}\varepsilon_{a,m}\varepsilon_{a,m'} \tag{15}$$

$$\begin{aligned}\hat{\beta}_1 = \beta_1 &- \sum_{m=1}^{M}\sum_{m'=1}^{M}\left\{\sum_{j=2}^{L}\frac{4\left(\mathbf{q}_j^H \mathbf{h}_m^H \mathbf{h}_m \mathbf{q}_1\right)^H \mathbf{q}_j^H \mathbf{h}_{m'}^H \mathbf{h}_{m'} \mathbf{q}_1}{M^2(\beta_1 - \beta_j)(\mathbf{q}_j^H \mathbf{R}_{d\ \mathrm{or}\ t} \mathbf{q}_j)^2}\right\}\varepsilon_{a,m}\varepsilon_{a,m'} \\ &- \beta_1^2 \sum_{n=1}^{N}\sum_{n'=1}^{N}\left\{\sum_{j=2}^{L}\frac{4\left(\mathbf{q}_j^H \mathbf{h}_n^H \mathbf{h}_n \mathbf{q}_1\right)^H \mathbf{q}_j^H \mathbf{h}_{n'}^H \mathbf{h}_{n'} \mathbf{q}_1}{N^2(\beta_1 - \beta_j)(\mathbf{q}_j^H \mathbf{R}_{d\ \mathrm{or}\ t} \mathbf{q}_j)^2}\right\}\varepsilon_{a,n}\varepsilon_{a,n'} \\ &+ 2\beta_1 \operatorname{Re}\left[\sum_{m=1}^{M}\sum_{n=1}^{N}\left\{\sum_{j=2}^{L}\frac{4\left(\mathbf{q}_j^H \mathbf{h}_m^H \mathbf{h}_m \mathbf{q}_1\right)^H \mathbf{q}_j^H \mathbf{h}_n^H \mathbf{h}_n \mathbf{q}_1}{MN(\beta_1 - \beta_j)(\mathbf{q}_j^H \mathbf{R}_{d\ \mathrm{or}\ t} \mathbf{q}_j)^2}\right\}\varepsilon_{a,m}\varepsilon_{a,n}\right]\end{aligned} \tag{16}$$

Eq. (15) and (16) implies the only magnitude error of each output channel contributes to the perturbation of brightness/contrast. The interest thing is that there is no contribution of the phase error of transfer function in perturbation of brightness/contrast. This is because the acoustic brightness/contrast is defined based on the acoustic energy quantity. In practice, $\varepsilon_{a,m(\mathrm{or}\,n)}$ is random and unpredictable so that it is meaningful to see the sensitivity of the mean of relative brightness/contrast to the variance of magnitude error. The mean of relative brightness/contrast is expressed as

$$E\left[\frac{\hat{\alpha}_1}{\alpha_1}\right] = 1 + \sigma_a^2 \sum_{j=2}^{L}\sum_{m=1}^{M}\frac{-4\left|\mathbf{q}_j^H \mathbf{h}_m^H \mathbf{h}_m \mathbf{q}_1\right|^2}{M^2 \alpha_1(\alpha_1 - \alpha_j)(\mathbf{q}_j^H \mathbf{q}_j)^2}, \tag{17}$$

$$E\left[\frac{\hat{\beta}_1}{\beta_1}\right] = 1 + \sigma_a^2 \sum_{j=2}^{L}\left\{\frac{\sum_{m=1}^{M}\frac{-4w_{d\ \mathrm{or}\ t}\left|\mathbf{q}_j^H \mathbf{h}_m^H \mathbf{h}_m \mathbf{q}_1\right|^2}{M^2} + \beta_1^2 \sum_{n=1}^{N}\frac{-4\left|\mathbf{q}_j^H \mathbf{h}_n^H \mathbf{h}_n \mathbf{q}_1\right|^2}{N^2}}{\beta_1(\beta_1 - \beta_j)(\mathbf{q}_j^H \mathbf{R}_{d\ \mathrm{or}\ t} \mathbf{q}_j)^2}\right\} \tag{18}$$

where $w_d = 1$, $w_t = 1 - 2\beta_1 \frac{M}{N}$, m and n are the index of sampled points in bright zone and dark zone, and M and N are the total number of sampled points in bright zone and dark zone (or the total zone of interest). Then, we define the sensitivity of brightness/contrast as the derivative of the mean of relative brightness/contrast to the variance of magnitude error as

$$S_{\sigma_a^2}^{\hat{\alpha}_1} \equiv \frac{\partial}{\partial \sigma_a^2} E\left[\frac{\hat{\alpha}_1}{\alpha_1}\right], \tag{19}$$

$$S_{\sigma_a^2}^{\hat{\beta}_1} \equiv \frac{\partial}{\partial \sigma_a^2} E\left[\frac{\hat{\beta}_1}{\beta_1}\right] \tag{20}$$

According to this definition, we can get the detail expression from Eqs. (17) and (18). These are

$$S_{\sigma_a^2}^{\hat{\alpha}_1} = \sum_{j=2}^{L}\frac{\frac{-4}{M}\sum_{m=1}^{M}\frac{\left|\mathbf{h}_m \mathbf{q}_j\right|^2 \left|\mathbf{h}_m \mathbf{q}_1\right|^2}{M}}{\alpha_1(\alpha_1 - \alpha_j)(\mathbf{q}_j^H \mathbf{q}_j)^2}, \tag{21}$$

$$S_{\sigma_a^2}^{\hat{\beta}_1} = \sum_{j=2}^{L}(-4)\cdot\frac{\frac{w_{d\ \mathrm{or}\ t}}{M}\sum_{m=1}^{M}\frac{\left|\mathbf{h}_m \mathbf{q}_j\right|^2 \left|\mathbf{h}_m \mathbf{q}_1\right|^2}{M} + \frac{1}{N}\sum_{n=1}^{N}\frac{\left|\mathbf{h}_n \mathbf{q}_j\right|^2 \left|\mathbf{h}_n \mathbf{q}_1\right|^2}{N}}{\beta_1(\beta_1 - \beta_j)(\mathbf{q}_j^H \mathbf{R}_{d\ \mathrm{or}\ t} \mathbf{q}_j)^2}. \tag{22}$$

SENSITIVITY ANALYSIS OF ACOUSTIC BRIGHTNESS / CONTRAST

In Eq. (21), the sensitivity of acoustic brightness to the variance of magnitude error is related with the correlation function between squared pressure via optimal filter and via non-optimal filter $\left(\sum_{m=1}^{M}\left|\mathbf{h}_m \mathbf{q}_j\right|^2 \left|\mathbf{h}_m \mathbf{q}_1\right|^2 \Big/ M\right)$ in a bright zone, brightness α_1, the difference between the maximum eigenvalue and the j-th eigenvalue ($\alpha_1 - \alpha_j$), the total number of sampled points in a bright zone (M), and control effort ($\mathbf{q}_j^H \mathbf{q}_j$). For simplicity, we denote $\alpha_1 - \alpha_j$ as $d_{\alpha,1j}$, $\sum_{m=1}^{M}\left|\mathbf{h}_m \mathbf{q}_j\right|^2 \left|\mathbf{h}_m \mathbf{q}_1\right|^2 \Big/ M$ as $r_{b,1j}$, and the term corresponding to the index j in $S_{\sigma_a^2}^{\hat{\alpha}_1}$ as $S_{\sigma_a^2}^{\hat{\alpha}_1}(j)$. Then Eq. (21) can be rewritten as

$$S_{\sigma_a^2}^{\hat{\alpha}_1} = \sum_{j=2}^{L} S_{\sigma_a^2}^{\hat{\alpha}_1}(j) \quad \left(S_{\sigma_a^2}^{\hat{\alpha}_1}(j) = \frac{-4}{\alpha_1(\mathbf{q}_j^H \mathbf{q}_j)^2}\cdot\frac{1}{M}\cdot\frac{r_{b,1j}}{d_{\alpha,1j}}\right). \tag{23}$$

Eq. (23) implies the sensitivity of acoustic brightness to the variance of magnitude error decreases as α_1, $d_{\alpha,1j}$ and M increase, and as $r_{b,1j}$ decreases. α_1 increases as the number of sources (L) increase and M increases as we sample a bright zone with more microphones. In other words, the more number of input and output channels for a bright zone we have, the smaller sensitivity of brightness to the variance of magnitude error we get. $d_{\alpha,1j}$ is depending on the selection of bright zone, source arrangement, and frequency and also physically correlated with $r_{b,1j}$.

By the same token, we can express the sensitivity of acoustic contrast to the variance of magnitude error as

$$S_{\sigma_a^2}^{\hat{\beta}_1} = \sum_{j=2}^{L} S_{\sigma_a^2}^{\hat{\beta}_1}(j)$$

$$\left(S_{\sigma_a^2}^{\hat{\beta}_1}(j) = \frac{-4}{\beta_1(\mathbf{q}_j^H \mathbf{R}_{d \text{ or } t}\mathbf{q}_j)^2} \cdot \frac{\frac{w_{d \text{ or } t}}{M} r_{b,1j} + \frac{1}{N} r_{d,1j}}{d_{\beta,1j}} \right). \qquad (24)$$

where $d_{\beta,1j}$ is $\beta_1 - \beta_j$ and $r_{d,1j}$ is $\sum_{n=1}^{N} \frac{|\mathbf{h}_n \mathbf{q}_j|^2 |\mathbf{h}_n \mathbf{q}_1|^2}{N}$. In Eq. (24), the sensitivity of acoustic contrast to the variance of magnitude error ($S_{\sigma_a^2}^{\hat{\beta}_1}$) decreases as β_1, $d_{\beta,1j}$, M an d N increase, and as $r_{b,1j}$ and $r_{d,1j}$ decrease. β_1 increases as the number of sources (L) increases.

Simulation condition

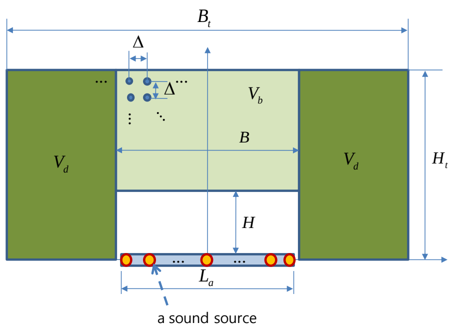

Figure 3. System configuration for numerical simulation (L_a : source array aperture, Δ : sampling distance between sensors, $H_t - H$: vertical width of bright zone, B : horizonetal width of bright zone, $V_{b(\text{or } d)}$: bright(or dark) zone)

To do the sensitivity analysis of brightness/contrast, numerical simulation is taken in case of private audio system[5] as shown in Fig. 3.

Figure 3 shows the system configuration for numerical simulation. In this simulation, we consider sound source is as a monopole. There are L sound sources within the aperture of L_a in free field condition. H_t, H, B and L_a are determined as $0.6m$, $0.2m$, $0.32m$ and $0.32m$ in this simulation which are reasonable size for private audio system. The interestic frequency is from 20Hz to 10kHz in this simulation The sensitivity of brightnes/contrast is investigated for three parameters: frequency (f) or characteristic size of bright zone ($\sqrt{B(H_t - H)}$), sampling distance(Δ) and number of sound sources (L)

Sensitivity of acoustic brightness

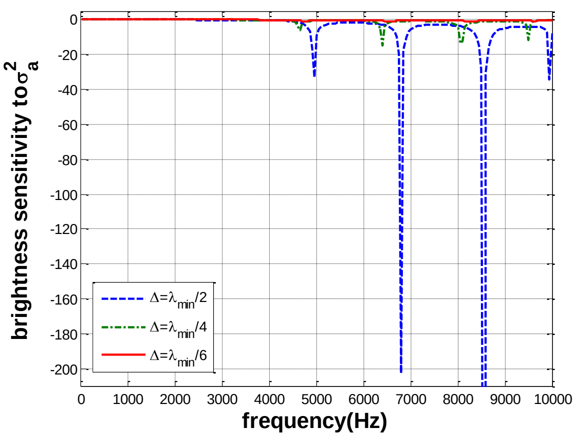

Figure 4. Acoustic brightness sensitivity ($S_{\sigma_a^2}^{\hat{\alpha}_1}$) for difference sampling distance(Δ) with respect to frequency(f) ($\lambda_{\min} = \lambda_{10kHz}$, $L = 9$)

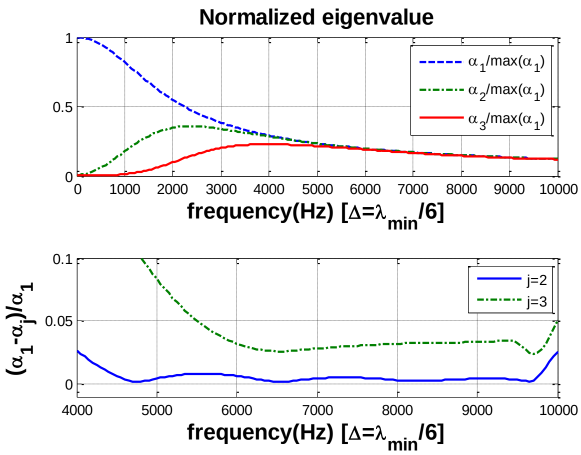

Figure 5. Normalized eigenvalues($\alpha_1/\max(\alpha_1)$ and $\alpha_j/\max(\alpha_1)$) and relative eigenvalue difference($d_{\alpha,1j}/\alpha_1$) with respect to frequency ($L = 9$)

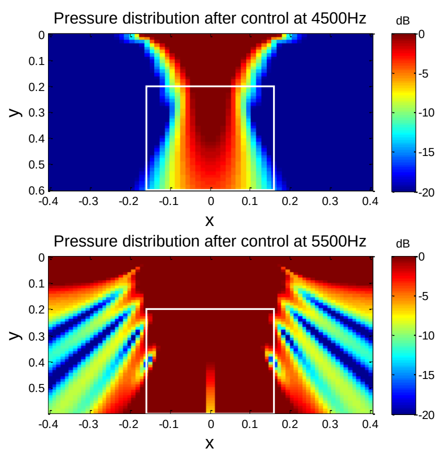

Figure 6. Pressure distribution after brightness control at 4.5kHz and 4.5kHz

Figure 4 shows the acoustic brightness sensitivity for difference sampling distance with respect to frequency. In Figure 4, we can observe the brightness is less sensitive to magnitude error of transfer function as sampling distance decreases. For example, at $\Delta = \lambda / 6$, there are several notchs around 5kHz, 6.7kHz, 9.6kHz and 10kHz. This phenomena is because the second maxium eigenvalue (α_2) becomes closer to the maximum eigenvalue (α_1) as shown in Figure 5 (bottom). Notch implies the change of focused shape in a bright zone around notch frequency. For instance, there is a change of focused shape around 5kHz as shown in Figure 5. It implies that careful selection of bright zone is needed around the notch frequencies for reducing the effect of transfer function error.

Figure 7 shows the acoustic brightness sensitivity for difference number of source per a fixed array aperture with respect to frequency. From Figure 6, we can observe the first notch frequency increases as the number of sources per a fixed array aperture increase. This implies that the available size of bright zone with small brightness sensitivity becomes larger as the gap between a source and the nearest sources becomes smaller.

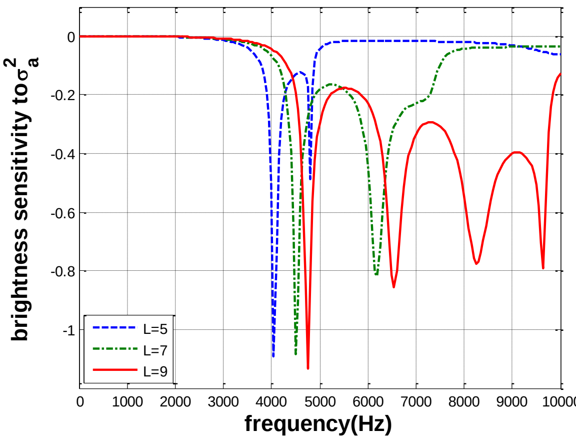

Figure 7. Acoustic brightness sensitivity ($S^{\hat{\alpha}_1}_{\sigma_a^2}$) for different number of sources (L) per constant array aperture (L_a) with respect to frequency (f)

Sensitivity of acoustic contrast

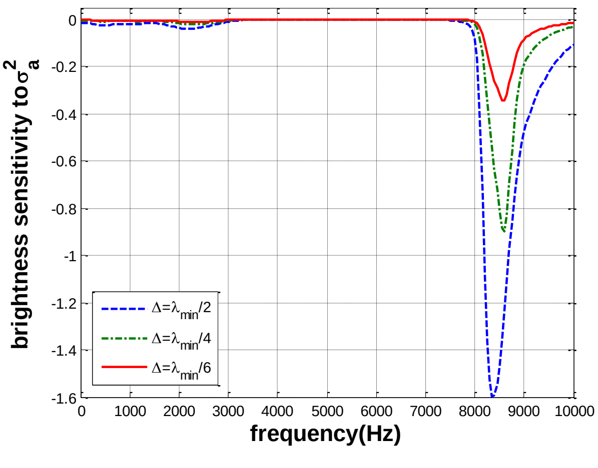

Figure 8. Acoustic contrast sensitivity ($S^{\hat{\beta}_1}_{\sigma_a^2}$) for difference sampling distance(Δ) with respect to frequency(f) [$\beta_1 = \mathbf{q}_1^H\mathbf{R}_b\mathbf{q}_1 / \mathbf{q}_1^H\mathbf{R}_d\mathbf{q}_1$]($L=9$)

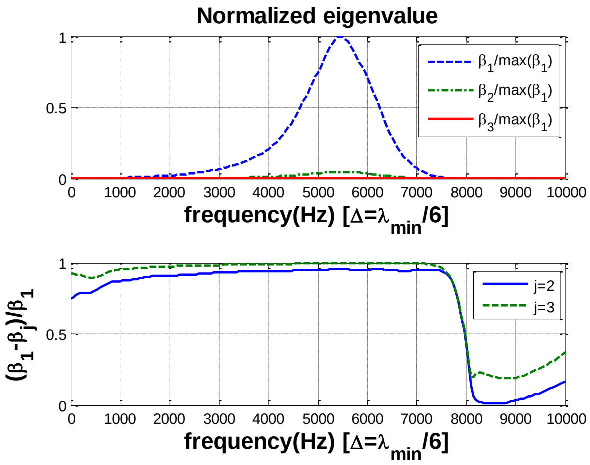

Figure 9. Normalized eigenvalues($\beta_1/\max(\beta_1)$ and $\beta_j/\max(\beta_1)$) and relative eigenvalue difference($d_{\beta,1j}/\beta_1$) with respect to frequency

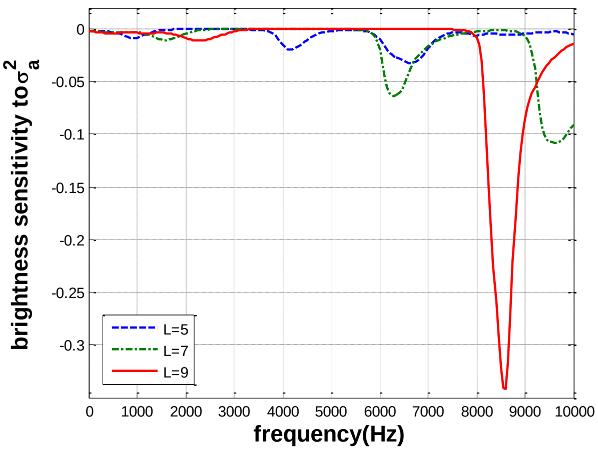

Figure 10. Acoustic contrast sensitivity ($S^{\hat{\beta}_1}_{\sigma_a^2}$) for different number of source (L) per constant array aperture (L_a) with respect to frequency(f) [$\beta_1 = \mathbf{q}_1^H\mathbf{R}_b\mathbf{q}_1 / \mathbf{q}_1^H\mathbf{R}_d\mathbf{q}_1$]

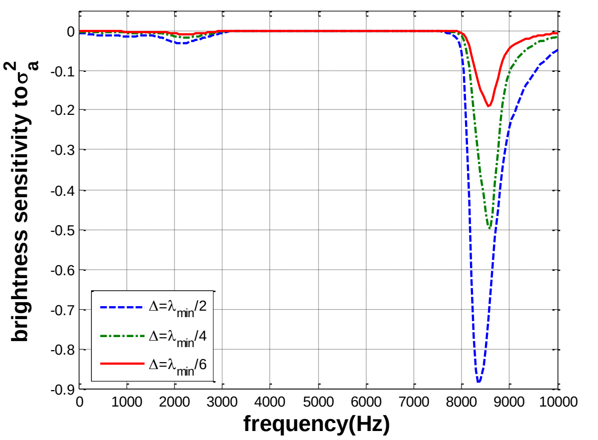

Figure 11. Acoustic contrast sensitivity ($S^{\hat{\beta}_1}_{\sigma_a^2}$) for difference sampling distance(Δ) with respect to frequency(f) [$\beta_1 = \mathbf{q}_1^H\mathbf{R}_b\mathbf{q}_1 / \mathbf{q}_1^H\mathbf{R}_t\mathbf{q}_1$]($L=9$)

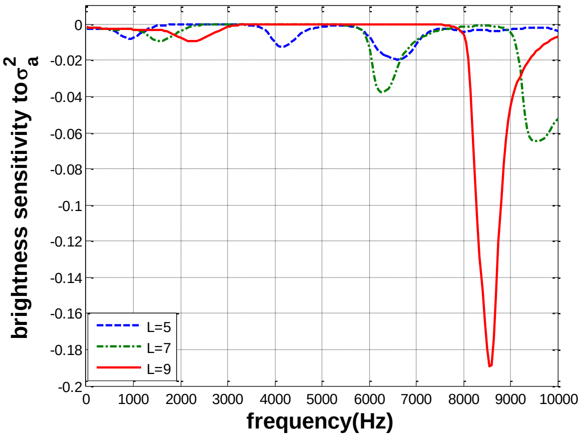

Figure 12. Acoustic contrast sensitivity ($S^{\hat{\beta}_1}_{\sigma_a^2}$) for different number of sources (L) per constant array aperture(L_a) with respect to frequency(f) [$\beta_1 = {\mathbf{q}_1^H \mathbf{R}_b \mathbf{q}_1} / {\mathbf{q}_1^H \mathbf{R}_t \mathbf{q}_1}$]

From Figure 6 to Figure 12, the acoustic contrast sensitivity has the same tendency with acoustic brightness sensitivity for three parameters: frequency (f), sampling distance (Δ) and number of sound sources (L). But we can observe contrast control when we define a contrast as the energy density ratio of bright zone to total zone of interest is less sensitive than contrast control when we define a contrast as the energy density ratio of bright zone to dark zone. This is because the acoustic energy density for a dark zone is smaller than the acoustic energy density for the total zone of interest.

CONCLUSIONS AND DISCUSSIONS

We investigate the sensitivity of acoustic brightness/contrast to the variance of transfer function error. Transfer function is defined in complex domain as the magnitude error and the phase error and we can get the result that there is no contribution of phase error in the sensitivity of brightness/contrast. Through the numerical simulation, acoustic brightness sensitivity and contrast sensitivity have the same tendeny for frequency (or characteristic size of bright zone), sampling distance and number of sound sources per a fixed array aperture. Brightness and contrast are less sensitive to magnitude error of transfer function as sampling distance decreases, and we need careful selection of bright zone around the notch frequencies for reducing the effect of transfer function error because there is a change of focused shape. Furthermore, for generating wider bright zone with small brightness sensitivity and contrast sensitivity, the gap between a source and the nearest sources should become smaller.

In this study, the sensitivity of acoustic brightness and contrast to the variance of magnitude error of transfer function has been investigated. However, it may not be enough to describe the degradation of control performance because acoustic brightness and contrast are spatially averaged quantity. Therefore, it is needed to be considered another measure for the performance degradation according to the purpose of control.

APPENDIX A : DERIVATION OF $\delta \mathbf{q}_1$

$$\tilde{\mathbf{R}}_b \tilde{\mathbf{q}}_1 = \beta_1 \tilde{\mathbf{R}}_{d\,ort} \tilde{\mathbf{q}}_1 \quad \text{(A1)}$$

$$\begin{aligned}(\mathbf{R}_b + \delta\mathbf{R}_b)(\mathbf{q}_1 + \delta\mathbf{q}_1) \\ = (\beta_1 + \delta\beta_1)(\mathbf{R}_{d\,ort} + \delta\mathbf{R}_{d\,ort})(\mathbf{q}_1 + \delta\mathbf{q}_1)\end{aligned} \quad \text{(A2)}$$

$$\begin{aligned}\delta\mathbf{R}_b\mathbf{q}_1 + \mathbf{R}_b\delta\mathbf{q}_1 \\ = \beta_1\delta\mathbf{R}_{d\,ort}\mathbf{q}_1 + \beta_1\mathbf{R}_{d\,ort}\delta\mathbf{q}_1 + \delta\beta_1\mathbf{R}_{d\,ort}\mathbf{q}_1\end{aligned} \quad \text{(A3)}$$

If all eigenvalues are simple, then we may assume that $\delta\mathbf{q}_1 = \sum_{j=2}^{L} c_{d\,ort,1j}\mathbf{q}_j$.

$$\begin{aligned}\delta\mathbf{R}_b\mathbf{q}_1 + \mathbf{R}_b\sum_{j=2}^{L} c_{d\,ort,1j}\mathbf{q}_j \\ = \beta_1\delta\mathbf{R}_{d\,ort}\mathbf{q}_1 + \beta_1\mathbf{R}_{d\,ort}\sum_{j=2}^{L} c_{d\,ort,1j}\mathbf{q}_j + \delta\beta_1\mathbf{R}_{d\,ort}\mathbf{q}_1\end{aligned} \quad \text{(A4)}$$

Multiply $\mathbf{q}_k^H$ in Eq. (A4)

$$\begin{aligned}\mathbf{q}_k^H\delta\mathbf{R}_b\mathbf{q}_1 + \sum_{j=2}^{L} c_{d\,ort,1j}\mathbf{q}_k^H\mathbf{R}_b\mathbf{q}_j \\ = \beta_1\mathbf{q}_k^H\delta\mathbf{R}_{d\,ort}\mathbf{q}_1 + \beta_1\sum_{j=2}^{L} c_{d\,ort,1j}\mathbf{q}_k^H\mathbf{R}_{d\,ort}\mathbf{q}_j + \delta\beta_1\mathbf{q}_k^H\mathbf{R}_{d\,ort}\mathbf{q}_1\end{aligned} \quad \text{(A5)}$$

If $k = j$, then

$$\begin{aligned}\mathbf{q}_j^H\delta\mathbf{R}_b\mathbf{q}_1 + c_{d\,ort,1j}\mathbf{q}_j^H\mathbf{R}_b\mathbf{q}_j \\ = \beta_1\mathbf{q}_j^H\delta\mathbf{R}_{d\,ort}\mathbf{q}_1 + c_{d\,ort,1j}\beta_1\mathbf{q}_j^H\mathbf{R}_{d\,ort}\mathbf{q}_j + \delta\beta_1\mathbf{q}_j^H\mathbf{R}_{d\,ort}\mathbf{q}_1 \\ c_{d\,ort,1j}(\beta_1 - \beta_j)\mathbf{q}_j^H\mathbf{R}_{d\,ort}\mathbf{q}_j = \mathbf{q}_j^H(\delta\mathbf{R}_b - \beta_1\delta\mathbf{R}_{d\,ort})\mathbf{q}_1\end{aligned} \quad \text{(A6)}$$

$$c_{d\,ort,1j} = \frac{\mathbf{q}_j^H(\delta\mathbf{R}_b - \beta_1\delta\mathbf{R}_{d\,ort})\mathbf{q}_1}{(\beta_1 - \beta_j)\mathbf{q}_j^H\mathbf{R}_{d\,ort}\mathbf{q}_j} \quad \text{(A7)}$$

For brightness control, we can obtain $c_{b,1j}$ using Eq. (A6) because it is equivalent to the case when $\delta\mathbf{R}_{d\,ort} = 0$, $\mathbf{R}_{d\,ort} = \mathbf{I}$ and $\beta_{1\text{ or }j} = \alpha_{1\text{ or }j}$. That is

$$c_{b,1j} = \frac{\mathbf{q}_j^H\delta\mathbf{R}_b\mathbf{q}_1}{(\alpha_1 - \alpha_j)\mathbf{q}_j^H\mathbf{q}_j} . \quad \text{(A8)}$$

APPENDIX B : DERIVATION OF $\hat{\alpha}_1$

$$\begin{aligned}\hat{\alpha}_1 = \frac{\tilde{\mathbf{q}}_1^H\mathbf{R}_b\tilde{\mathbf{q}}_1}{\tilde{\mathbf{q}}_1^H\tilde{\mathbf{q}}_1} \\ = \alpha_1 + \delta\alpha_1\end{aligned} \quad \text{(B1)}$$

Eq. (B1) can be rewritten as

$$\hat{\alpha}_1\tilde{\mathbf{q}}_1^H\tilde{\mathbf{q}}_1 = \tilde{\mathbf{q}}_1^H\mathbf{R}_b\tilde{\mathbf{q}}_1 \quad \text{(B2)}$$

$$\begin{aligned}(\alpha_1 + \delta\alpha_1)(\mathbf{q}_1 + \delta\mathbf{q}_1)^H(\mathbf{q}_1 + \delta\mathbf{q}_1) \\ = (\mathbf{q}_1 + \delta\mathbf{q}_1)^H\mathbf{R}_b(\mathbf{q}_1 + \delta\mathbf{q}_1)\end{aligned} \quad \text{(B3)}$$

In Eq. (B3), if we neglect the terms with over the second order, it can be expressed as

$$\begin{aligned}\delta\alpha_1\mathbf{q}_1^H\mathbf{q}_1 + \alpha_1(\mathbf{q}_1 + \delta\mathbf{q}_1)^H(\mathbf{q}_1 + \delta\mathbf{q}_1) \\ = (\mathbf{q}_1 + \delta\mathbf{q}_1)^H\mathbf{R}_b(\mathbf{q}_1 + \delta\mathbf{q}_1)\end{aligned} \quad \text{(B4)}$$

$$\delta\hat{\alpha}_1\mathbf{q}_1^H\mathbf{q}_1 + \alpha_1(\mathbf{q}_1^H\mathbf{q}_1 + \delta\mathbf{q}_1^H\mathbf{q}_1 + \mathbf{q}_1^H\delta\mathbf{q}_1 + \delta\mathbf{q}_1^H\delta\mathbf{q}_1) = \mathbf{q}_1^H\mathbf{R}_b\mathbf{q}_1 + \delta\mathbf{q}_1^H\mathbf{R}_b\mathbf{q}_1 + \mathbf{q}_1^H\mathbf{R}_b\delta\mathbf{q}_1 + \delta\mathbf{q}_1^H\mathbf{R}_b\delta\mathbf{q}_1 \quad \text{(B5)}$$

$$\delta\hat{\alpha}_1\mathbf{q}_1^H\mathbf{q}_1 + \alpha_1\delta\mathbf{q}_1^H\delta\mathbf{q}_1 = \delta\mathbf{q}_1^H\mathbf{R}_b\delta\mathbf{q}_1 \quad \text{(B6)}$$

$$\delta\alpha_1 = \frac{\delta\mathbf{q}_1^H(\mathbf{R}_b - \alpha_1\mathbf{I})\delta\mathbf{q}_1}{\mathbf{q}_1^H\mathbf{q}_1} \quad \text{(B7)}$$

$$\hat{\alpha}_1 = \alpha_1 + \frac{\delta\mathbf{q}_1^H(\mathbf{R}_b - \alpha_1\mathbf{I})\delta\mathbf{q}_1}{\mathbf{q}_1^H\mathbf{q}_1} \quad \text{(B8)}$$

APPENDIX C : DERIVATION OF $\hat{\beta}_1$

$$\hat{\beta}_1 = \frac{\tilde{\mathbf{q}}_1^H\mathbf{R}_b\tilde{\mathbf{q}}_1}{\tilde{\mathbf{q}}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\tilde{\mathbf{q}}_1} = \beta_1 + \delta\beta_1 \quad \text{(C1)}$$

$$\hat{\beta}_1\tilde{\mathbf{q}}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\tilde{\mathbf{q}}_1 = \tilde{\mathbf{q}}_1^H\mathbf{R}_b\tilde{\mathbf{q}}_1 \quad \text{(C2)}$$

$$(\beta_1 + \delta\beta_1)(\mathbf{q}_1 + \delta\mathbf{q}_1)^H\mathbf{R}_{d\,\mathrm{or}\,t}(\mathbf{q}_1 + \delta\mathbf{q}_1) = (\mathbf{q}_1 + \delta\mathbf{q}_1)^H\mathbf{R}_b(\mathbf{q}_1 + \delta\mathbf{q}_1) \quad \text{(C3)}$$

In Eq. (C3), if we neglect the terms with over the second orde

r, it can be expressed as

$$\delta\beta_1\mathbf{q}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\mathbf{q}_1 + \beta_1(\mathbf{q}_1 + \delta\mathbf{q}_1)^H\mathbf{R}_{d\,\mathrm{or}\,t}(\mathbf{q}_1 + \delta\mathbf{q}_1) = (\mathbf{q}_1 + \delta\mathbf{q}_1)^H\mathbf{R}_b(\mathbf{q}_1 + \delta\mathbf{q}_1) \quad \text{(C4)}$$

$$\delta\beta_1\mathbf{q}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\mathbf{q}_1 + \beta_1\delta\mathbf{q}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\delta\mathbf{q}_1 = \delta\mathbf{q}_1^H\mathbf{R}_b\delta\mathbf{q}_1 \quad \text{(C5)}$$

$$\delta\beta_1 + \beta_1\delta\mathbf{q}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\delta\mathbf{q}_1 = \frac{\delta\mathbf{q}_1^H(\mathbf{R}_b - \beta_1\mathbf{R}_{d\,\mathrm{or}\,t})\delta\mathbf{q}_1}{\mathbf{q}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\mathbf{q}_1} \quad \text{(C6)}$$

$$\hat{\beta}_1 = \beta_1 + \frac{\delta\mathbf{q}_1^H(\mathbf{R}_b - \beta_1\mathbf{R}_{d\,\mathrm{or}\,t})\delta\mathbf{q}_1}{\mathbf{q}_1^H\mathbf{R}_{d\,\mathrm{or}\,t}\mathbf{q}_1} \quad \text{(C7)}$$

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