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Critical temperature ratio needed for a spontaneous gas oscillation in a miniature thermoacoustic engine

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ABSTRACT

A thermoacoustic engine consists of a resonance tube and a stack that is composed of many narrow tubes; the stack is located inside the resonator. When the ratio of the temperatures at two ends of the stack exceeds a critical value, a gas in the resonance tube spontaneously oscillates and the thermoacoustic engine works. Recently, a miniaturization of thermoacoustic engines has attracted much attention due to its simplicity and high-efficiency potential. In this study, the stability limit of the spontaneous gas oscillation in a miniature thermoacoustic engine is numerically calculated by using the thermoacoustic theory. In this calculation, the stack length is taken as one of parameters. This is because when one designs a miniature thermoacoustic engine, one wants to make a stack length as long as possible to reduce thermal conduction loss along the stack. As a result of the calculation, it was fond that the length ratio largely affects the critical temperature ratio needed for causing the spontaneous gas oscillation.

INTORODUCTION

When an acoustic wave propagates in a tube, pressure, temperature and displacement of a gas inside the tube oscillates. By using these oscillations, one can construct an engine that is called thermoacoustic engine. [1]

As shown in Fig.1, a thermoacoustic engine is essentially composed of a tube and a differentially heated stack that has many narrow flow channels. When the ratio of temperatures at the two ends of the stack exceeds a critical value, the gas inside the engine spontaneously oscillates and then, heat input from the hot end of the stack is converted into acoustical work. Due to its simplicity, the thermoacoustic engine has attracted much attention.

In 2004, Symko et al. developed a miniature thermoacoustic engine: the length of a conventional thermoacoustic engine is some meters but that of their engine is a few centi-meters.[2] The investigation of a miniature thermoacoustic engine has been progressed. In this study, we numerically investigate the critical temperature ratio needed for the operation of a miniature thermoacoustic engine with varying the length of a stack.

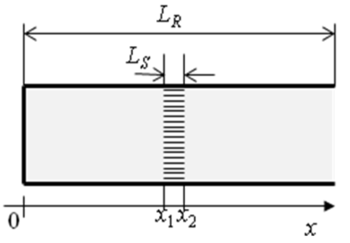

Fig. 1 Thermoacoustic engine.

CLACULATION

Model

A thermoacoustic engine shown in Fig. 1 is numerically investigated. The length of the resonator tube, L_R, is set to the value of 30 mm. One end of the resonator is opened and the other end is closed. At the midpoint of the resonator, a stack is located. Heat exchangers are neglected for simplicity. The length of the stack is denoted as L_S and it is varied. The radius of the resonator tube, r_R, is set to 10 mm and the flow channel radius in the stack, r_S, is also varied. Nitrogen at 101 kPa is used as a working gas.

The time averaged temperature of the gas, T_m, depends only on the axial direction along the tube, x. The origin of x is set at the closed end of the resonator tube. [see Fig. 1] The temperature T_m from $x = 0$ to x_1 is maintained at T_H and T_m from $x = x_1$ to L_R at T_C =300 K, where $x_1 = L_R / 2 - L_S / 2$ and $x_2 = L_R / 2 + L_S / 2$. In the stack ($x_1 < x < x_2$), T_m linearly decreases.

Calculation method

In this subsection, the calculation method of the critical temperature ratio and angular frequency of the spontaneous gas oscillation in the thermoacoustic engine is briefly described. The calculation is based on the two Rott's thermoacoustic differential equations.[3] They can be written in the matrix form as

$$\frac{d}{dx}\begin{pmatrix} P \\ U \end{pmatrix} = \begin{pmatrix} 0 & \dfrac{-i\omega\rho_m}{1-\chi_\upsilon} \\ \dfrac{-i\omega}{\gamma P_m}[1+(\gamma-1)\chi_\alpha] & \dfrac{\chi_\alpha-\chi_\upsilon}{(1-\chi_\upsilon)(1-\sigma)}\dfrac{1}{T_m}\dfrac{dT_m}{dx} \end{pmatrix}\begin{pmatrix} P \\ U \end{pmatrix} \tag{1}$$

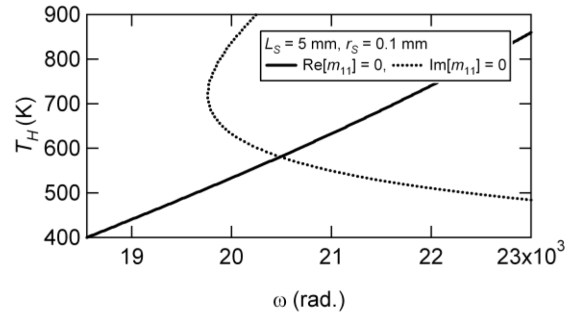

Fig. 2. The relation of the solutions of Eq. (4) (dotted line) and Eq. (5) (solid line).

where P and U are the oscillatory pressure and velocity, respectively; P_m, ρ_m, γ, and σ are the mean pressure, mean density, specific heat ratio, and Prandtl number of the working gas, respectively; χ_υ and χ_α are the thermoacoustic functions that depend on the ratio of the tube (channel) radius to δ. (see Eq. (15) of the reference[3]). These equations were changed to difference equations and they were numerically integrated. [4] As a result, we obtained transfer matrix M for P and U. By using M, the oscillatory pressure and velocity at the closed end of the resonator tube (P_{closed} and U_{closed}) are related to those at the open end of the resonator (P_{open} and U_{open}) as

$$\begin{pmatrix} P_{\text{open}} \\ U_{\text{open}} \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} P_{\text{closed}} \\ U_{\text{closed}} \end{pmatrix} \qquad (2)$$

where m_{ij} is the element of M. For the present case, $U_{\text{closed}} = 0$ and $P_{\text{open}} = 0$. Therefore if the element m_{11} is zero, U_{open} and P_{closed} become nonzero. In other words, the solution of the equation

$$m_{11} = 0 \qquad (3)$$

gives the critical temperature ratio α and angular frequency ω of the spontaneous thermoacoustic oscillation.[4]

Since m_{11}has imaginary and real parts, Eq. (3) can be divided into two equations;

$$\text{Im}[m_{11}] = 0 \qquad (4)$$

$$\text{Re}[m_{11}] = 0 \qquad (5)$$

We computationally calculated the solutions of Eq. (4) and (5), respectively. In the present calculation, ω, T_H, r_S, and L_S are parameters. As an example, the obtained ω and T_H satisfying Eq. (4) and those satisfying Eq. (5) for the case r_S = 0.1 mm and L_S = 10 mm are plotted in Fig. (2) by dotted and solid lines, respectively. As can be seen from this figure, the lines intersect at the pint (ω ~ 20500 rad., T_H ~ 580 K). Hence, the critical temperature T_{criti} and angular frequency ω_1 of the spontaneous gas oscillation in the thermoacoustic engine with r_S = 0.1 mm and L_S = 10 mm are 580 K and 20500 rad., respectively. This is because Eq. (4) and Eq. (5) must be simultaneously satisfied.

RESULT

In Fig. 3, the critical temperature ratio $\alpha = T_{\text{criti}} / T_C$ in the cases of L_S = 1, 3, 5, and 10 mm is shown as a function of r_S / δ. This figure shows that α depends on r_S / δ and takes the minimum value α_{min} in the region $2 < r_S / \delta < 4$ in each case of L_S. This tendency is the same as that of the large scale thermoacoustic engine. [4]

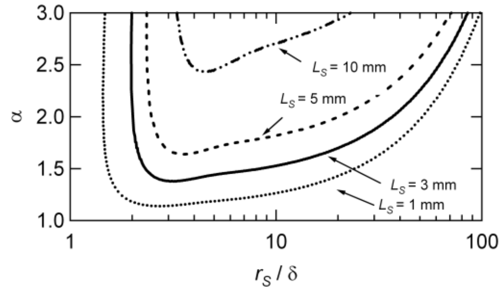

Fig. 3. The critical temperature ratio α for the spontaneous gas oscillation occurring in the miniature thermoacoustic engine.

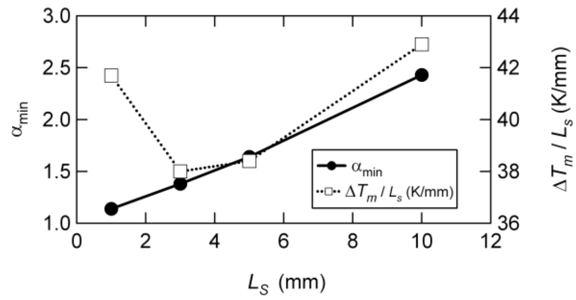

Fig. 4. Dependencies of the minimum critical temperature ratio α_{min} and temperature gradient $\Delta T_m / L_S$.

In order to discuss the effect of the value of L_S, we plot α_{min} as a function of L_S in Fig. 4 by closed circles. As shown this figure, the value α_{min} monotonically decreases with decreasing L_S. Hence, if one wants to decrease the onset temperature of a thermoacoustic engine, one should make a stack as short as possible.

There is the possibility that the decrease in the stack length causes the increase in the value of the temperature gradient along the stack. The value of the temperature gradient is very important because it is proportional to the thermal conduction loss that is one of the main losses of a thermoacoustic engine. In Fig. 4, the temperature gradient $\Delta T_m / L_S$ with α_{min} is shown as a function of L_S by open squares; $\Delta T_m = (\alpha_{\text{min}} - 1)\, T_C$. It was found that $\Delta T_m / L_S$ takes the minimum value at L_S = 3 mm. Therefore, we can say that when one wants to decrease the thermal conduction loss, one should appropriately determine the stack length; in the present case, the length of the stack is a tenth of that of the resonator tube.

Summary

We numerically investigated the critical temperature ratio of the spontaneous gas oscillation occurring in the miniature thermoacoustic engine. It was found that the temperature ratio decreases with decreasing the stack length and that there is the optimum stack length to reduce temperature gradient.

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