Proceedings of 20th International Congress on Acoustics, ICA 2010

23-27 August 2010, Sydney, Australia

Experimental analysis of the radiated noise from a small propeller

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PACS: 43.28, 43.50

ABSTRACT

The acoustic signature of unmanned aerial vehicles (UAVs) is one of the limiting factors facing the expanding use of these platforms for both civil and military uses. The overall propeller noise signature can be reduced by firstly reducing the motor noise and the blade passage noise, which is a result of the propellers rotational speed, diameter and shape. However, once these are optimised only modifications to the propeller self noise will help to further reduce the platforms noise signature. This investigation presents one method that will reduce the propeller self noise through tripping the boundary layer on a small propeller (diameter ~250mm) with a short chord length (15~30mm) with blades operating at low Reynolds numbers.

Laminar separation bubbles commonly occur on propellers of this size as a result of the low Reynolds number conditions existing on blades. Experiments have shown that boundary layer tripping not only reduces that drag of the blade but, when a laminar separation bubble on the suction surface of the propeller blade is eliminated, a noise reduction occurs as well. The reasons for this noise reduction were not initially clear, and so its characteristics were examined experimentally on a rotating propeller in both static and wind tunnel conditions. These experiments have helped to show that a number of aerofoil noise mechanisms are at work simultaneously and do not necessarily occur as the simple turbulent or laminar boundary layer noise models as traditionally believed. Analyse of the spectral peaks using a method developed to approximate the 3D rotating blade as a 2D aerofoil section has exhibited characteristics of laminar boundary layer noise, even with the presence of a laminar separation bubble which would promote boundary layer transition to occur on the blade surface. Comparisons with literature models such as the semi-empirical aerofoil self noise model of Brooks, Pope, et al [1], and the laminar boundary layer noise models of Tam [2], Akishita [3] and Paterson [4] have also shown agreement with laminar boundary layer noise characteristics, but still not clearly explain the role of the laminar separation bubble.

NOMECLATURE

BPF	Blade Passage Frequency, Hz
c	chord, m
I	Acoustic Intensity, W/m^2
L	Scale Length, m
l	length, mm
LAeq	A weighted Equivalent Noise level, dB
M	Mach Number
mAV	mini Aerial Vehicle
p	Acoustic pressure, Pa
r	general or section radius, m
r_e	distance to receiver, m
R	Blade radius, m
r/R	Radial Position/Blade Radius
SPL	Sound Pressure Level, dB
St	Strouhal Number
SWL	Sound Power Level, dB
U	freestream or section velocity, m/s
U_{tunnel}	Wind tunnel freestream velocity, m/s
UAV	Unmanned Aerial Vehicle
v	viscosity, m^2/s
δ	boundary layer thickness, m
δ^*	boundary layer displacement thickness, m
δ^*_s	suction surface boundary layer displacement thickness, m
δ^*_p	pressure surface boundary layer displacement thickness, m

INTRODUCTION

Experiments undertaken from 2006 to 2008 have shown that an overall noise reduction can be produced on a Master Airscrew 10x5 propeller operating in both static and wind tunnel conditions over 3500-8000RPM by using a leading edge boundary layer trip on the suction surface of the blade. The analysis of the recorded spectrums from the various propeller configurations have been conducted to try and determine the noise generating mechanisms on the propeller, and what noise source is being modified through the leading edge boundary layer trip.

The problem of propeller noise and the use of boundary layer tripping to reduce aerofoil self noise is not at all a new problem. The typical explanation for the noise reduction via tripping is the elimination of the laminar boundary layer noise, as a result of the forced transition of the laminar boundary layer to turbulent. The turbulent boundary layer generates noise differently to the laminar boundary layer on an aerofoil. The laminar boundary layer exhibits strong and loud tonal noise which is the result of an aeroacoustic feedback loop formed within the laminar boundary layer as a result of boundary layer oscillations being reinforced by noise radiated from the trailing edge at the same frequency. The forced transition to turbulence breaks down this aeroacoustic feedback loop and replaces it with broadband noise radiated from the trailing edge. This forced transition can be analogous to the modification of the boundary layer of a golf ball as a result of the use of dimples, or the improved performance of an aeroplane wing with vortex generators installed. However, the unique problem found in the experimental setup being examined in this current study was the presence of a laminar separation bubble on the suction surface of the propeller blade. As a result elements of this typical explanation did not fit completely with the experimental findings, and it is this that will be further explored within this paper.

Traditionally the common techniques for analysing the boundary layer and aerofoil self-noise of wind turbines and propeller blades takes place in anechoic wind tunnels and involves examining 2D blades sections. Measurements of full rotating wind turbine blades in operation are used to examine the overall total noise produced by the wind turbine, and usually are a culmination point of extensive 2D wind tunnel testing [5]. However, in the present investigation the noise reduction method was discovered during the measurements of a 3D rotating propeller. It was not initially clear as to what was the underlying cause of the noise reduction. In trying to determine the underlying cause it was concluded that the potentially strong 3D effects occurring on the propeller under investigation would involve too many assumptions to validate any findings before more was known about what the aerodynamic properties of the actual 3D rotating blades. Instead, an insitu examination took place to examine the characteristics of the noise reduction. The following techniques were used in an attempt to combine 2D theoretical and 3D experimental results to examine the aerofoil self-noise sources on the rotating blade. An example of the experimental setup is shown in Figure 1.

A combination of surface visualisation studies and the 3D rotating blade and computational modelling of the boundary layer of the blade using the 2D Panel Method XFOIL [6] were used to determine key boundary layer properties. This approach was used to try and determine whether the source of the noise that was being reduced could be defined as either turbulent boundary layer trailing edge noise (TBL-TE) or laminar boundary layer vortex shedding noise (LBL-VS).

Figure 1. Wind Tunnel Test Rig Setup

METHODOLOGY

Modelling a 3D rotating propeller blade using 2D aerodynamic assumptions

The majority of theoretical models examining turbulent and laminar boundary layer noise are based upon 2D aerofoil sections. To examine the 3D rotating propeller results it was required to formulate and validate a method, if possible, to compare the complete 3D rotating blade noise spectrum against these 2D aerofoil theoretical models.

One method that was used was based upon the aerofoil self-noise model developed in Brooks, Pope et al [1] (BPM). This has been used in a number of studies [5] to assess the boundary layer noise components of 2D aerofoil sections, but was developed with the potential application of being used as a blade element model to assess complete rotating blade noise. Successful comparisons against helicopter rotor noise experiments were included in the appendix of the Brooks, Pope et al study [1], with similar methods used to assess wind turbine noise applied in more recent studies [5].

However, even though the overall noise measured noise spectrum can be compared against the entire BPM prediction, which is made up of individual noise mechanisms, it was difficult to use this model in isolation to validate any conclusions about the underlying dominant noise mechanisms. This meant that other methods other than the BPM method had to be used, and many others required a 2D aerofoil section and had linear models developed around the 2D section parameters such as section velocity, and section trailing edge boundary layer displacement thickness. To use these various models to analyse the propeller noise during operation a number of assumptions had to be made so that the flow over the rotating blade section could be approximated as closely to 2D flow as reasonably possible. For the 2D analysis, a location of 0.8r/R was assumed, as previsouly explained, to be the source of the peak broadband frequency.

One of the key justifications for being able to use XFOIL as an approximation method for these 3D rotational tests was based upon the findings in Lindenburg [7] examining wind turbine aerofoils. In the study it was found that for spanwise locations larger than 80%, no correction for rotation was required. The lack of rotational flow close to the top is due to the large aspect ratio and small chord length of the blade [7]. Findings in this study and in Leslie [8] concluded that the last 25% of the blade is responsible for 75% of the radiated noise. Based upon these findings, it was decided to focus upon 0.8r/R as the key analysis location along the blade as it would resemble the closest to 2D flow, while capturing the characteristics of one of the blade sections producing the most noise. Additionally, this location was chosen as it was still inboard of the tip, and as such would help to minimise the impact of tip effects on the local section aerodynamics and the subsequent noise results.

Using this technique it was possible and appropriate to use XFOIL, and the e^n method, which is based upon 2D boundary layer transition to approximate this 3D rotating problem for analysis purposes. However, it should be noted that the 2D analysis was not used blindly, and that 3D rotational effects such as centrifugal pumping were considered and taken into account when the full analysis was undertaken.

Visualisation results to determine boundary layer profile

In order to examine the noise radiated from a rotating propeller, the aerodynamic properties of the blade profile must firstly be ascertained. The majority of the blade self-noise results from the interaction of the boundary layer of the aerofoil section of the blade with the trailing edge. As it was not possible to completely determine the boundary layer properties experimentally, key experimental results combined with computational modelling were used to provide detailed boundary layer property information.

Boundary layer modelling programs such as XFOIL [6], used in this study, are useful to determine the boundary layer thicknesses analytically when experimental methods such as hotwire measurements are not feasible. This method has been seen commonly across other wind turbine noise studies [5]. However this modelling method assumes a 2D aerofoil section without any 3D span affects, which is not the case for a rotating propeller with a fully 3D boundary layer. The errors that can arise when dealing with 3D rotating blade boundary layers include aspects such as spanwise flow and coriolis forces, which result in effects such as a thinning of the boundary layer compared with a similar 2D aerofoil section [7], and different velocity spanwise wise flows within both the laminar separation bubble on the blades surface and the main attached boundary layer, all of which modify the characteristics of the boundary layer. An alternate method of analysis to account for these 3D affects was proposed for this study, and involved using characteristics of the existing laminar separation being present on the surface of the blade as unique boundary layer variables that could be used as marker points in the boundary layer.

The method employed in this study uses the titanium dioxide/kerosene visualisation method [9, 10], in conjunction with XFOIL [6] as a means of determining the rotating blade boundary layer thickness and characteristics. The method involves finding a unique match of the boundary layer characteristics in XFOIL to the separation and reattachment locations found through the visualisation examination on the experimental propeller. This is done by varying the parameter for inflow turbulence employed in XFOIL, which is the linear instability e^n method [11]. This method relies on the fact that 2D Tollmien-Schlichting waves are the dominant transition-initiating mechanism, and that their growth follows linear stability theory. The majority of studies that have looked at aerofoils at similar Reynolds number ranges (<Re=150000) to that of the present investigation have found this to be the dominant instability mechanism in the laminar boundary layer, such as show in Drela [12]. In many laminar boundary layer studies the T-S waves were found to be the dominant form of laminar boundary layer noise [13] and so, using this method of transition prediction, as implemented in XFOIL, appeared to be a appropriate model to use.

Two unique locations of separation and reattachment could be determined through a visualisation technique using titanium dioxide and kerosene. To define two clear marker points this method also relied on the separation and reattachment locations to have two unique clear locations chordwise along the blade (Figure 2.). The principle of the technique is that for similar unqiue locations to occur in the boundary layer modelling within XFOIL similar boundary layer growth to what actually exists must occur over the blade. It is from this unique match that the boundary layer thickness and other aerofoil boundary layer paramenters to use in the noise preductions of aerofoil self-noise modelling are taken.

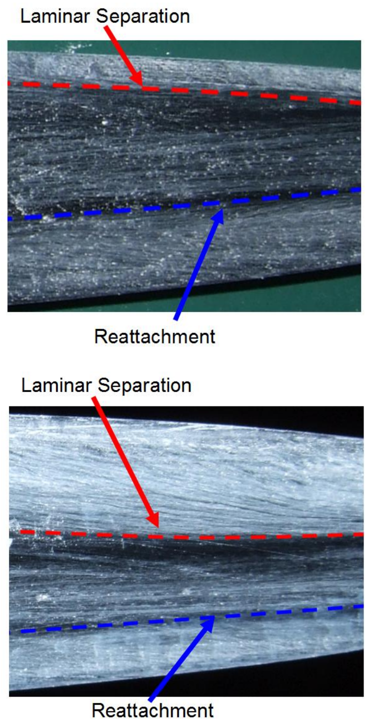

Figure 2. Example of visualisation results using titanium dioxide and kerosene. Static testing (top) and Wind Tunnel (bottom) testing, showing the laminar separation and reattachment locations.

Matching the observed separation and reattachment locations with the XFOIL predicted locations was the starting point for determining the turbulence intensity meeting the leading edge of the propeller blade (Figure 2.). Using this analysis method it was concluded for this initial set of measurements for the Master Airscrew 10x5 in the NAL Silent Airflow tunnel that the e^n value for static testing was close to n~10 for various RPM values, while for the U_{tunnel}=10m/s case, it was closer to n~7**.** As the inflow varied with tunnel speed and RPM increase, it was therefore determined that for a first approximation an average of e^n value of n=9 would be used for the XFOIL prediction models to determine characteristics such as trailing edge boundary layer displacement and momentum thicknesses. The highest turbulence level predicted through the XFOIL e^n method is a turbulent intensity of 2.982% [12]**,** which is much lower that the measured free stream turbulence intensity measured using a hotwire. However, the compounding factors of rotational blade flow increasing boundary layer stability, and a decrease of turbulence intensity in the propeller stream tube, means that the turbulence approximation from XFOIL is applied as a tool to model the boundary layer transition characteristics and not necessarily quantify the free stream turbulence impinging onto the blade.

Conversion of rotating blade spectrum data into 2D aerofoil noise Sound Pressure Levels

As previously outlined, 0.8r/R was the focus radial position on the blade for the study, and had been experimentally found to radiate the loudest noise aerofoil self noise [8]. Analysing the loudest point in the spectrum, was used as starting point to examine the aerofoil self noise, to see if the full propeller spectrum could be used to make any conclusions about the source of the blade self-noise.

To determine the peak level of the broadband frequency hump found in the experimental results of the base blade, the Blade Passage Frequency (BPF) peaks had to firstly be eliminated from the data to be analysed. To do this, a local minimum function was firstly applied to the data over the range of 3000-22000Hz, as the aerofoil self noise would be the lower limit of the total propeller blade noise, and is the dominant noise source for this frequency range. This range was for the majority of cases well above the main BPF and its harmonic peaks which also had to be avoided in the analysis, as shown in Figure 3. The results of this new function were then smoothed and 4th order polynomial was fitted to the data. The use of a 4th order polynomial was chosen after examining the polynomial fits from 2-10 for accurately locating the broadband hump peak over a large selection of experimental samples. The major assumption was then that the peak value for the fitted polynomial was taken as the SPL for the 0.8r/R section of blade, allowing all associated boundary layer properties at this location to then be used in the analysis along with the now associated SPL value. The peak values and frequencies were then able to be analysed and compared against current 2D aerofoil self-noise models. An example of this technique in full is shown in Figure 3. Upon an examination of the entire range of experimental data, it was found that a peak location could only be clearly identified for RPM values of 6000RPM and less, as above this RPM value there was no clear broadband hump in the mid-high range of the frequency spectrum.

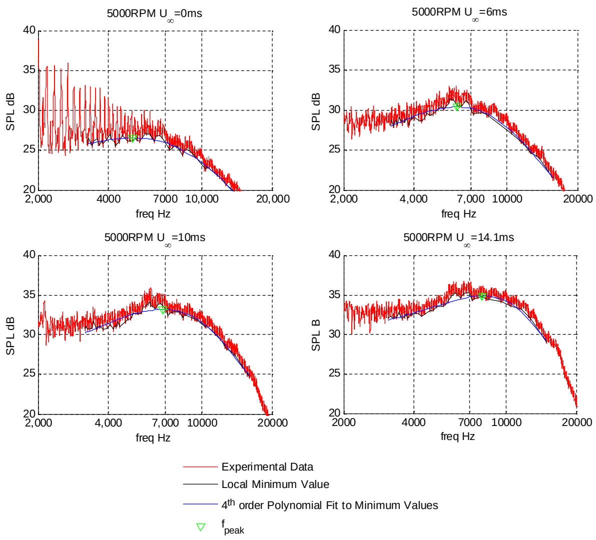

Figure 3. Estimation technique of peak frequency location in propeller blade spectrum

ANALYSIS

The analysis presented in this paper will initially briefly examine the general trends in the raw measurement data, followed by an examination of the trends as a result of the conversion of 3D propeller data into 2D aerofoil section data using the previously outlined technique.

Strouhal Scaling of Raw Measurements

Strouhal scaling allows for the comparison of a variety of tests at different operating conditions (angle of attack, airspeed). The Strouhal scaling method used is shown in Equations 1. and 2. as proposed by Brooks, Pope et al [1]. The scaling parameter used was originally for TBL-TE noise at non-zero angles of attack used in Brooks, Pope et al [1] and was based upon the trailing displacement thickness of the pressure surface (δ^*_p). For this study it has been extended to look at both blade surfaces, to try and determine the major noise contributor, and if the boundary layer tripping changes the relationship of the trailing edge displacement thickness to the noise generated by either the suction or pressure surfaces of the blade.

$$SPL_{1/3} = SPL_{1/3} - 10\log\left(M^5 \frac{\delta_s^* L}{r_e^2}\right) \qquad (1)[1]$$

For the level scaling,

$$St = \frac{f\delta_s^*}{U} \qquad (2)[1]$$

For the Strouhal frequency scaling.

The results of different scaling methods based upon both the pressure and suction sides scaling are shown in Figure 4. It should be noted that the results shown in Figure 4. are based upon the adjustment of the wind tunnel speed so that the resultant angle of attack on the blade for each RPM is approximately the same at the 0.8r/R location.

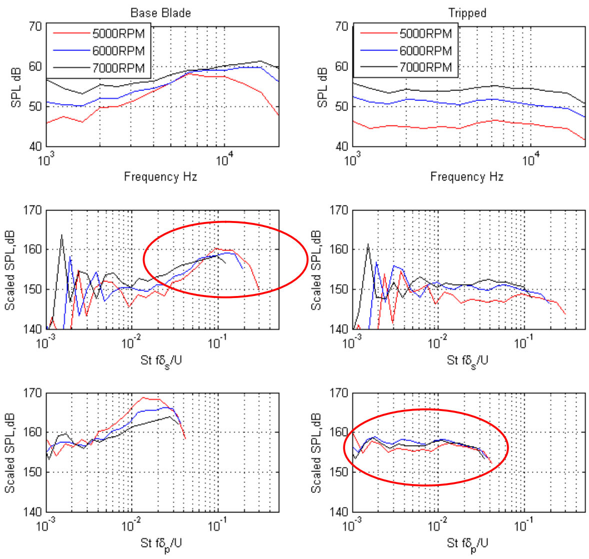

Figure 4. Experimental Blade Noise Results with Scaled SPL and Strouhal Frequency Scaling using Brooks, Pope Marcolini Method [1].

The scaling is based upon the local section velocity and boundary layer displacement thickness as predicted using XFOIL. The key features to note in Figure 4., circled in red, are the collapse of the peak values following the scaling. The results collapse well for the base blade when they are scaled based upon the suction surface boundary layer displacement thickness, while following the tripping of the boundary layer the results then are much more closely related to the pressure surface trailing edge displacement thickness. These results show that the suction surface dominates the source of aerofoil self noise for the unmodified based blade, while following tripping of the suction surface boundary layer this major source aerofoil self noise shifts so that the pressure surface becomes the dominant contributor.

Examination of the laminar boundary layer tonal noise peak frequencies

The conversion of the 3D experimental results into approximately equivalent 2D aerofoil section results allows for an analysis of both the frequency and SPL content of the propeller noise.

The examination of the raw results showed that the suction surface is the dominant noise source for the untripped blade configuration. As a result of this it was important to then focus upon the suction surface boundary layer properties when trying to understand the source of the broadband hump which was eliminated from the measured propeller spectrum through tripping.

One of the key identifiers of laminar boundary layer noise on aerofoil sections is that the tonal noise that is radiated which follows a trend of $f \approx U^{0.8}$. However, the nature of this tonal noise relationship is not a constant increase, but instead one which then jumps up in a 'ladder' like fashion as the section velocity increases, with an overall trend following $f \approx U^{1.5}$, while the individual ladder rungs themselves increase by $f \approx U^{0.8}$, as shown in Figure 5.

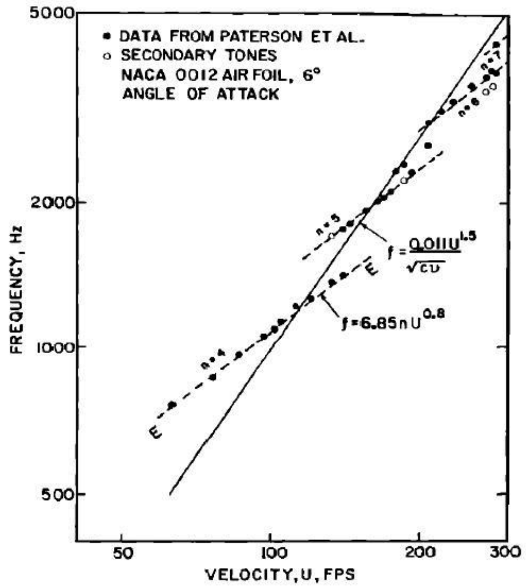

Figure 5. Laminar Boundary Layer Vortex Shedding Tones, NACA 0012, AoA=6 degrees [2]

Once the equivalent 2D aerofoil section results were determined for the 0.8r/R sections, the results were analysed as if this was a series of single 2D aerofoils. Figure 6. shows the analysis of the experimental f_{peak} values compared with the prediction models, similar to those shown in Figure 5., for both laminar and turbulent boundary layer noise. The aim was to determine the relationship between the propeller blade section speed, and the peak frequency. This helped to point towards whether either laminar or turbulent noise being the dominant noise mechanisms for the broadband hump. The models used for laminar, and turbulent boundary layer noise prediction, as shown in Figure 6., are similar to those shown in Figure 5.

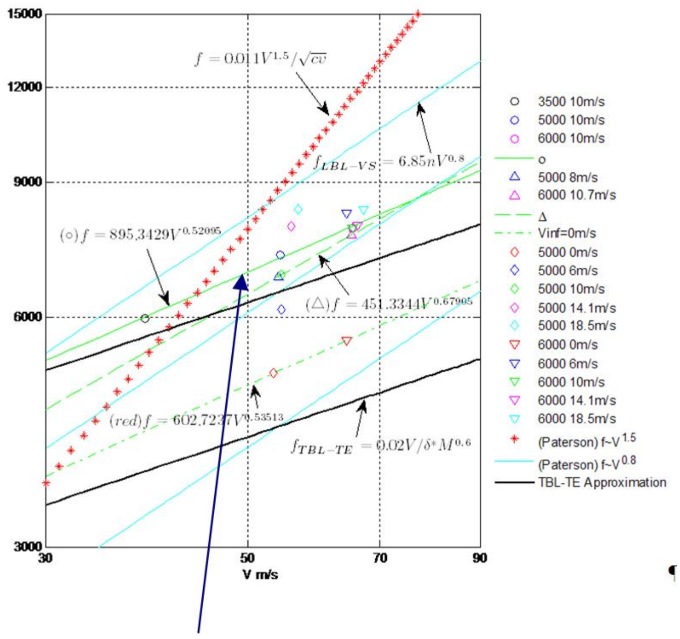

Figure 6. Comparison of experimental broadband 'hump' peak frequencies against predicted peak frequencies for both TBL-TE and LBL-VS noise sources. (Legend: RPM, U_{tunnel})

By examining Figure 6. the correlation between the peak locations at different RPM values with similar angles of attack (shown by the △ symbol) an increase which fits very closely to that of the theoretical increase for LBL-VS noise is found, with a trend of $f \sim U^{0.68}$. Similar trends can also be seen as the RPM value is varied and the inflow U_∞ is kept the same (shown by the ○ symbol) with a trend of $f \sim U^{0.52}$. Taking into consideration the possibility of errors, or discrepancies as a result of the f_{peak} calculation assumptions, or as a result of the assumptions used to approximate a section of the 3D rotating blade as a 2D section, the results fit remarkably closely to the literature trend for laminar boundary layer noise. While the overall trend with velocity (shown by the ✸ symbol) shows that the results fit in a similar fashion to the $f \sim U^{1.5}$ overall trend shown in literature, such as shown in Figure 5. Even with the large assumptions made to convert the 3D blade into an approximately equivalent 2D section, the result help to provide strong evidence for both the laminar boundary layer noise hypothesis and the evaluation method itself. In contrast it is also noticeable that the f_{peak} frequencies increase at a rate in excess of what would occur for TBL-TE noise. The 2D analysis of the broadband 'hump' peak frequencies shown in Figure 6. show trends that suggest a laminar boundary layer noise source mechanism, rather than a turbulent boundary layer one.

Examination of the laminar boundary layer tonal noise SPL

There is still presently uncertainty about the true relationship of relationship of LBL-VS SWL to section velocity. Variations in the finding within literature mean that general guidance is all that can be taken from these studies [2-4]. Paterson et al [4] presented that the acoustic intensity of the 'vortex shedding' noise of an isolated aerofoil at low Reynolds numbers increased with $I \alpha U^{5.6}$, while also stating that other studies had found U^6 (the typical relationship for a dipole source). However as shown in Tam [2] the SPL would increase in the typical U^6 (dipole source) trend for low velocities (<25m/s), but would reach a level at which the SPL would no longer increase (Figure 7.), reaching a maximum level and plateauing, straying away from any linear trend. This is in contrast to a turbulent boundary layer trailing edge noise where the velocity dependence is much more consistent for an aerofoil in flow, which scales according to $p^2 \approx U^5$ [14]. A more recent study by Akishita [3], it was found that the tones radiated from a NACA0012, with a laminar boundary layer, followed a power law for the overall sound pressure was 2.3~3.3, in comparison to the much higher U^6.

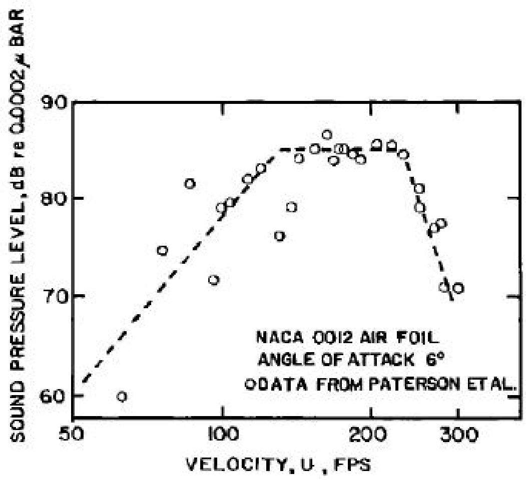

Figure 7. Laminar Boundary Layer Vortex Shedding SPL Saturation [2]

A comparison of the experimental data, normalised by the trailing edge boundary layer thickness, against both of these trends is shown in Figure 8. Normalising the results by δ* attempts to make the SPL values independent of angle of attack, and so therefore independent of inflow speed making the results directly comparable. Using the approximations for the equivalent 2D section values for f_{peak} it is a key finding to see that such a close relationship between the experimental trend of laminar boundary layer noise found in Akishita [3] and the results in the current study. The approximation for TBL-TE noise is also shown. It can be seen that with the overall trend of the increasing RPM values the trend is a much closer fit to the prediction model for laminar boundary layer noise, as found in of Akishita [3], than turbulent boundary layer noise trend of U^5.

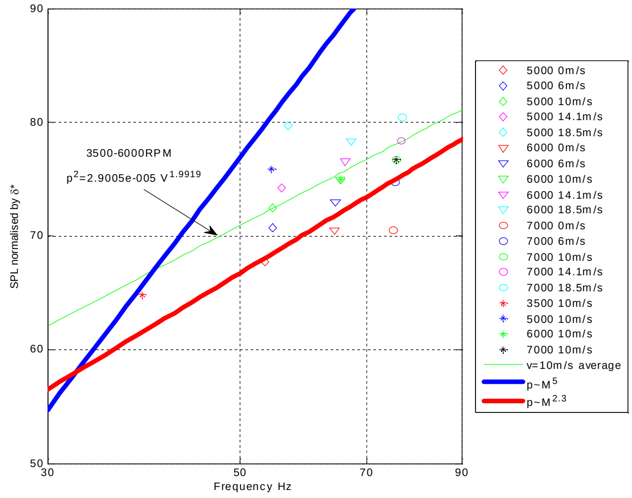

Figure 8. Peak Frequency SPL normalised by δ* velocity dependence for the untripped blade

CONCLUSION

The experimental findings of this investigation can initially be interperted as another good example of the acoustic benefits of transitioning a laminar boundary layer to turbulent so as to eliminate LBL-VS noise, removing the aeroacoustic feedback loop, which is the strong source of tonal noise. However, the key difference found in this investigation and the question that it raises is around the role of a laminar separation bubble on the suction surface of the blade in aerofoil self-noise.

The experimental results have shown the suction surface to be the major noise source in the untripped configuration. With the presence of a laminar separation bubble on the suction surface it could be assumed that the boundary layer should have already transitioned to turbulent by the trailing edge. Transition is one of the main reasons for boundary layer reattachement takes place, and helps to form the laminar separation bubble. As a result, the transition should have destroyed the aeroacoustic feedback loop and removed the resulting tonal LBL-VS tonal noise. However, the experimental findings raise the interesting point that the main noise source still appears to be LBL-VS noise, and is being radiated from the surface with the laminar separation bubble. This suggests the elimination of the LBL-VS has not occurred.

What is required now is to go back to basics. This investigation has provided strong evidence that LBL-VS noise is still present on the suction surface which has a laminar serparation bubble.

Further knowledge about the role of the laminar separartion bubble in aerofoil self-noise has to be developed separate to the current investigation. Further experimental examination of a similar 2D aerofoil with a laminar separation bubble present on the suction surface is required away from this experiment so as to shed light on the role of the laminar separation bubble in the aerofoil self noise of the blade. It is only when this further understanding is clearly developed that the current problem presented in this paper can be readdressed.

The current problem requires the combination of both the 3D blade self noise knowledge developed in this study, combined with an improved understanding of the role of the laminar separation bubble in the generation of aerofoil self-noise to be properly explained. It is only following this combined knowledge that it can be assess whether or not the laminar separation bubble actually plays a role in the generation of aerofoil self noise in the present problem. This study has raised an unexpliained issue of aerofoil self-noise, which could provide a good basis for acoustic analysis and problem solving for many of the low Reynolds number noise producing devices currently in development and use.

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