Proceedi nternational Congress on Acoustics, ICA 2010

ICA 2010

23-27 August 2010, Sydney, Australia

On conditions of equivalence between Curle's and non-uniform Kirchhoff equations of aeroacoustics

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PACS: 43.28.RA, 43.50.NM

ABSTRACT

The derivation of Curle's equation for the sound radiated by a flow near a rigid surface is reconsidered. It is shown that this equation and the non-uniform Kirchhoff equation previously derived by the author are equivalent if the sum of two integrals containing Lighthill's stress tensor over the rigid surface is zero. These two integrals are equivalent to the acoustic field radiated by sources determined by Lighthill's stress tensor and its spatial derivatives on the boundary. This leads to an immediate result that the two equations are equivalent if Lighthill's stress tensor vanishes altogether, for instance, for linear acoustical waves in an ideal fluid. The obtained criterion is formulated for a flow near an infinite rigid plane in a fluid. Two cases are considered: a weakly non-linear flow (with low Mach number) in an ideal fluid and a linear flow in a viscous fluid. It is shown that, in a weakly non-linear flow, the equations are equivalent if the plane is stationary. If the plane is vibrating as well as for a viscous fluid, the sum of the two integrals is, in general, non-zero, but a more detailed investigation is required for a definite conclusion. Some directions of future investigations of the equivalence of the two equations are suggested.

INTRODUCTION

The development and widespread use of jet aircraft in the 1950s caused significant interest in the prediction of sound generated by a fluid flow. The first significant contribution to this topic was made by Sir James Lighthill (1952). He showed that the sound radiated by a turbulent flow without boundaries was controlled by the wave equation with the source term determined by the Lighthill's stress tensor, which represents all non-acoustic stresses in the fluid. Lighthill also showed that the source term corresponds to quadrupole sound.

Curle (1955) extended Lighthill's theory to a flow with solid boundaries. Curle stated that, for an immoveable boundary, the radiated sound consisted of Lighthill's quadrupole sound as well as the dipole sound originating at the rigid boundary. The amplitude of the dipole sound was determined by the total force acting upon the fluid from the boundary including the viscous tangential force.

Ffowcs Williams and Hawkings (1969) extended Curle's theory to a flow with moving boundaries. They showed that the motion of the boundaries led to the appearance of a third term in the equation for the radiated sound amplitude. This term describes the monopole sound and is determined by the normal velocity of the boundary with respect to a stationary observer. For a stationary boundary, Ffowcs Williams and Hawkings (FW-H) equation is reduced to Curle's equation.

Since its derivation, the Ffowcs Williams and Hawkings equation has become the foundation for one of the most frequently used methods of prediction of sound radiated by fluid flow near rigid surfaces. A brief list of applications where the FW-H equation is utilised includes rotating helicopter blades, rotating fans, and flow near an airfoil. This equation is also used in the prediction of noise radiated by moving ships and ship propellers. (See Zinoviev (2007) for a list of references).

There were many attempts to verify Curle – Ffowcs Williams and Hawkings theory. However, from the point of view of the author, the results of these experiments are inconclusive, as many of them showed a discrepancy of a few decibels between theoretical predictions and experimental results. For example, early results by Clark and Ribner (1969) and by Heller and Widnall (1969) showed discrepancy of up to 5 dB. Bies (1992) investigated the noise produced by a circular saw and reported that the measured noise was 2.5 dB lower than predicted. More recent experiments (Eschricht et al 2007, Greschner et al 2007) still continue to show a discrepancy of a few decibels between the predictions of FW-H theory and experimental data.

In 2001, the author presented a result of his theoretical reconsideration of Curle's derivation (Zinoviev, 2001), where he pointed out that the divergence theorem in the derivation should have been used differently. Later Zinoviev and Bies (2004) have conducted a more detailed critical analysis of Curle's derivation. The authors have shown that, if the divergence theorem is used correctly, Curle's derivation leads not to Curle's equation, but to a different equation. This equation is called the non-uniform Kirchhoff equation in this paper, as it includes Lighthill's quadrupole sources from his non-uniform wave equation as well as surface distributions of dipole and monopole sources as described by Kirchhoff integrals (Stratton 1941). This equation differs from the FW-H equation by the appearance of the terms describing the sources at rigid boundaries.

The claim by Zinoviev and Bies that the FW-H equation is derived with an error and the non-uniform Kirchhoff equation should be used instead caused objections from some members of the aeroacoustical community (Farassat 2005, Farassat & Myers 2006), to which the authors responded (Zinoviev & Bies 2005, Zinoviev & Bies 2006). The discussion demonstrated that the FW-H equation produced results identical to those by the non-uniform Kirchhoff equation at least in some situations, for example, in acoustic scattering by solid objects.

The two equations describe the same physical process of sound generation by a fluid flow near a solid boundary. This fact, together with the identical predictions for acoustic scattering problems, raises a question of whether the two equations are identical and, in fact, are different forms of the same equation. The other possibility is that the equations are different and, therefore, at least one of them is not correct. If the former statement is true, it would reconcile the present author's conclusion about the derivation of Curle's equation with the raised objections. If the latter is true, that, in view of the present author, would still leave the question of correctness of both equations open.

This paper is devoted to the question of whether Curle's and the non-uniform Kirchhoff equations are indistinguishable. In the first section, both equations are shown and the condition of them being equivalent is formulated theoretically. In the second section, this condition is evaluated for acoustical problems and for the problem of vibrations of an infinite plane in a fluid with different flow conditions. The third section of the paper is devoted to examples, which, in view of the present author, may demonstrate the equivalence or otherwise of the two equations.

CRITERION OF THE EQUIVALENCE OF CURLE'S EQUATION AND THE NON-UNIFORM KIRCHHOFF EQUATION

Curle's equation

Using the fundamental laws of mass and momentum conservation for the motion of a fluid, Lighthill (1952) showed that sound generation and propagation in a turbulent fluid flow without boundaries was determined by the following wave equation with respect to the fluid density, ρ:

$$\frac{\partial^2 \rho}{\partial t^2} - c_0^2 \nabla^2 \rho(\mathbf{x},t) = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \quad i, j = 1,2,3. \qquad (1)$$

In Eq. (1), c_0 is the sound speed and T_{ij} is Lighthill's stress tensor given by:

$$T_{ij} = \rho v_i v_j + p_{ij} - c_0^2 \rho \delta_{ij}, \qquad (2)$$

where $\mathbf{v}$ is the fluid velocity vector, δ_{ij} is Kronecker's delta, and p_{ij} is the compressive stress tensor determined as follows:

$$p_{ij} = p\delta_{ij} + \mu \left\{ -\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i} + \frac{2}{3}\left(\frac{\partial v_k}{\partial x_k} \right) \delta_{ij} \right\}, \quad k = 1,2,3. \qquad (3)$$

where p is the pressure and μ is the viscosity of the fluid.

Curle (1955) starts his derivation with a formulation of a general solution of Eq. (1) near solid boundaries. As Eq. (1) is a non-uniform wave equation, its solution can be formulated as a sum of a term describing the sound due to the volume distribution of sound sources and a term containing Kirchhoff integrals over the boundaries. This solution can be written as follows:

$$4\pi\rho'(\mathbf{x},t) = \frac{1}{c_0^2}\int_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \right] \frac{d\mathbf{y}}{r} + \int_S \left\{ \frac{1}{r}\left[\frac{\partial \rho}{\partial n} \right] + \frac{1}{r^2}\frac{\partial r}{\partial n}[\rho] + \frac{1}{c_0 r}\frac{\partial r}{\partial n}\left[\frac{\partial \rho}{\partial t} \right] \right\} dS(\mathbf{y}). \qquad (4)$$

In Eq. (4), $\rho' = \rho - \rho_0$, ρ_0 is the value at equilibrium, V is the total fluid volume, S is the surface of solid boundaries, $r = |\mathbf{x} - \mathbf{y}|$, $\mathbf{x} = (x_1, x_2, x_3)$ is the radius-vector of the observation point, $\mathbf{y} = (y_1, y_2, y_3)$ is the radius-vector of the source point, $\mathbf{n} = (l_1, l_2, l_3,)$ is the outward normal from the fluid, and square brackets denote dependence on retarded time, $t - r/c_0$.

After using the divergence theorem twice, Curle obtained the following equation determining the interchange of derivatives in the volume integral of Lighthill's stress tensor:

$$\int_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \right] \frac{d\mathbf{y}}{r} = \frac{\partial^2}{\partial x_i \partial x_j}\int_S \frac{\left[T_{ij} \right]}{r} d\mathbf{y} + \frac{\partial}{\partial x_i}\int_S l_j \left[T_{ij} \right] \frac{dS(\mathbf{y})}{r} + \int_S l_i \left[\frac{\partial T_{ij}}{\partial y_j} \right] \frac{dS(\mathbf{y})}{r} \quad . \qquad (5)$$

Note that the surface integrals are taken over the surface which is in immediate contact with the fluid.

The momentum conservation equation for the fluid takes the form of

$$l_i \frac{\partial}{\partial y_j}\left(\rho v_i v_j + p_{ij} \right) = -l_i \frac{\partial}{\partial t}\left(\rho v_i \right). \qquad (6)$$

By substituting Eqs. (5) and (6) into Eq. (4), Curle obtained the following equations for the density fluctuations due to a flow near solid boundaries:

$$4\pi c_0^2 \rho'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j}\int_V \frac{\left[T_{ij} \right]}{r} d\mathbf{y} + \frac{\partial}{\partial x_i}\int_S l_j \frac{1}{r}\left[\rho v_i v_j + p_{ij} \right] dS(\mathbf{y}) - \frac{\partial}{\partial t}\int_S l_i \frac{1}{r}\left[\rho v_i \right] dS(\mathbf{y}). \qquad (7)$$

Denoting the total force per unit area acting upon the fluid from the boundary, $P_i = \rho v_i v_j + p_{ij}$, one can rewrite Eq. (7) as follows:

$$4\pi c_0^2 \rho'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j}\int_V \frac{\left[T_{ij} \right]}{r} d\mathbf{y} + \frac{\partial}{\partial x_i}\int_S \frac{1}{r}\left[P_i \right] dS(\mathbf{y}) - \frac{\partial}{\partial t}\int_S l_i \frac{1}{r}\left[\rho v_i \right] dS(\mathbf{y}). \qquad (8)$$

By assuming that the boundary has zero normal velocity, Eq. (8) can be reduced to the original Curle's equation (Curle 1955):

$$4\pi c_0^2 \rho'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i \partial x_j} \int_V \frac{[T_{ij}]}{r} d\mathbf{y} + \frac{\partial}{\partial x_i} \int_S \frac{1}{r} [P_i] dS(\mathbf{y}). \quad (9)$$

The first term in the right-hand part of Eq. (9) determines Lighthill's quadrupole sound generated in the fluid volume, whereas the second term determines the dipole sound generated at the rigid surface due to force acting between the surface and the fluid.

In the analysis below, as Eq. (8) has also been obtained by Curle just before substituting zero boundary velocity, Eq. (8) is referred to as Curle's equation.

The non-uniform Kirchhoff equation

Zinoviev and Bies (2004) conducted a critical analysis of Curle's derivation. In the view of these authors, the divergence theorem in the derivation should be used in a way different from that utilised by Curle. The authors showed that, as a result, the interchange of derivatives in the integrals should be described not by Eq. (5), but by the following equation:

$$\int_V \left[\frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} \right] \frac{d\mathbf{y}}{r} = \frac{\partial^2}{\partial x_i \partial x_j} \int_S \frac{[T_{ij}]}{r} d\mathbf{y}. \quad (10)$$

Substitution of Eq. (10) into the general solution of Lighthill's equation (Eq. (4)) leads to the following equation for the acoustic wave amplitude radiated by the flow near solid boundaries:

$$\begin{aligned} 4\pi\rho'(\mathbf{x},t) &= \frac{1}{c_0^2} \frac{\partial^2}{\partial x_i \partial x_j} \int_V [T_{ij}] \frac{d\mathbf{y}}{r} + \\ &\int_S \left\{ \frac{1}{r}\left[\frac{\partial \rho'}{\partial n}\right] + \frac{1}{r^2}\frac{\partial r}{\partial n}[\rho'] + \frac{1}{c_0 r}\frac{\partial r}{\partial n}\left[\frac{\partial \rho'}{\partial t}\right] \right\} dS(\mathbf{y}). \end{aligned} \quad (11)$$

After regrouping its terms and using

$$\frac{\partial}{\partial n}\left\{ \frac{1}{r}\rho'(t - r/c_0) \right\} = -\left\{ \frac{1}{r^2}[\rho'] + \frac{1}{c_0 r}\left[\frac{\partial \rho'}{\partial t}\right] \right\} \frac{\partial r}{\partial n}, \quad (12)$$

Eq. (11) can be re-written as follows:

$$\begin{aligned} 4\pi c_0^2 \rho'(\mathbf{x},t) &= \frac{\partial^2}{\partial x_i \partial x_j} \int_V [T_{ij}] \frac{d\mathbf{y}}{r} - \\ &c_0^2 \int_S \frac{\partial}{\partial n} \frac{[\rho']}{r} dS(\mathbf{y}) + c_0^2 \int_S \frac{1}{r}\left[\frac{\partial \rho'}{\partial n}\right] dS(\mathbf{y}). \end{aligned} \quad (13)$$

As Eq. (13) is a general solution of the non-uniform wave equation (1) and it contains Kirchhoff integrals, this equation is referred to in this analysis as the non-uniform Kirchhoff equation.

Difference between non-uniform Kirchhoff and Curle's equations.

It can be clearly seen that, in Eqs. (8) and (13), the first term in the right-hand parts is identical. This term determines Lighthill's quadrupole sound generated in the fluid volume. The second and third terms in both equations determine sound radiated by layers of dipoles and monopoles on the rigid boundary respectively. These terms in the two equations are different not only in their appearance, but also in their physical meaning as the strength of the monopole and dipole acoustic sources is determined differently in both equations. In Eq. (8), the strength of the dipole sources depends on the total force per unit area acting on the boundary, and the strength of the monopole sources is described by the total normal velocity of the boundary. On the contrary, the dipole term in Eq. (13) is determined by density fluctuations on the boundary, which are proportional to acoustic pressure fluctuations. The monopole term in Eq. (13) is determined by the normal derivative of the density fluctuations, which is proportional to the acoustic (potential) velocity. Therefore, whereas the source terms in Eq. (8) contain the full force and the full fluid velocity on the boundary, the terms in Eq. (13) contain only potential components of the force and velocity.

The criterion of equivalence of both equations

The only difference between the derivations by Curle (1955) and Zinoviev & Bies (2004) is in the appearance of the formula for the interchange of derivatives in the volume integrals. Whereas Curle used this formula in the form of Eq. (5), the latter authors utilised Eq. (10) for this purpose. While the differences in the derivation of Eqs. (5) and (10) are outside the scope of this paper, it is clear that the equations are equivalent if the sum of the second and third terms in the right-hand part of Eq. (5) is zero, i.e. if the following condition is satisfied:

$$\frac{\partial}{\partial x_i} \int_S l_j [T_{ij}] \frac{dS(\mathbf{y})}{r} + \int_S l_i \left[\frac{\partial T_{ij}}{\partial y_j}\right] \frac{dS(\mathbf{y})}{r} = 0. \quad (14)$$

It can be seen from Eq. (14) that its two terms represent the dipole and monopole sound generated on the rigid boundary due to the force determined by Lighthill's stress tensor and its spatial derivatives.

If the condition described by Eq. (14) is true, then the derivations by Curle and Zinoviev & Bies lead to the same equation, which can be represented in the form of either Eq. (8) or Eq. (13). This statement represents the main contribution of this work.

CRITERION OF EQUIVALENCE FOR LINEAR ACOUSTIC WAVES IN AN INVISCID FLUID

It is clearly seen that the condition determined by Eq. (14) is true if Lighthill's stress tensor, T_{ij}, vanishes altogether. For example, this condition is satisfied for generation, propagation and scattering of acoustic waves of small amplitude if the fluid viscosity can be ignored. Indeed, in this case the non-linear term $\rho u_i u_j$ in Lighthill's stress tensor (Eq. (2)) as well as the viscous term in the compressive stress tensor (Eq. (3)) can be neglected. Also, for linear acoustic waves, $p = \rho c_0^2$, and, as a result, T_{ij} = 0. This leads to the conclusion that Eqs. (8) and (13) are equivalent for linear acoustic waves in an inviscid fluid.

CRITERION OF EQUIVALENCE FOR A FLOW NEAR AN INFINITE RIGID PLANE

Formulation of the criterion

Consider a general fluid flow near an infinite rigid plane parallel to the plane (y_2, y_3). The y_1-axis is normal to the plane. In this case, the terms of Eq. (14) can be written as follows:

$$\frac{\partial}{\partial x_i}\int_S l_j\left[T_{ij}\right]\frac{dS(\mathbf{y})}{r} = \frac{\partial}{\partial x_1}\int_S\left[T_{11}\right]\frac{dS(\mathbf{y})}{r} + \frac{\partial}{\partial x_2}\int_S\left[T_{21}\right]\frac{dS(\mathbf{y})}{r} + \frac{\partial}{\partial x_3}\int_S\left[T_{31}\right]\frac{dS(\mathbf{y})}{r}. \tag{15}$$

$$\int_S l_i\left[\frac{\partial T_{ij}}{\partial y_j}\right]\frac{dS(\mathbf{y})}{r} = \int_S\left(\left[\frac{\partial T_{11}}{\partial y_1}\right]+\left[\frac{\partial T_{12}}{\partial y_2}\right]+\left[\frac{\partial T_{13}}{\partial y_3}\right]\right)\frac{dS(\mathbf{y})}{r}. \tag{16}$$

A slightly non-linear inviscid flow near a stationary plane

Lighthill (1952) showed that, if viscous stresses can be ignored and Mach number is low, the equation for the tensor T_{ij} can be reduced to the following equation:

$$T_{ij} \approx \rho_0 v_i v_j. \tag{17}$$

If the plane is assumed to be stationary, then $v_1 = 0, \quad T_{i1} = 0$, and

$$\frac{\partial}{\partial x_i}\int_S l_j\left[T_{ij}\right]\frac{dS(\mathbf{y})}{r} = 0. \tag{18}$$

It can be easily proven that the derivatives of Lighthill's stress tensor vanish:

$$\frac{\partial T_{11}}{\partial y_1} = 2\rho_0 v_1\frac{\partial v_1}{\partial y_1} = 0, \tag{19}$$

$$\frac{\partial T_{12}}{\partial y_2} = \rho_0\left(v_1\frac{\partial v_2}{\partial y_2}+\frac{\partial v_1}{\partial y_2}v_2\right) = 0, \tag{20}$$

$$\frac{\partial T_{13}}{\partial y_3} = \rho_0\left(v_1\frac{\partial v_3}{\partial y_3}+\frac{\partial v_1}{\partial y_3}v_3\right) = 0. \tag{21}$$

It follows from Eqs. (19) – (21) that the second integral in Eq. (14) is also zero:

$$\int_S l_i\left[\frac{\partial T_{ij}}{\partial y_j}\right]\frac{dS(\mathbf{y})}{r} = 0. \tag{22}$$

As a result, it can be concluded that both the non-uniform Kirchhoff equation and Curle's equation are equivalent for a slightly non-linear inviscid flow near a stationary rigid infinite plane.

A slightly non-linear inviscid flow near a vibrating plane

Assume now that the rigid plane vibrates while remaining parallel to the (x_2, x_3) plane. In this case, the components of Lighthill's stress tensor (Eq. (17)) are, in general, non-zero. As the vertical fluid velocity v_1 on the rigid plane is equal to the velocity of the plane, its derivatives over horizontal coordinates y_2 and y_3 vanish. As a result, the spatial derivatives take the following form:

$$\frac{\partial T_{11}}{\partial y_1} = 2\rho_0 v_1\frac{\partial v_1}{\partial y_1}, \tag{23}$$

$$\frac{\partial T_{12}}{\partial y_2} = \rho_0\left(v_1\frac{\partial v_2}{\partial y_2}+\frac{\partial v_1}{\partial y_2}v_2\right) = \rho_0 v_1\frac{\partial v_2}{\partial y_2}, \tag{24}$$

$$\frac{\partial T_{13}}{\partial y_3} = \rho_0\left(v_1\frac{\partial v_3}{\partial y_3}+\frac{\partial v_1}{\partial y_3}v_3\right) = \rho_0 v_1\frac{\partial v_3}{\partial y_3}. \tag{25}$$

The spatial derivatives determined by Eqs. (23) - (25) are also, in general, non-zero. However, this cannot be the basis for the conclusion that the criterion of the equivalence (Eq. (14) is not satisfied in the case under consideration, as the two terms may still vanish after integration or cancel each other. A more detailed investigation of this issue is outside the scope of this work and, therefore, can be a subject of future research. A consideration of particular situations where an exact solution for the fluid flow is known can be of special value. For example, vibrations of the plane with no external flow can be one such situation.

A linear viscous flow near a rigid plane

Assume now that the flow near the plane is linear (i.e. the term $\rho_0 v_i v_j$ can be neglected). Assume also that the fluid viscosity is non-zero. In this case Lighthill's stress tensor takes the following form:

$$T_{ij} = \mu\left\{-\frac{\partial v_i}{\partial x_j}-\frac{\partial v_j}{\partial x_i}+\frac{2}{3}\left(\frac{\partial v_k}{\partial x_k}\right)\delta_{ij}\right\}. \tag{26}$$

Taking into account that the fluid velocity vector on the plane equals that of the plane, the components of T_{ij} and its spatial derivatives on the plane can be written as follows:

$$T_{11} = \mu\left\{-2\frac{\partial v_1}{\partial y_1}+\frac{2}{3}\left(\frac{\partial v_1}{\partial y_1}+\frac{\partial v_2}{\partial y_2}+\frac{\partial v_3}{\partial y_3}\right)\right\} = -\frac{4}{3}\mu\frac{\partial v_1}{\partial y_1}, \tag{27}$$

$$T_{21} = \mu\left(-\frac{\partial v_2}{\partial y_1}-\frac{\partial v_1}{\partial y_2}\right) = -\mu\frac{\partial v_2}{\partial y_1}, \tag{28}$$

$$T_{31} = \mu\left(-\frac{\partial v_3}{\partial y_1}-\frac{\partial v_1}{\partial y_3}\right) = -\mu\frac{\partial v_3}{\partial y_1}. \tag{29}$$

$$\frac{\partial T_{11}}{\partial y_1} = -\frac{4}{3}\mu\frac{\partial^2 v_1}{\partial y_1^2}, \tag{30}$$

$$\frac{\partial T_{12}}{\partial y_2} = \frac{\partial T_{21}}{\partial y_2} = 0, \tag{31}$$

$$\frac{\partial T_{13}}{\partial y_3} = \frac{\partial T_{31}}{\partial y_3} = 0. \tag{32}$$

The two integrals in Eq. (14) take the following form in this case:

$$\frac{\partial}{\partial x_i}\int_S l_j\left[T_{ij}\right]\frac{dS(\mathbf{y})}{r} = -\mu\left\{\frac{\partial}{\partial x_1}\int_S\left[\frac{4}{3}\frac{\partial v_1}{\partial y_1}\right]\frac{dS(\mathbf{y})}{r} + \frac{\partial}{\partial x_2}\int_S\left[\frac{\partial v_2}{\partial y_1}\right]\frac{dS(\mathbf{y})}{r} + \frac{\partial}{\partial x_3}\int_S\left[\frac{\partial v_3}{\partial y_1}\right]\frac{dS(\mathbf{y})}{r}\right\}. \tag{33}$$

$$\int_S l_i \left[\frac{\partial T_{ij}}{\partial y_j}\right]\frac{dS(\mathbf{y})}{r} = -\frac{4}{3}\mu\int_S \left[\frac{\partial^2 v_1}{\partial y_1^2}\right]\frac{dS(\mathbf{y})}{r}. \quad (34)$$

Opposite to the non-linear inviscid flow considered above, the stationary plane for the linear viscous flow presents no special case, as the integrals are determined by the first and second derivatives of the fluid velocity rather than the velocity as such. At the same time, this case is similar to the previous one in the impossibility to make a conclusion about the correctness of the criterion of the equivalence of the two equations described by Eq. (14) without detailed consideration of the velocity and pressure fields.

FUTURE RESEARCH – PROVING THE CRITERION OF EQUIVALENCE

If Curle's equation and the non-uniform Kirchhoff equation are identical, it will mean that the criterion of their equivalence (Eq. (14)) should *always* be satisfied, i.e. the sum of the two integrals should *always* vanish. The task of proving this for a very general fluid flow (in fact, for *all possible* fluid flows) is not straightforward.

However, if the two equations are not equivalent, it should be possible to show that the criterion is not correct at least for some particular flow situations. Examples of situations which could be considered in this respect are presented below.

First of all, the case of a fluid flow near an infinite rigid plane considered above in this paper presents a good opportunity to verify the criterion. With some additional simplifying assumptions it is possible to obtain an exact solution of the equations of fluid dynamics for the fluid velocity and pressure and evaluate the integrals in Eq. (14) based on this exact solution. Such a simplified fluid flow situation could be, for instance, fluid motion only due to vibrations of the plane, i.e. without any external flow.

Another case of a fluid flow, where the exact solution can be obtained and, therefore, the criterion of equivalence can be verified, is the sound radiation due to the motion of a rigid sphere or a cylinder along a closed circular path. If the fluid is stationary, this motion will generate sound as a rotating acoustic dipole (Morfey & Tanna 1971). However, if a rigid body is embedded into a rotating fluid, it is obvious from common sense that no sound will be radiated, as the body can be considered to be simply a fluid particle. Since the monopole term in Curle's equation takes into account the *total* velocity of the boundary with respect to a *stationary* observer, it would be very interesting, in the view of the present author, to evaluate both equations, as well as the criterion of their equivalence, in this case.

The author previously considered the case of a stationary sphere in a variable velocity field, which can be realised if the sphere is immersed into a vortex street (Zinoviev 2007). It has been concluded that this situation is equivalent to the problem of sound radiation by a vibrating sphere in a stationary fluid and, therefore, the radiated sound amplitude in this case should be equal to that radiated by the vibrating sphere. Whereas obtaining the exact solution for the fluid flow would be more difficult than in the previous example, a consideration of possible equivalence of the two equations in this situation would also be worthwhile.

CONCLUSIONS

In this paper, the original derivation of Curle's equation is reconsidered and compared with the derivation suggested previously by the present author. It is shown that the equations resulting from the two procedures are equivalent if the sum of two integrals over rigid boundaries is zero. The terms of the sum correspond to the sound generated on the boundary due to Lighthill's stress tensor and its spatial derivatives.

The obtained criterion of equivalence is applied to different flow situations. It is shown that the criterion is satisfied for linear acoustic waves in an inviscid fluid, as Lighthill's stress tensor vanishes altogether in this case. Also, a flow near an infinite rigid plane is considered for two kinds of fluid flows: a slightly non-linear inviscid flow and a linear viscous flow. It is shown that, in the former case, the criterion of equivalence is satisfied if the plane is stationary. The criterion is formulated for the vibrating plane and it is shown that a more detailed investigation of the flow is required to make a conclusion about the criterion in this case.

Some flow situations where the criterion of the equivalence of the two equations can be evaluated are suggested.

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