

Three Formulas of Generalized Gamma Functions Occurring in Analytical Acoustics

Wonju Jeon

National Institute for Mathematical Sciences, Daejeon, South Korea

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ABSTRACT

A wide range of problems concerning important areas of analytical acoustics are associated with applications of special functions. For instance, Bessel function plays a key role in acoustic problems defined in cylindrical coordinate, and therefore has been enormously studied and used by physicists and engineers as well as mathematicians. In this paper, the author presents a special function called generalized gamma function occurring in mathematical theory of diffraction. The paper consists of two parts. In Part One, the generalized gamma function is defined in its original form firstly introduced by Kobayashi in 1991. And then, the appearance of this special function in analytical acoustics is briefly explained by formulating the Wiener-Hopf integral equation for a famous diffraction problem by a finite strip or a single slit. Part Two is started with the derivation of a new and exact formula of the generalized gamma function in its specific form occurring in finite diffraction theory. The characteristics of present formula is graphically illustrated and numerically compared with existing two formulas. Firstly, the present formula is compared with the Kobayashi's asymptotic formula (1991) with a discussion about the lower bound of available argument yielding the relative error less than 0.0001. Secondly, the present formula is compared with the Srivastava's exact formula (2005) from the viewpoint of computational accuracy and efficiency for large argument. And, finally, the author discuss about the limitation of present formula and the future work concerning a further mathematical improvement as well as the practical applications of the generalized gamma function.