Proceedings of 20th International Symposium on Music Acoustics (Associated Meeting of the International Congress on Acoustics)

25-31 August 2010, Sydney and Katoomba, Australia

Numerical Modal Analysis of a Recorder Fluid

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PACS: 43.20.Mv, 43.28.Ra, 43.75.Qr

ABSTRACT

In this talk the three-dimensional numerical modal analysis of a fluid inside and around a recorder is presented. The fluid inside and close to the recorder is meshed by Lagrangian tetrahedral finite elements. Complex conjugated Astley-Leis infinite elements are used to obtain results in the far field of the recorder. The numerical method for solving problems in unbounded domains and the characteristics of the formulation of the eigenvalue problem are explained before the results of those computations are discussed.

As three-dimensional model, a soprano recorder with German fingering, which is tuned to 442 Hz, is used. A modal analysis of all playable notes, except the ones with half open tone holes, is accomplished. The results of the numerical modal analysis are compared to the values of the MIDI-table. Graphical results of the eigenvectors as well as the convergence behaviour of different tones are presented.

INTRODUCTION

For the model of a fluid inside and around a recorder, a modal analysis is performed for all playable notes, except the ones with half-open tone holes. The results are validated by comparison to the MIDI-table.

For the following computations, a three-dimensional model of a soprano recorder with german fingering and tuning to 442 Hz is used.

ACOUSTIC RADIATION PROBLEM

Assuming that the pressure is harmonic in time

$$p(\boldsymbol{x},t) = Re\left\{p(\boldsymbol{x})e^{-i\omega t}\right\} \tag{1}$$

the boundary value problem can be described by the Helmholtz equation

$$-\Delta p(\boldsymbol{x}) - k^2 p(\boldsymbol{x}) = 0 \quad \text{with} \quad \boldsymbol{x} \in \Omega\ , \tag{2}$$

the Neumann boundary condition on the recorder boundary

$$\frac{\partial p(\boldsymbol{x})}{\partial \boldsymbol{n}} = 0 \quad \text{with} \quad \boldsymbol{x} \in \Gamma\ , \tag{3}$$

and the Sommerfeld radiation condition [3]

$$R\left\{\frac{\partial p}{\partial R} - ikp\right\} \to 0 \quad \text{for} \quad R \to \infty\ . \tag{4}$$

The fluid inside and close to the recorder is described by finite elements (FE), while infinite elements (IE) are used to describe the far field. The Sommerfeld radiation condition ensures that only outward propagating components exist at large distance from the radiating body. As finite elements second order Lagrange tetrahedral elements are used and as infinite elements complex conjugated Astley-Leis elements are chosen [1, 2].

The matrix formulation of the entire problem, consisting of finite and infinite elements, is

$$(\boldsymbol{K} - ik\boldsymbol{D} - k^2\boldsymbol{M})\boldsymbol{p} = \boldsymbol{b}\ , \tag{5}$$

with $\boldsymbol{K}$, $\boldsymbol{D}$ and $\boldsymbol{M}$ as stiffness, damping and mass matrix. These matrices are real, unsymmetric and independent of the wave number k. The damping matrix contains only values from the infinite elements. For the eigenvalue problem considered in this paper, the right-hand-side vector b is equal to zero.

MODAL ANALYSIS

To compute eigenvectors and eigenfrequencies, equation (5) is transformed in the state space equation [4]

$$\left(\begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{K} \end{bmatrix} - \lambda \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{M} & \boldsymbol{D} \end{bmatrix}\right)\begin{bmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Psi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}, \tag{6}$$

with $\boldsymbol{\Phi} = \lambda\boldsymbol{\Psi}$ and $\lambda = -ik$. To solve this linear eigenvalue problem it has to be transformed in the standard formulation

$$\frac{1}{\lambda}\begin{bmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Psi} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{K} \end{bmatrix}^{-1}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{M} & \boldsymbol{D} \end{bmatrix}\begin{bmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\Psi} \end{bmatrix}. \tag{7}$$

The three-dimensional finite element model of the recorder fluid is build in Ansys 11.0 and read into a noncommercial code that was developed at our institute. In this Fortran 90 code the infinite elements are added before starting with the computations.

The modal analysis is accomplished for all playable notes, except the ones with half-open tone holes. In this paper, the results of notes c$''$, b$''$ and d$'''$ are presented. In Figures 1, 2 and 3 the first eigenfrequency and the first harmonic of c$''$, b$''$ and d$'''$ can be seen.

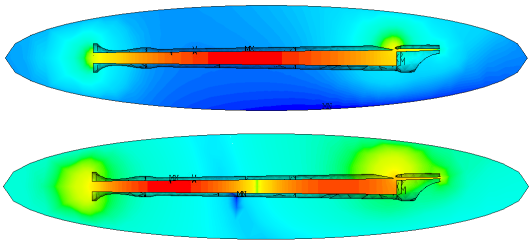

Figure 1: c′′: First eigenfrequency (above) and first harmonic (below).

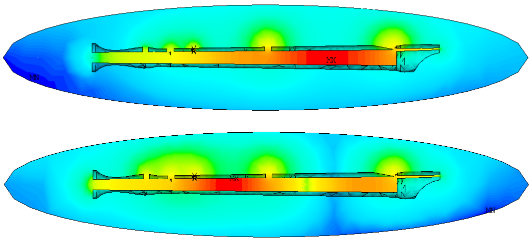

Figure 2: b′′: First eigenfrequency (above) and first harmonic (below).

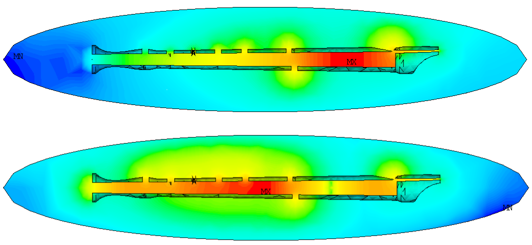

Figure 3: d′′′: First eigenfrequency (above) and first harmonic (below).

To gain information about the convergence behaviour of the computed frequencies, comparisons are made between the numerically computed values and the ones from the MIDI-table. The values from the MIDI-table have to be converted for a tuning of 442 Hz.

The numeric modal analysis of those notes is accomplished for eight different meshes, with different degrees of freedom, in each case. The exact frequencies from the MIDI-table are

$$\begin{aligned} c'' &= 525.630\,Hz \\ b'' &= 936.565\,Hz \\ d''' &= 1179.998\,Hz\,. \end{aligned} \tag{8}$$

Figures 4–6 present the convergence of each first eigenfrequency over the degree of freedom.

At first sight, all three notes show a similar covergence behaviour, but when taking a closer look it can be seen that for note c′′ there is only 1 Hz between coarsest and finest mesh, while for the other two notes the coarse meshes give significantly worse results than the fine meshes.

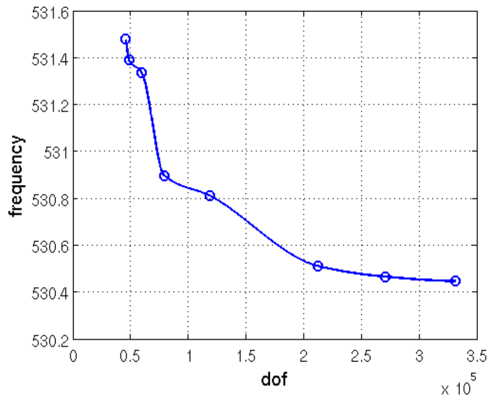

Figure 4: Convergence of c′′.

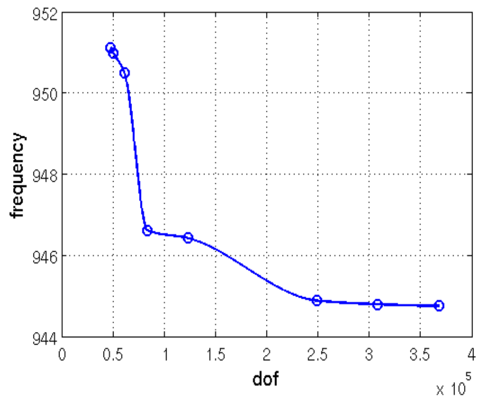

Figure 5: Convergence of b′′.

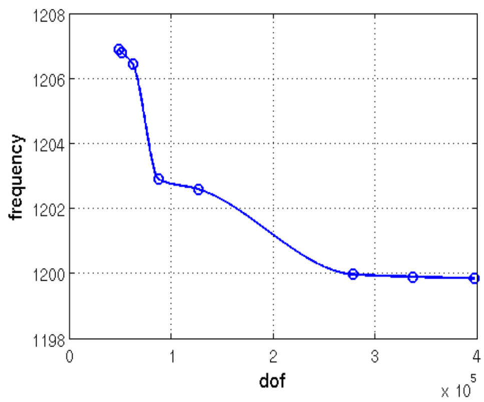

Figure 6: Convergence of d′′′.

CONCLUSION

The examinations of the convergence behaviour for different notes showed that the eigenfrequencies approach the expected values from above. For some notes, the difference between numerically computed and exact values can be several Herz, even for fine meshes. This deviation is too high for a music instrument. We suspect that we obtain such a deviation due to neglecting the volume flow inside the recorder.

When playing a recorder, the air column inside the instrument starts to oscillate due to the inserted air flow. The musician is able to influence the frequency of a note by varying the blowing pressure and therewith a fine-tuning of the sound is possible. Due to that, the characteristic flow profile inside a recorder will be taken into consideration in future studies.

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