Proceedings of 20th International Symposium on Music Acoustics
(Associated Meeting of the International Congress on Acoustics)

25-31 August 2010, Sydney and Katoomba, Australia

Voice of the dragon: the mystery of the missing fundamental mode

D. Tonon (1), G. Nakiboglu (1), J. Golliard (2) and A. Hirschberg (1)

(1) Department of Applied Physics, Eindhoven University of Technology, Eindhoven, The Netherlands
(2) TNO Science and Industry, Delft, The Netherlands

PACS: 43.75.-z

ABSTRACT

When a corrugated pipe is swirled it produces a musically interesting sound. By increasing its rotational speed one can produce a series of frequencies corresponding to the modes of the open-open pipe. An interesting issue, raised since the early studies on the whistling of corrugated pipes, is that the fundamental acoustic mode is not whistling. This aspect has been related in the literature to the onset of turbulence in the pipe flow. In the present paper we provide a critical literature review and a physical model for the sound production, which contradicts the explanation of the missing fundamental mode presented in the literature.

INTRODUCTION

A short corrugated open-open tube, known as the "Voice of the Dragon" in Japan [1] and the "Hummer" in the United States [2], was a widely diffused musical toy since the early 70's. When swirled in a circular motion, it produces such amusing tonalities that it has been used as a musical instrument by several composers [1, 2]. It is referred to by Schickele [3] as the "Lasso d'Amore". The singing corrugated pipes are attractive because of their technical simplicity, high expressive potential, and unexpected sonic qualities. We consider a flexible corrugated pipe of $L = 90$ cm length with an internal diameter $D = 2.5$ cm. The pitch of the corrugations is about $Pt = 6$ mm (Figure 1).

When the corrugated pipe is hold with a hand and swirled in a circular motion, a pleasant whistling is obtained. As the rotation is accelerated one can produce a series of tones with frequencies corresponding to open-open resonant modes of the pipe:

$$f_n = nc_{eff}/(2L)\,(n = 2, 3, 4, \ldots) \qquad (1)$$

where c_{eff} is the speed of sound in the tube. The value of c_{eff} is lower than the speed of sound in air [4]. The whistling of a specific mode f_n can be obtained by blowing through the pipe with a velocity U corresponding to a critical Strouhal number $Sr_{Pt} = f_n Pt/U$. For our flexible pipe $Sr_{Pt} \approx 0.5$. The specific sound quality of the pipe can be explained as a result of the interference between the sound radiated from the moving pipe termination and the sound radiated from the fixed pipe termination [1].

One wonders why the first mode $n = 1$ is not whistling. In the present paper we give a review of our current understanding of the physics of the instrument and try to answer this question. While doing so, we will propose some corrections to the physical models proposed in the literature.

FLUID DYNAMICS

The whistling of the corrugated pipe is induced by the flow through the pipe driven by its rotation. This can be demonstrated by closing the pipe termination which is held with the hand. Placing the thumb in the tube is a convenient way to do so. This suppresses the whistling. Another way to demonstrate that it

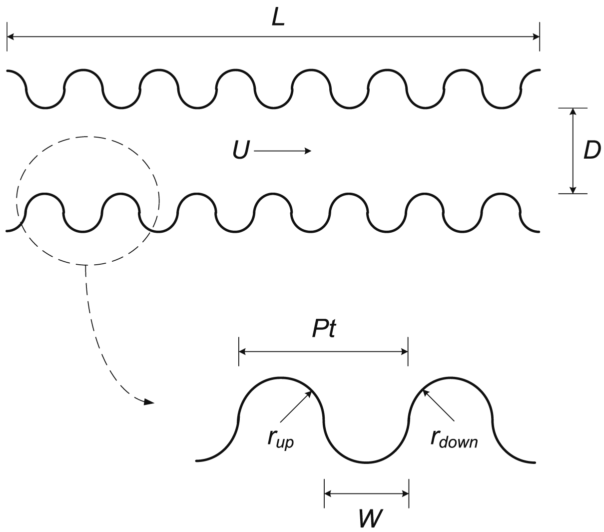

Figure 1: Geometry of the corrugated pipe.

is the flow through the corrugated pipe that sustains whistling, is to blow through the pipe. Our lung capacity is not sufficient to make a typical hummer whistle. However, one can take a narrower corrugated pipe and make it whistle. A corrugate pipe with diameter $D = 1$ cm used as protection jacket for electrical cables in buildings of $L = 1$ m length, whistles nicely at a rather high pitch. A problem here is that our lungs do not behave as an acoustically open end, so that a quantitative interpretation of this result is difficult.

The flow velocity U through the swinging pipe can be estimated by assuming a steady frictionless flow. As the velocities are low compared to the speed of sound, the pressure difference across the pipe is very small compared to the atmospheric pressure. One can therefore neglect the density variation in the steady component of the flow. The fact that the air is almost incompressible implies that, in a steady flow, the volume flux Q along the tube must be independent of the position along the tube x, measured from the fixed open end. If we neglect changes in the velocity profile, the flow velocity remains constant along the pipe. This velocity is given by $U = 4Q/\left(\pi D^2\right)$. Because of the swinging motion, the tube is rotating with an angular velocity Ω. A fluid particle, corresponding to a slice of the tube of length dx, will undergo a centrifigual force $\rho_0 \left(\pi\, dx\, D^2/4\right) \Omega^2 x$. As the fluid velocity is constant this force should be balanced by the pressure forces $-\left[p\left(x+dx\right)-p\left(x\right)\right]\pi D^2/4 = -dp\left(\pi D^2/4\right)$. This yields the differential equation:

$$dp = \rho_0 \Omega^2 x dx \qquad (2)$$

Integration between the non-moving tube inlet $x = 0$ and the moving tube outlet $x = L$ yields:

$$p\left(L\right) - p\left(0\right) = \frac{1}{2}\rho_0 \Omega^2 L^2 \qquad (3)$$

Note that this equation has the opposite sign from the equation used by Silverman and Cushman [1] and Serafin and Kojs [2]. This is due to the fact that Silverman and Cushman[1] ignored the impact of the centrifugal force on their measurement of the pressure difference and made the erroneous assumption that the inlet pressure $p\left(0\right)$ should be equal to atmospheric pressure p_{atm}. In fact, as a result of flow separation, a free jet is formed at the swinging outlet of the pipe. Like in the plume flowing out of a chimney, the pressure $p\left(L\right)$ in this free jet is equal to the surrounding atmospheric pressure p_{atm}. The low pressure at the inlet:

$$p\left(0\right) = p_{atm} - \frac{1}{2}\rho_0 \Omega^2 L^2 \qquad (4)$$

is actually sucking the surrounding air into the pipe. Assuming a steady incompressible frictionless flow around the inlet one finds, from the conservation of mechanical energy (Bernoulli):

$$p_{atm} = p\left(0\right) + \frac{1}{2}\rho_0 U^2 \qquad (5)$$

which combined with Eq. 4 yields the very simple result:

$$U = \Omega L \qquad (6)$$

In this simple model we assume that the friction is negligible (except for flow separation at the outlet) and that the velocity in the pipe is uniform. As a result of friction the velocity in the pipe will be lower near the walls than in the middle, so that a velocity profile will develop.

At high flow rates the velocity field inside the pipe can display a complex unsteady chaotic motion called turbulence. The transition from a laminar (smooth) velocity field toward a turbulent (chaotic) flow is determined by the ratio of inertial to viscous forces. A measure for this is the Reynold number $Re_D = UD/\nu$, where ν is the kinematic viscosity (for air at room conditions $\nu = 1.5 \times 10^{-5}$ m^2/s). For a smooth pipe below $Re_D = 2300$ turbulence cannot be maintained. Depending on the inflow conditions a laminar flow can however be maintained in a smooth pipe up to very high values of Re_D [5]. In the case of rough walls (such as for a corrugated pipe) turbulence is commonly observed for $Re_D \geq 4000$ [5]. Transition can occur already for $Re_D \geq 2300$.

As we will explain later, the whistling frequency is related to the flow velocity by a critical Strouhal number (dimensionless frequency) $Sr_{Pt} = fP_t/U$ of the order of 0.5. The lowest mode of the pipe, whistling at the frequency $f \approx 170$ Hz corresponds to a flow velocity $U \approx 2$ m/s and $Re_D \approx 3 \times 10^3$. Hence, the flow is typically transitional. For the next mode $Re_D \approx 5 \times 10^3$ and the flow is certainly turbulent. It is assumed in the literature that turbulence triggers the whistling [6, 7]. The absence of the first mode would be explained by the absence of turbulence.

Experiments carried out by Elliot [4] on a pipe with $D =$ 1.07 cm, $Pt = 2$ mm and $L = 1$ m show that the first mode to be excited is the ninth mode ($n = 9$) with $f = 1.5$ kHz. This corresponds to a flow velocity $U = 8.6$ m/s, so that $Re_D \approx 6.3 \times 10^3$. Mode $n = 8$ has a Reynolds $Re_D \approx 5.6 \times 10^3$ and does not sound. Elliot [4] carried out experiments with both a sharp inlet (promoting turbulence) and a smooth inlet nozzle (retarding turbulence). He did not observe any difference between the two measurements series. In both cases the first whistling mode was the ninth mode. This experiment indicates that the absence of turbulence is not likely to be the essential factor determining whether a mode does not whistle.

SOUND SOURCES

Whistling occurs at each acoustic mode f_n around a specific value of the Strouhal number $Sr_{Pt} = f_n Pt/U$. The exact value of Sr_{Pt} appears to depend on the geometry of the corrugations. Experiments show that [8]:

$$Sr_{Pt} = 0.58 \frac{Pt}{W + r_{up}} \left(\frac{W + r_{up}}{D}\right)^{0.2} \qquad (7)$$

where W is the cavity width as defined in Figure 1 and r_{up} is the radius of curvature of the upstream edge of the cavity. In particular, it is observed that the critical flow velocity for a specific acoustic mode does not change when one increases the pitch Pt while keeping $W + r_{up}$ constant. This demonstrates that the whistling is due to a local hydrodynamic phenomenon at each cavity. Note that the factor $\left[\left(W + r_{up}\right)/D\right]^{0.2}$ appears to be related to the shape of the velocity profile and it is valid only for turbulent flows.

It is well known from literature that the grazing flow over a cavity is unstable [9, 10]. In the case of the corrugated pipe this instability is triggered by the velocity perturbation associated with the acoustic standing wave along the pipe. For a given acoustic standing wave p'_n:

$$p'_n\left(x,t\right) = A_n sin\left(\frac{\pi x n}{L}\right) cos\left(2\pi f_n t\right) \qquad (8)$$

where A_n is the mode amplitude, the acoustic velocity u'_n can be calculated from the equation of motion $\rho_0\left(\partial u' \partial t\right) = -\left(\partial p'/\partial x\right)$:

$$u'_n\left(x,t\right) = -\frac{A_n}{\rho_0 c_{eff}} cos\left(\frac{\pi x n}{L}\right) sin\left(2\pi f_n t\right) \qquad (9)$$

This velocity is the average of the oscillating acoustic velocity across the section of the pipe at position x.

The coupling of the flow instability with the acoustic flow results into periodic vortex shedding. This unsteadiness of the hydrodynamic flow results into an unsteady periodic force of the flow on the walls of the pipe. This unsteady periodic force is associated with a reaction force from the walls on the flow. It can be shown that such an unsteady force is a source of sound [11, 12]. It is essential to realize that this sound production does not involve wall vibrations. This can be verified by varying the wall stiffness.

As we found that the hydrodynamic instability is a local phenomenon at each cavity, one can try to describe the phenomenon by carrying out a numerical simulation of the flow within a single cavity. Since Pt is much smaller than the acoustic wavelength c_{eff}/f_n of the produced sound wave, one can assume that wave propagation is locally negligible. This corresponds to the assumption that the flow is locally incompressible. We follow here a procedure inspired by the work of Martínez-Lera et al. [13].

While three-dimensional turbulent flow simulations are still extremely difficult, incompressible two-dimensional (axisymmetrical) simulations are feasible. Such flow simulations are laminar, but one can assume a velocity profile $\vec{u} = (u(r), 0, 0)$ just upstream of the cavity that corresponds to a time-average turbulent velocity profile. From these simulations we predict the oscillating pressure difference $\Delta p'$ induced along the pipe by the cavity oscillation driven by an imposed acoustic oscillation u', which is uniform across the section of the pipe just upstream of the cavity. As the viscous effects are not accurately described, we correct the simulations by subtracting the pressure difference $\Delta p'_{visc}$ obtained from simulations of the flow in a uniform pipe segment without cavity [14]. The acoustic energy produced by the source is given in first approximation by:

$$P_{source} = S\,u' \left(\Delta p' - \Delta p'_{visc}\right) \qquad (10)$$

A first consequence of this theory is that sound in a corrugated pipe is generated mainly around pressure nodes of the acoustic standing wave, where the acoustic velocity has the largest amplitude. Experiments do indeed confirm this. When cavities around pressure nodes are plugged the corrugated pipe stops whistling [15, 16, 18], while it is much less sensitive to plugging of cavities around velocity nodes.

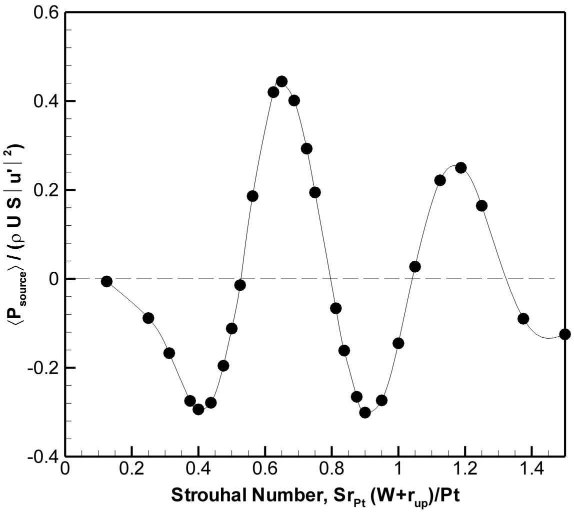

Figure 2: Dimensionless source power as function of the Strouhal number for a cavity with rounded edges and $|u'|/U = 0.05$

.

If we want to predict whistling we should consider an energy balance [19] for a specific mode between the time-average of the energy production $\langle P_{source}\rangle$ and the losses $\langle P_{loss}\rangle$. A typical result of the calculation of $\langle P_{source}\rangle$ is shown in Figure 2 as function of $Sr_{Pt}\left(W + r_{up}\right)/Pt$ for $|u'|/U = 0.05$. We observe that for some values of Sr_{Pt} the produced power is positive, which is a necessary condition for whistling. This occurs for $Sr_{Pt}\left(W + r_{up}\right)/Pt < 0.1$, for $0.5 < Sr_{Pt}\left(W + r_{up}\right)/Pt < 0.8$ and for $1.1 < Sr_{Pt}\left(W + r_{up}\right)/Pt < 1.3$. Each range for which $\langle P_{source}\rangle$ is positive is called an hydrodynamic mode.

A first success of our theory is that it does predict an optimal sound production for the second hydrodynamic mode around $Sr_{Pt}\left(W + r_{up}\right)/Pt = 0.6$, which is the mode observed in the experiments [15, 18]. The theory actually predicts Sr_{Pt} within 10% [14]. A further conclusion from this theory is that turbulence is not essential to promote whistling. Actually a velocity profile with thin laminar boundary layers will correspond to a larger source power than a flow with a fully developed turbulent velocity profile [8].

BALANCING SOURCES AND LOSSES

We now focus on the maximum $\langle P_{source}\rangle_{max}$ of $\langle P_{source}\rangle$ predicted, around $Sr_{Pt}\left(W + r_{up}\right)/Pt = 0.6$. In Figure 3 we show a dimensionless representation of $\langle P_{source}\rangle_{max}$ as a function of the amplitude $|u'|/U$. We observe that:

$$\frac{D}{W + r_{up}} \frac{\langle P_{source}\rangle_{max}}{\rho_0 U S |u'|^2} \qquad (11)$$

is almost constant for $|u'|/U < 10^{-2}$, which corresponds to a linear behavior ($\Delta p' \propto u'$). For larger amplitudes $|u'|/U > 10^{-2}$ the dimensionless source decreases rapidly. This non-linear saturation behavior allows us to predict a finite amplitude as a result of an equilibrium between linear losses, scaling as $\langle P_{loss}\rangle \propto |u'|^2$, and the non-linear sound production $\langle P_{source}\rangle$.

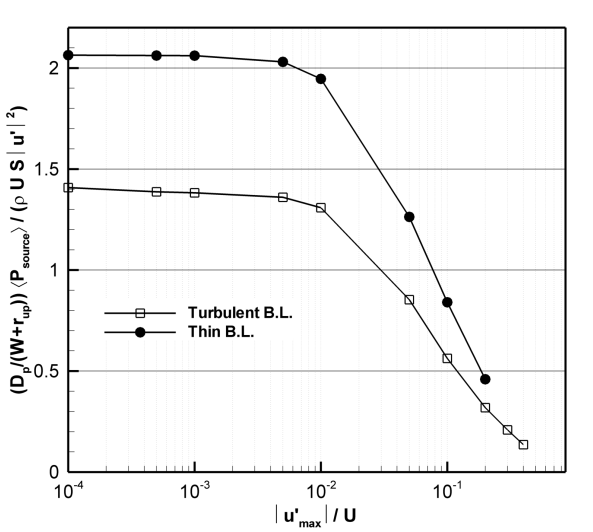

Figure 3: Maximum dimensionless source power as function of the dimensionless amplitude of the acoustic velocity. Results of the numerical simulations carried out with different flow profiles upstream of the cavity.

We now reconsider our initial question: why does the fundamental mode of the corrugated pipe not whistle?

At low frequencies it appears that visco-thermal losses are dominant, as in most wind instruments [20]. Radiation losses at the pipe terminations can be neglected for the lowest modes, when we consider the prediction of the oscillation amplitude in the pipe. The visco-thermal losses of a traveling acoustic wave are described by a damping coefficient α defined by:

$$\alpha = \frac{1}{p'} \frac{dp'}{dx} \qquad (12)$$

The dimensionless losses in a standing wave are given by:

$$\frac{D}{W} \frac{\langle P_{losses}\rangle}{\rho_0 U S |u'|^2} = \frac{1}{2} \frac{\alpha Pt}{M} \frac{c_0}{c_{eff}} \frac{D}{W} \qquad (13)$$

Assuming thin acoustic visco-thermal boundary layers, one expects that $\alpha \propto \sqrt{f_n}$ [21], while $U \propto f_n$ (because $Sr_{Pt} = constant$). Hence, the dimensionless losses are expected to decrease with increasing mode number. From Figure 3 we see that if mode $n = 2$ just whistles, the increase of dimensionless losses with respect to the dimensionless production by a factor $\sqrt{2}$ between mode $n = 2$ and mode $n = 1$ easily explains the impossibility to make the tube sound at the fundamental mode $n = 1$.

There is a small additional reason why a corrugated pipe has troubles when whistling at the fundamental mode. The regions of sound production correspond to the pressure nodes of the standing wave. These nodes are actually outside the pipe as a result of the inertia of the oscillating flow around the open pipe termination. This corresponds to the so called end-corrections which should be added to the pipe length to predict the mode frequency [20]. We are missing a length $0.6D$ of corrugations as a result of these end-corrections. Also, a hummer has often a smooth pipe segment (few centimeters) at its inlet, used to hold the pipe. This definitively tend to kill the whistling on the fundamental mode.

It is difficult to provide a definitive answer to the problem because the predicted sound source depends strongly on the imposed main flow velocity profile. Also, we do not have yet an accurate prediction for the damping coefficient α.

CONCLUSIONS

The absence of whistling of the "Voice of the Dragon" and the "Hummer" on their fundamental mode has been attributed in the literature to the absence of turbulence in the pipe at low rotation speeds. While the Reynolds numbers corresponding to the whistling of the first mode are in the transitional range $2300 \leq Re_D \leq 4000$ for the onset of turbulence, there is no indication that turbulence is essential. Actually, theory predicts that a flow with thin laminar boundary layers will whistle more efficiently than a turbulent flow.

Assuming that acoustic losses are dominated by visco-thermal losses we conclude that for lower modes there is a relative decrease of losses with respect to the production by a factor $\sqrt{2}$, when moving from the first mode $n = 1$ to the second mode $n = 2$. If the second mode just whistles, the first mode will certainly not whistle, because of this strong relative increase in losses. The chance that the first mode will whistle is further reduced by a reduction of the source region at the pipe terminations due to end-corrections and smooth pipe segments. It is not until we have a more quantitative description for the acoustic losses that a definitive answer to the problem can be given.

ACKNOWLEDGMENTS

This work is being sponsored by European Commission Marie Curie RTN Project AETHER (contract nbr MRTN-CT-2006-035713) and by the contributions of STW Technologiestichting (project STW 08126), BP, Bureau Veritas, ExxonMobil, Statoil and UK Health and Safety Executive. The authors wish to thank Dr. A. Houtsma for encouraging us to explore the potential of our model to explain the missing fundamental mode in a hummer.

REFERENCES

[1] M.P. Silverman and G.M. Cushman, Voice of the Dragon: the rotating corrugated resonator, *Eur. J. Phys.* 10 (1989) 298-304.

[2] S. Serafin and J. Kojs, Computer models and compositional applications of plastic corrugated tubes, *Organised Sound* 10 (2005) 67-73.

[3] P. Schickele, *The Definitive Biography of P.D.Q. Bach*, Random House, New York, 1976.

[4] J.W. Elliot, *Corrugated Pipe Flow* in Lecture notes on the mathematics of acoustics, M.C.M. Wrigth, Imperial College Press, London, 2005.

[5] R.D., Blevins, *Applied Fluid Dynamics Handbook*, Van Nostrand Reinhold Company, 1984.

[6] F.S. Crawford, Singing corrugated pipes, *Am. J. Phys.* 42 (1974) 278-288.

[7] L.H. Cadwell, Singing corrugated pipes revisited, *Am. J. Phys.* 62 (1994) 224-227.

[8] G. Nakiboglu, S.P.C. Belfroid, J. Golliard and A. Hirschberg, On the whistling of periodic systems: corrugated pipes and multiple side branch system, *Submitted to the Journal of Fluid Mechanics* (2010)

[9] D. Rockwell and E. Naudascher, Review - Self-sustaining oscillations of flow past cavities, *Journal of Fluids Engineering* 100 (1978) 152-165.

[10] D. Rockwell, Oscillations of impinging shear layers, *AIAA Journal* 21 (1983) 645-664.

[11] L. Gutin, in *On the sound field of a rotating propeller*. Original in Russian: Zhurnal Tekhnicheskoi Fiziki 1936; 12:76-83: NACA TM No.1195.

[12] N. Curle, The influence of solid boundaries upon aerodynamic sound, *Proc. R. Soc. London A* 231 (1955) 505-514.

[13] P. Martínez-Lera, C. Schram, S. Föller, R. Kaess and W. Polifke, Identification of the aeroacoustic response of a low Mach number flow through a T-joint, *Journal of the Acoustical Society of America* 126 (2009) 582-586.

[14] G. Nakiboglu and A. Hirschberg, A numerical study of the aeroacoustic interaction of a cavity with a confined flow: effect of edge geometry in corrugated pipes, FEDSM2010-ICNMM2010, *ASME 2010 Fluids Engineering Summer Meeting*, Montreal, Canada.

[15] D. Tonon, B.J.T. Landry, S.P.C. Belfroid, J.F.H. Willems, G.C.J. Hofmans and A. Hirschberg, Whistling of a pipe system with multiple side branches: Comparison with corrugated pipes, *Journal of Sound and Vibration* 329 (2010) 1007-1024.

[16] J. Golliard and D. Tonon, Whistling of short corrugated pipes: experimental investigation of the source location, AIAA-2010-3928 *16th AIAA/CEAS Aeroacoustics Conference*, 2010, Stochkolm, Sweden.

[18] G. Nakiboglu, S.P.C. Belfroid, J.F.H. Willems and A. Hirschberg, Whistling behavior of periodic systems: corrugated pipes and multiple side branch system, *International Journal of Mechanical Sciences*, Article in press.

[19] D. Tonon, A. Hirschberg, J. Golliard and S. Ziada, Aeroacoustics of pipe systems with closed branches, *Submitted to the International Journal of Aeroacoustics* (2010).

[20] C.J. Nederveen, *Acoustical aspects of woodwind instruments*, Northern Illinois University Press, DeKalb, 1998.

[21] G. Kirchhoff, Über den EinfluSS der Warmteleiyung in einem Gase auf die Schallbewegung, *Annalen der Physik und Chemie* 134 (1868) 177-193.