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Numerical simulation of wolf-note in string instruments using string-body coupled model

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Large size string instrument, such as cello and contrabass, can ingenerate phenomenon called ‘wolf-note’. When we play a note of specific pitch, the body of the instrument strongly vibrates and the bow leaps on a string continuously, which makes it difficult to bow stably. In order to avoid this phenomenon, small equipments so called `wolf-killer’ are often utilized. Such equipments have been empirically devised and it causes some troubles like timbre degradation. Consequently, we are trying to figure out the cause of the wolf-note and control it without significant effect. In this paper, we consider a model in which a string and a body are coupled at a bridge. Wolf-note is numerically reproduced using this model. The model is experimentally validated.

1. INTRODUCTION

Large size string instruments, cello and contrabass, sometimes generate fine beat tone, called ‘wolf-note’, when we play a note of specific pitch. It occurs regardless of player’s mind and it degrades quality of performance. To suppress wolf-note, an equipment called ‘wolf-killer’ is commercialized. The wolf killer changes the quality of sound compare to original quality. It is very serious problem for performer. Our work aims to find out the mechanism of wolf-note and control it without changing the quality of sound.

Therefore, we assumed a simple model, in which a string is coupled with body at a bridge, named string-body coupled model. Using this model, we simulated wolf-note under plucked string condition, compared to actual measurement result, and verified the validity of this model. Because wolf-note is generated in the condition that string is rubbed by bow, we have to consider an effect of bowed string. And we also simulated of bowed string with force as a ‘stick & slip’ function connect with wolf-note.

2. STRING-BODY COUPLED MODEL

In this paper, we intended for cello as a research object among string instruments. Because cello is comparatively large in violin-family and always generates wolf-note. Figure 1 shows a schematic view of cello used for the experiment. The order of string is also shown, starting from the left the first string (A string), the second string (D string), the third string (G string), and the fourth string (C string), getting thicker and lower pitch sound. The strings are set up on boxing body, and vibration of string propagates to table and back of body through a bridge. The amplified sound is radiated from surface. The body has natural frequency which depends on its volume, stiffness, and the shape and size of f-hole. In manufacturing process, these parameters are selected not to cause wolf note [1].

The string-body coupled model is focused on the body’s resonant vibration with respect to the bridge which vibrates with strings. Figure 2 shows a string-body coupled model. In this model, one end of a string is fixed and the other end is attached to mass-spring system.

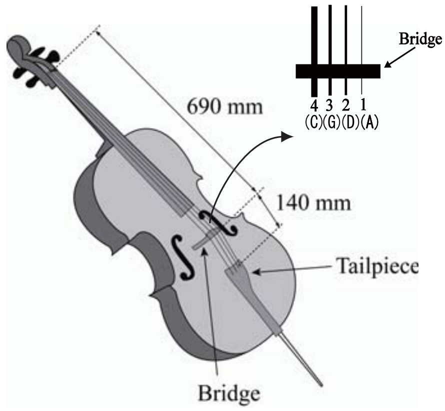

Figure 1. Schematic view of cello

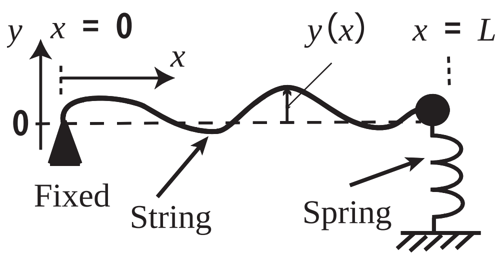

Figure 2. String-body coupled model

In real cello, one end of string is fixed on a peg, and the other is verged on a bridge which is a resonant body. In this model, the equation which expresses mass-spring system is given as,

$$M\frac{\partial^2 y}{\partial t^2} + ky = -T\frac{\partial y}{\partial x}, \quad (1)$$

where, M, k, and T express mass and rate of spring, and string tension, respectively.

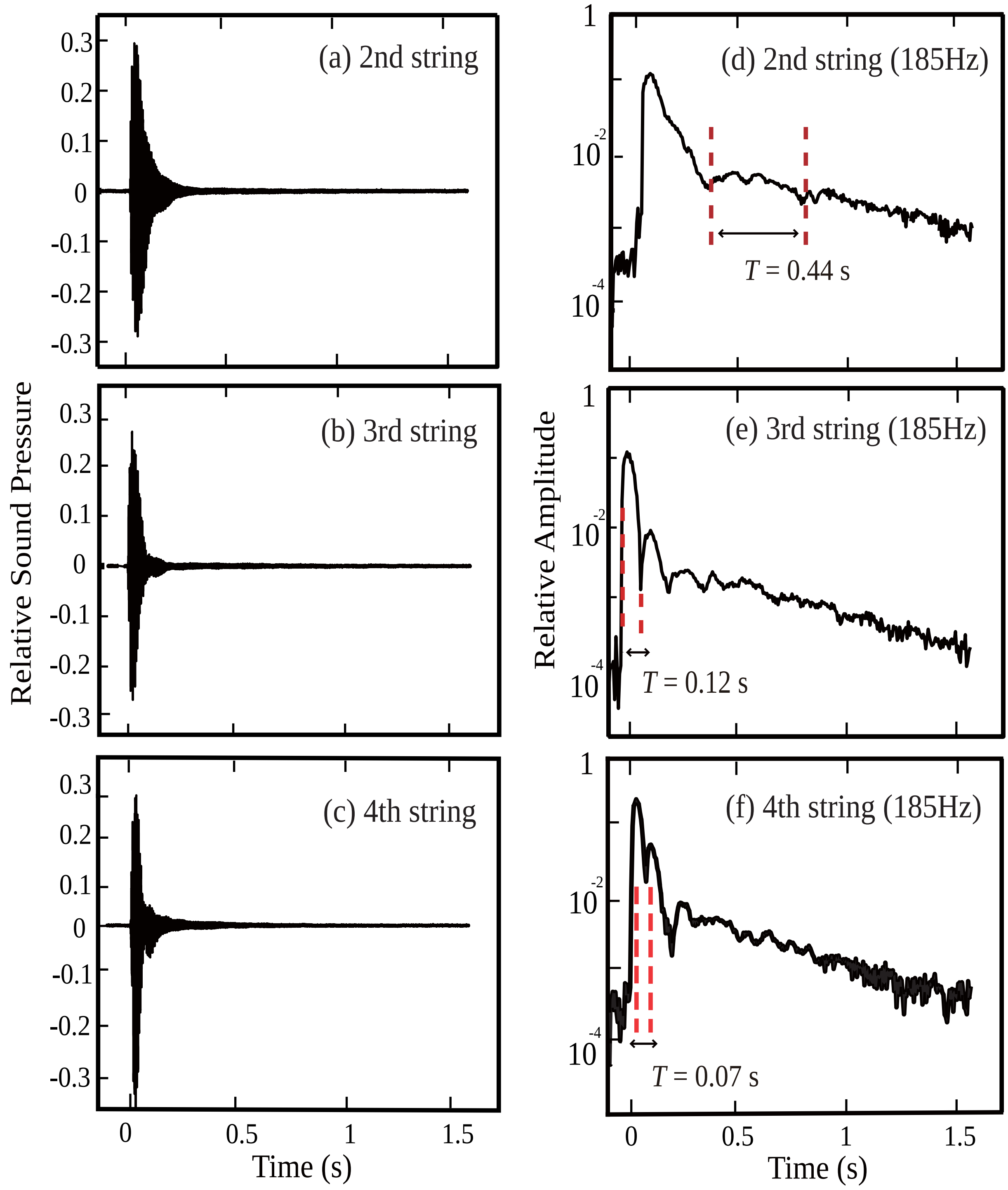

Figure 3. Observation result

3. ACTUAL MEASUREMENT

In fact, each period of wolf-note is different by line density of the string, and it becomes shorter as string gets thicker. However, the phenomenon of bowed string is very complicated. To figure out a behavior of combined vibration system, we assumed plucked string.

Figure 3 shows a waveform observed in the experiments. We recorded the sound of wolf-note under plucked string condition using digital recorder (PCM-D50, SONY) (sampling frequency: 96 kHz, quantization bit rate: 24 bit).

To measure th effect of line-density on combined vibration system, we tentioned four strings, A, D, G, and C, at the third-string point.

Here, we except for the first string because it is not used for performing the pitch of a sound which generates wolf note.

Figures 3 (a), (b) and (c) show waveforms of ‘the first string (A string)’, ‘the second string (D string)’, ‘the third string (G string)’ (set a full scale as ±1), and (d),(e), and (f) show waveforms (185Hz) which are extracted from (a),(b), and (c) using synchronized detection. Here, vertical axis uses logarithmic scale.

The obtained results show the two-phased attenuation that expresses the slow attenuation after the rapid attenuation. A reason includes that after plucking a string, vibration of longitudinal direction starts decaying and vibration of vertical direction

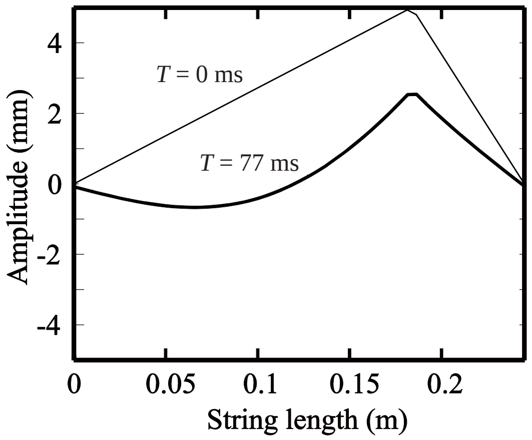

Figure 4. Motion of 4th string

keeps own vibration. This observation results shows that each beat period of third and fourth strings are T = 0.12 and 0.07(s), respectively. About the second string, it is difficult to judge whether beat tone is generated or not only from the amplitude change, but if we assume that second peak is generated by beat tone, the period is T = 0.44 s. As above, we figure out that the periods of beat tone are different by each string under plucked string condition.

4. SIMULATION WITH STRING-BODY COUPLED MODEL

4-1. Condition

We simulated the wolf-note using string-body coupled model. The observation result shows that the real instrument also generates beat tone under plucked string condition. Here, the initial condition is $x = 3/4L$ and y = 5 mm, and initial velocity is set as 0 m/s. Under this condition, we solved one-dimensional wave equation, eq.(2), by difference method. We set time interval as ΔT= 54.7μs and space interval as Δx= 0.5 cm, to minimize dispersion.

We simulated wolf-note of each string by changing a line density and string length under this condition. We made fundamental frequency of string conform to natural frequency of body.

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}. \qquad (2)$$

4-2. Vibration mode and force to bridge

Figure 4 shows the initial state of the fourth string (T = 0 s) and the shape after 77 ms under plucked string condition.We found out from this simulation result that amplitude is getting small with time and the string shape is rounded by energy exchange from body. We regarded the sound from body as the change of force to bridge. Figure 5 shows the force to bridge after the each string that the 2nd(D), the 3rd(G), and the 4th(C), are plucked. The horizontal axis shows time and the vertical axis shows the force to bridge. As this result, we figured out that the force to bridge expressed by the distinct beat tone. This beat tone is generated by coupling between string vibration and spring system. The period of this beat tone depends on each string, the 2nd(D), the 3rd(G), and the 4th(C), the thicker the string is, the faster the beat becomes.

One of the reason considered that specific impedance is changed as the line density is changed, and degree of coupling between mass-sprig system and string. In this simulation, we decided the spring constant, k, and the mass, m, with conversion of the beat tone of each string. However they should be decided from observation results. The period of beat tone differs from real value, but it is possible to decide the value of k and m to approach the observation result. But in this case, by increase of line density, the period of beat tone was increased less than actual measurement. Now we are trying to identify cause of this point. Figure 6 shows a result of the vibration at the bridge obtained by solving eq.(2). From this simulation, we found out that the vibration at the bridge itself has the distinct beat tone when wolf-note is generated. Fundamental frequency component of the mass-sprig system is more stressed than string vibration because the mass-spring system has simple harmonic motion.

5. BOWED STRING

5-1. Bowed string model

Our intended purpose is to simulate wolf-note using string-body coupled model, so the stable vibration should be simulated under the condition that both ends are fixed. So we check up two parameters (velocity of bow, V_b, and pressure of bow, f_b) which can bow the string stably.

In this simulation, position of the bow on the string is L= 0.25 m from the bridge shown in Figure 7. And width of bow is 5 mm. Here, we assumed that velocity of bow is constant and gave the force, F, relates to the speed of bow. Wave equation is given as,

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2} + \frac{F}{\rho}, \qquad (3)$$

Where, ρ expresses line density. And F is a function as follows

$$F = -[\mu_d + (\mu_s - \mu_d)\exp\left\{\frac{(V_r + V_c)}{\tau V_b}\right\}] f_b \qquad (V_r \le -V_c)$$

(4)

$$F = \mu_s \left(\frac{V_r}{V_c}\right) f_b \qquad (-V_c \le V_r \le V_c)$$

(5)

$$F = [\mu_d + (\mu_s - \mu_d)\exp\left\{-\frac{(V_r - V_c)}{\tau V_b}\right\}] f_b \qquad (V_r \ge V_c)$$

(6)

where Vr is relative velocity of the bow with respect to string ($V_r = V_b - V_s$). The value of V_c, μ_s, and μ_d are critical velocity, static friction coefficient, and kinetic friction coefficient, respectively. Figure 8 shows the characteristic of kinetic friction. [5]. Table 1 shows parameters of this simulation.

5-2. Result of simulation

Figure 9 shows the shape of fourth string. In this result, the top of locus of wave spins around, and we found out different shape changing two parameters. Figure 10 shows the simulation result using the value of V_b and f_b as 0.5 and 0.4, respectively. This is different from the simulation using the value of Table.1 and this vibration generates over tone.

In the real instrument, the sound sometimes turns to octave when speed of bow is fast or pressure of bow is weak, and we could duplicate it in this simulation. However we do not con-

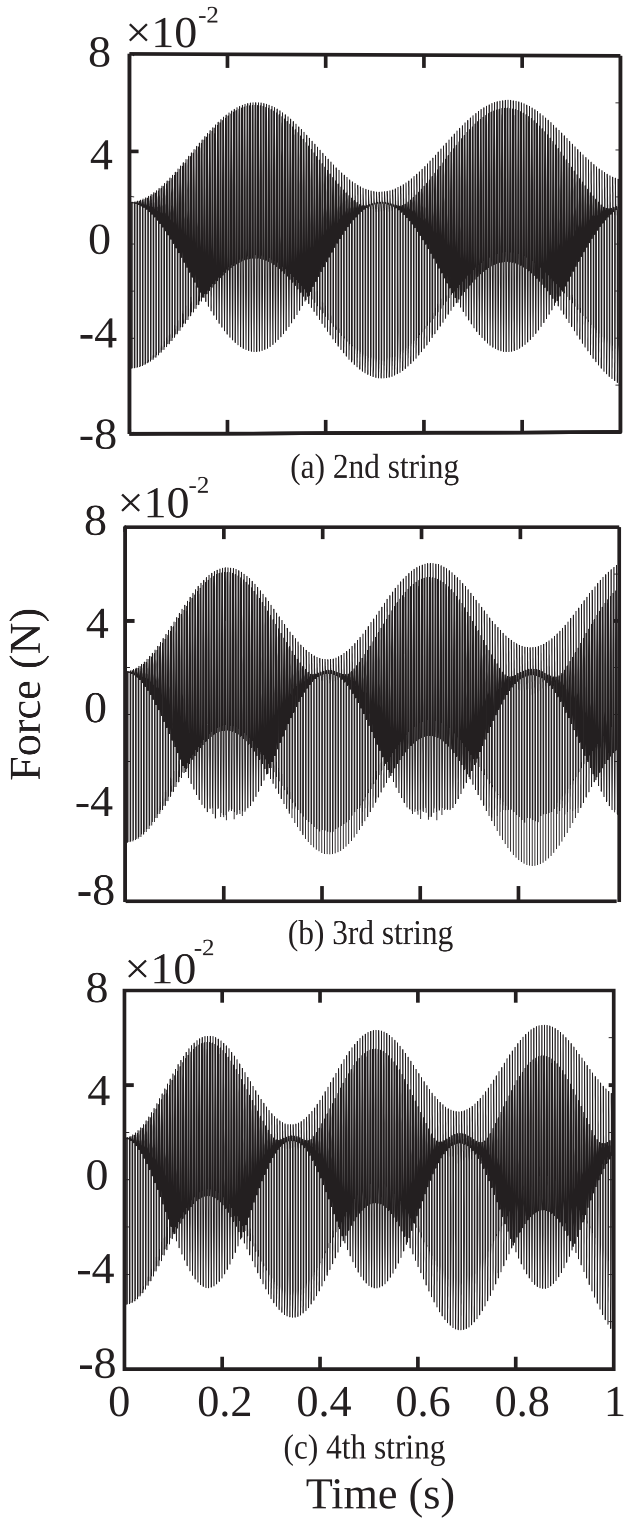

Figure 5. Force applied to bridge

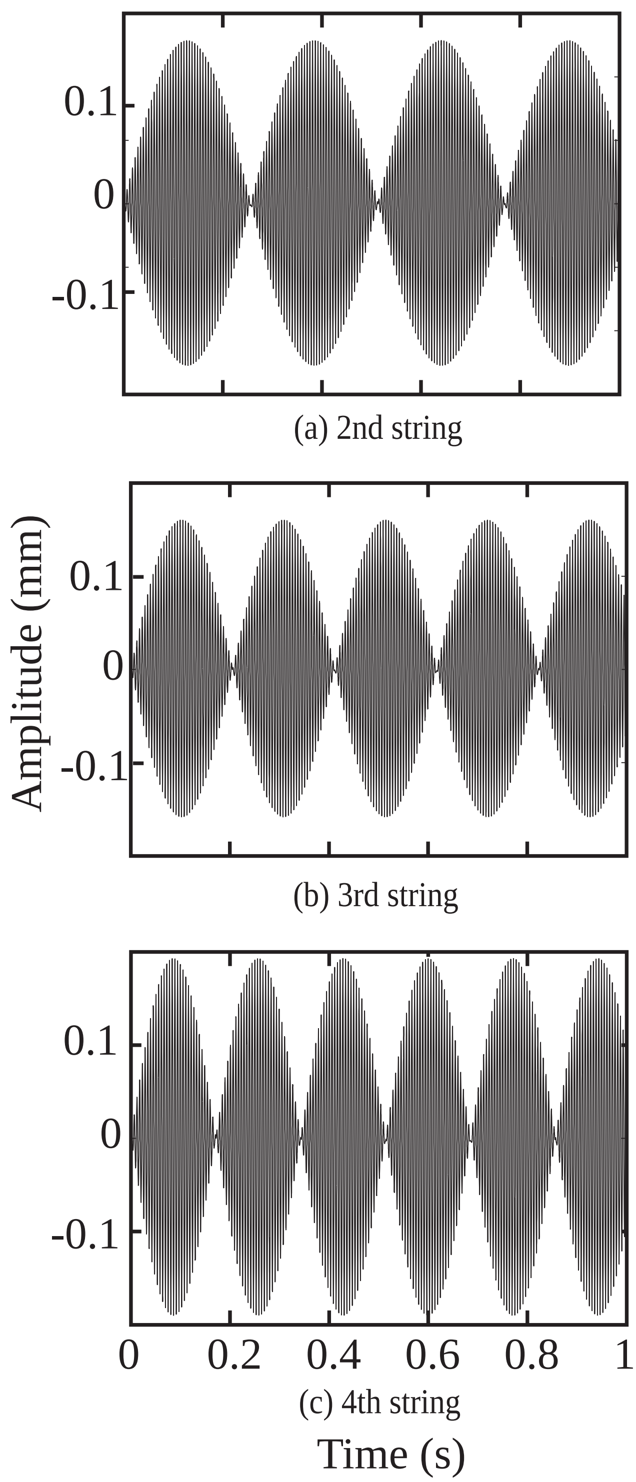

Figure 6. Bidge vibration

Table 1. Simulation condition

dynamic friction coefficient : μ_d	0.1
static friction coefficien : μ_s	1.2
time constant : τ	2.5
velocity of bow : V_b	3.7×10^{-3} m/s
pressure of bow : f_b	1.5 N
relative velocity of bow: V_r	3.0×10^{-4} m/s
critical velocity : V_c	3.7×10^{-5} m/s

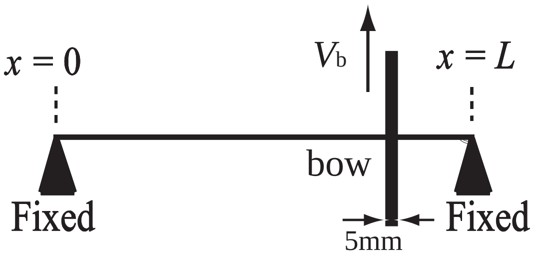

Figure 7. Model of string rubbed by bow

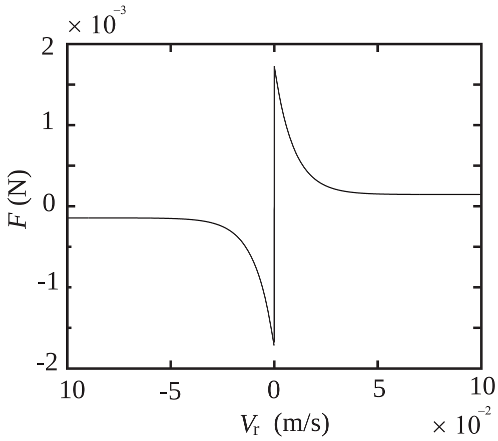

Figure 8. Input function to bow

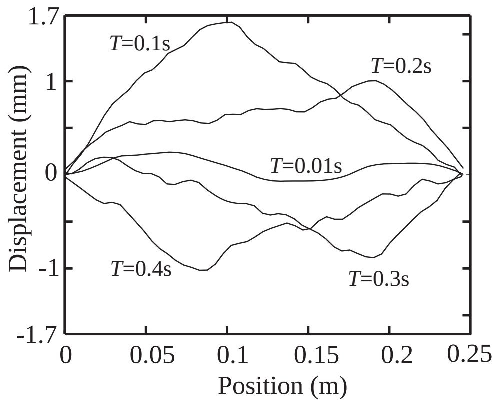

Figure 9. Motion of 4th string rubbed by bow

(Fundamental oscillation)

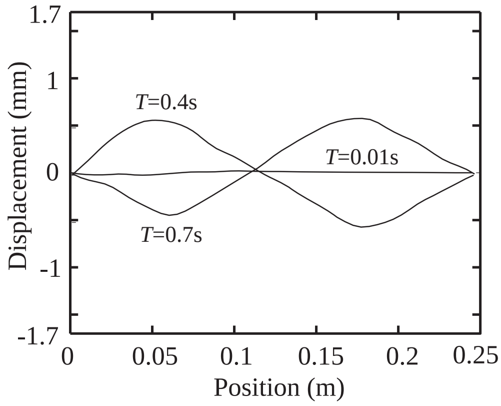

Figure 10. Motion of 4th string rubbed by bow

(Overtone oscillation)

sider information about flexure of bow. And some fine wave is observed in the string shape. One of the reasons is that the relative velocity of V_b with respect to Vs is changing and vibrating finely while the bow is sticking the string.

CONCLUSION

In this paper, we involved the bowed string model in the string-body coupled model and check up bow's parameters which can rub the bow stably. As a result, stably Helmholtz motion and surface sound were duplicated by the way to decide on parameters.

The future challenge is to derive rising time, peak size of partial tone, pitch and time jitter, and make a map including these information.

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