**Proceedings of the International Symposium on Room Acoustics, ISRA 2010**

29-31 August 2010, Melbourne, Australia

# Reflection Density and Attenuation on Stages and Rehearsal Halls

**John O'Keefe**

Aercoustics Engineering Limited, 50 Ronson Drive, #165, Toronto Canada

**PACS:** 43.55.Br, 43.55.Gx, 43.55.Mc, 43.58.Gn,

## ABSTRACT

A novel method of counting reflections and reflection density is presented. The method is borrowed from non-linear studies of fractal systems where time delayed versions of a signal are plotted against each other. In this case, the sound pressure p(t) of an impulse response function is plotted against its time derivative, dp/dt. Each resulting circle represents a reflection. Reflection counts are much lower than would be expected from theoretical predictions. In this study they're primarily in the hundreds rather than thousands. This tool was then used to study measurements on 25 stages and rehearsal halls. Previous work by the author demonstrated that sound levels on stage attenuate in a linear fashion, with regression coefficients in the range of 0.9. The hypothesis of the current study was that linear regression analysis might be a better predictor of sound levels on a stage than Revised Theory. The reasons for this are primarily the proximity of the sound source and early reflecting surfaces. In stage impulse response functions, where non-diffuse early energy is stronger than it is in the audience, the assumption of diffuse sound has been violated; hence the inappropriateness of the reverberant field theory that Revised Theory is based on. The study's hypothesis has proved incorrect, at least partially. Revised Theory is indeed a good predictor of sound levels on stage, provided that the direct sound is included. When the direct sound is eliminated, which is not uncommon on an occupied stage, linear regression is a better predictor of sound levels.

## INTRODUCTION

This paper re-visits stage acoustic measurements performed by the author over the last 20 years. The measurement procedure has been described in [1] and [2], among others, and has also been briefly summarised in the Appendix attached to this paper. A new method for counting reflections will also be introduced, as will a definition of the border between discrete and diffuse reflections zones.

## HYPOTHESIS

In [2] it was found that the behaviour of sound levels on a stage fit very well to linear regressions. The hypothesis of this study is that linear regression might explain sound levels on a stage better than Revised Theory. Revised Theory [3] is based on the assumption of spherical divergence of the direct sound and reverberant field theory. But these two conditions, notably the latter, may not always be present on a stage. If the sound field is made up mostly of discrete reflections as opposed to a diffuse reverberant field – in essence, if the basic assumptions that Revised Theory are based on have not yet been achieved – one might not expect it to perform as well as it does further away in the audience chamber. So, the question arises: How does one determine when the zone of discrete reflections ends and when the reverberant field begins? This is where counting reflections might come in handy.

## REFLECTION COUNTER

Most natural systems are of fractal dimension. That is to say, anything from a leaf to a continent does not have an integer dimension like 2 or 3. It's typically somewhere in between or sometimes fractionally above. The same is true for time signals. One of the ways that has been developed to determine the fractal dimension of a signal is to plot it against a time delayed version of itself [4]. In so doing, one might be able to determine a pattern of order in an otherwise chaotic signal. It was while applying these techniques to impulse response functions that the reflection counter was developed. One of the simplest ways to create a time delayed version of a signal, for example p(t), is to take its time derivative dp/dt. When the two are plotted against each other, the result is a series of circles spinning down the time axis. Each circle represents a reflection. To count the reflections a very simple Matlab routine was developed to count the circles and, hence, the number of reflections. One of the plots from the routine is shown in Figure 1. Note that the vertical axis is time so the beginning of the impulse response is at the bottom of the figure. By using a vertical time axis, the impulse response function really does look like a good impulse should – a Christmas Tree!

One of the more notable results from reflection counting is the vast difference between theoretical predictions and actual measurements. Bolt, et al. developed the following formula in 1950 [5]. It has since been widely accepted.

$$N = \frac{4\pi c^3 t^3}{3V} \quad (1)$$

where: N is the number of reflections
c is the speed of sound (m/s)
t is time (s)
V is room volume ($m^3$)

A comparison between the predictions of Equation 1 and measurements is shown in Figure 2. These results are fairly typical but, if anything, the measured reflection count in this room is higher than others.

There are three things to note from Figure 2:

At the right end of the graph, there are quite obviously much fewer reflections than predicted. One reason for this might be wave interference effects. Equation 1 may actually predict the correct number of reflections created in a room but it is unlikely that they could ever be measured due to interference. It is also possible that interference may be more important later in the signal, in the diffuse reverberant field.

On the left side of the graph, the measured reflection count is higher than the prediction. This is harder to explain. It could be because all the measurements were performed in the close confines of a stage. It could also be an artefact of the sound source used. Hardly a point source, it was a 12 pointed source; a dodecahedron!

Finally, note the shape of the measured curve. It certainly doesn't follow the cubic time relationship of Equation 1. But neither is it a straight line. It's more of a straight line at the beginning and the end, with a curve in the middle.

A comment is required here about the length of clean time signals available for counting. For all of the measured data in this study, the reflection count ends 10 dB above the noise floor. The temporal location of 10 dB above the noise floor was determined with a Matlab routine found in Grillon [6]. This typically resulted in a signal of 1.0 seconds or shorter. If the dynamic range of the signal allowed for 2.0 seconds of clean data, the reflection count would be twice as long. This however would not change the discrepancy between predicted and measured data, as is obvious if the curves in Figure 2 were extrapolated out to 2.0 seconds.

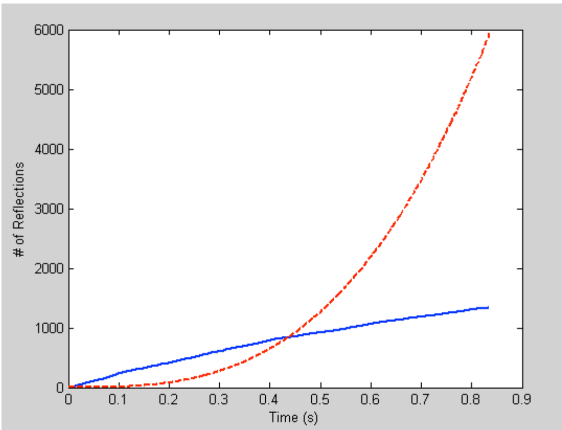


Figure 2. Comparison between measured reflection counts (solid line) and predictions from Equation 1 (dashed line). The measured data came from the Centre in the Square, source at the Soloist position, receiver at Bass.

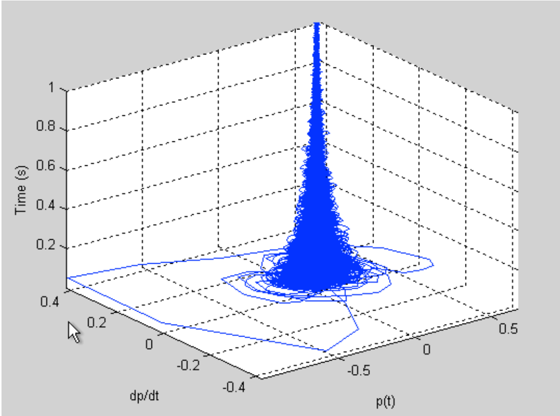


Figure 1. A 3 dimensional view of an impulse response function. The signal p(t) is plotted against time and its time derivative dp/dt. Note that the vertical axis is time. Each circle of this "Christmas Tree" impulse response represents a reflection.

## THE DISCRETE/REVERBERANT BORDER

As stated, in almost all cases, the reflection counting curves have three parts: a straight line at the beginning, a curve in the middle and a straight line at the end. This is convenient, and can be used to advantage, because we know that the first part of the impulse is made up of discrete reflections, we know that the last part is certainly diffuse reflections and, somewhere in between, there is a border between the two.

A procedure has been developed to locate the border between the discrete and diffuse reflections zones. It can be best explained by making reference to Figure 3. A linear regression is performed on the first 100 ms of the reflection curve. A straight line is generated from this. Then a linear regression is performed on the last 200 ms and from that a similar line is drawn. The intersection point of the two lines is called the Border Time between the two zones.

It could be argued, of course, that there is no single point where the discrete zone changes immediately into the diffuse zone. It is, rather, a progression from one zone to the next. But, similar to subjective parameters like D50 or C80, it's convenient to draw the line somewhere and to do so in a logical fashion.

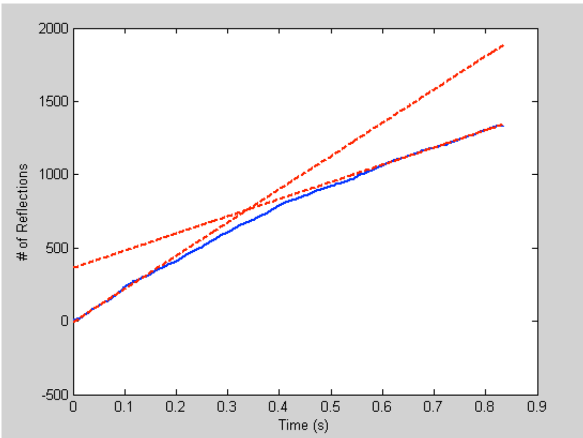


Figure 3. A larger scale version of Figure 2 including linear regressions to the beginning and end of the measured reflections curve (dashed lines). The border between the early discrete reflection zone and the late diffuse zone is located at the intersection at the intersection of the two regression lines, in this case at 337 ms.

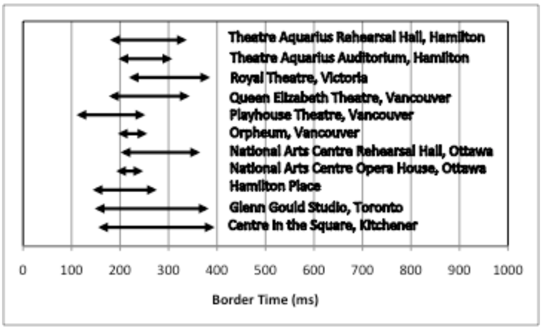


Figure 5 The range of Border Times for the 11 rooms used in this study. 10 measurements from each stage or rehearsal hall were studied here. The arrows indicate the Border Time on a given stage. For example, in the Theatre Aquarius Rehearsal Hall, shown at the top, the shortest Border Time is just below 200 ms and the longest is just above 300 ms.

## WINDOWED STRENGTH

If the hypothesis is that Revised Theory won't be accurate until the point in time when the diffuse field has been achieved, the study cannot be limited to the standard measurements such as G50 or G80. The Border Time ranges, shown in Figure 5, suggest that the diffuse field isn't achieved until after both of these.

G values were studied, instead, using a series of incrementally longer Strength Windows. Starting at G20, G40 and so on, the length of the window was increased by 20 ms each step all the way to G400. This, according to the range of Border Times shown in Figure 5, covers all possibilities. These Strength Windows were applied to measured data and then were compared to Revised Theory predictions of the same. Calculating Revised Theory of the Strength Windows was done according to Equation 2, below.

$$I_{t_1}^{t_2} = \frac{31200Te^{-0.04r/T}}{V}\left[1-(1-e^{-K_1/T})-e^{-K_2/T}\right]_{t_1}^{t_2}$$
$$K_1 = 13.82t_1 \qquad (2)$$
$$K_2 = 13.82t_2$$

where: I is the acoustic intensity (W/m$^2$)
T is the Reverberation Time (s)
$t_1$ is time at the beginning of the window (s)
$t_2$ is time at the end of the window (s)

## MEASURED VENUES

Measurements were performed in 25 venues. Venue types included concert halls, recital halls, proscenium arch theatres and rehearsal halls. For consistency, only 12 of the halls have been reported here. Reasons for this are explained in Appendix A. Some venues also had to be excluded because information on room volume was not available. In several venues, the availability of flexible acoustics encouraged multiple sets of measurements. Only one of the sets per room is reported here. The list of venues is shown in Table 1.

**Table 1**

| Venue | City[1] | Vol. (m$^3$) |
|---|---|---|
| Centre in the Square[2] | Kitchener | 16,632 |
| Glenn Gould Studio | Toronto | 4,587 |
| Hamilton Place[3] | Hamilton | 29,907 |
| NAC Opera House[3] | Ottawa | 37,452 |
| NAC Rehearsal Hall | Ottawa | 1,398 |
| O'Keefe Centre Rehearsal Hall[4] | Toronto | 1,918 |
| Orpheum[5] | Vancouver | 19,200 |
| Playhouse Theatre | Vancouver | 6,533 |
| Queen Elizabeth Theatre[6] | Vancouver | 32,452 |
| Royal Theatre | Victoria | 15,240 |
| Theatre Aquarius | Hamilton | 12,526 |
| Theatre Aquarius Rehearsal Hall | Hamilton | 1,822 |

1. All venues are in Canada
2. Concert mode configuration
3. Theatre mode configuration
4. Now known as the Sony Centre
5. Prior to the 1995 renovation
6. Prior to the 2009 renovation

## RESULTS

A fairly typical example of the comparison between measured data and predictions is shown in Figure 4. The data is taken from Hamilton Place. Revised Theory is seen to be a slightly better predictor than the linear regression. The rms error between measurements and predictions is 1.91 dB for Revised Theory and 2.96 dB for linear regression. This contradicts the original hypothesis of the study.

One explanation why this might be is that, at these close distances, the direct field might dominate the reverberant field, depending, of course, on how close the receiver is to the source. To study this further the direct component was removed from both measured and predicted data. For the measured data, the first 5ms of the impulse response function was removed. Two versions of the Revised Theory calculation were performed. The first simply removed the direct component from the calculation. The second, using Equation 2, removed both the direct sound and the first 5ms. It was thought that the latter calculation was a more appropriate comparison with the measurements which, by necessity, had to have both the direct sound and the first 5ms removed.

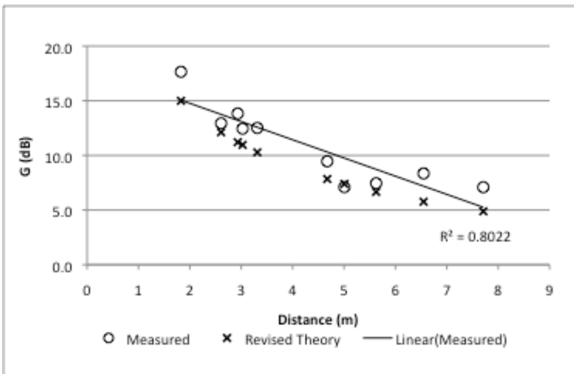


Figure 4. Typical comparison between measured data (circles) Revised Theory (x) and linear regression (line). The measurements were performed at Hamilton Place.

To continue with the Hamilton Place example shown in Figure 4, the rms error between measurements and linear regressions when the first 5ms have been removed is 1.61 dB. The same comparison for Revised Theory without the direct component is 5.94 dB. For Revised Theory without both the direct sound and the first 5 ms, the rms error is 6.11 dB. To summarise, without the direct component, linear regression is the better predictor. With the direct sound, Revised Theory is better.

The rms errors associated with Figure 4 (i.e. with direct sound) were described as typical – and they are. The same errors for Revised Theory without direct sound were not typical; they were on the high side. Please see Figure 6. The data point from Hamilton Place is indicated with an arrow.

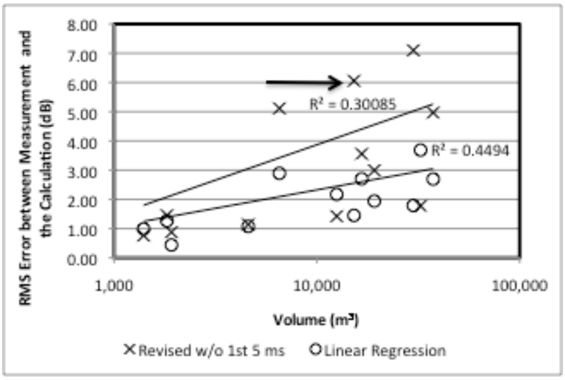


Figure 6. RMS errors between calculations and measurements plotted against Volume. In these comparisons, the first 5ms of the measured data and the Revised Theory have been removed. RMS errors for linear regression prediction shown in circles, RMS errors for Revised Theory shown with x's. The data point for Hamilton Place is indicated with an arrow.

As the data analysis was developing, it appeared that larger volume rooms had larger rms errors. Hence the development of studies such as that seen in Figure 6. And while this is true in some cases, the correlations are by no means conclusive. What Figure 6 does demonstrate however is that (i) there is a trend and (ii) Revised Theory, at least in few cases, has rms errors comparable to linear regression.

Still, the question remains, with or without the direct sound, does Revised Theory need to be in a diffuse sound field to make accurate predictions? This was studied at length using graphs such as the one shown in Figure 7. This one comes from Centre in the Square and, again, is representative of the other 11 venues. It is for data that *includes* the direct sound. The graph requires some explanation.

The circles and x's indicate the average rms error between measurements and predictions for linear regression and Revised Theory respectively. The average is taken over all 10 measurement locations. Each rms error data point is for a Strength Window, as defined above. Thus, for example, the point for 100 ms corresponds to G100.

The range defined by the arrows in the middle of the graph comes from Figure 5. It indicates the range of Border Times at all 10 measurement locations on stage. The Border Time tells us when the diffuse sound field starts. Thus, the earliest time that a diffuse field is achieved on the Centre in the Square stage is 154 ms and the latest is 395 ms. (The vertical position of the arrow is insignificant, it was arbitrarily placed in the middle of the graph for the sake of clarity.)

The circles tell us that, when the direct component is included, linear regression predictions are always worse than Revised Theory. That is, their rms errors are always higher, regardless of whether diffuse field has been achieved or not.

The x's tell us that the accuracy of Revised Theory improves as the time signal progresses and, indeed, achieves its optimum accuracy around the same time that the diffuse field has been established. (Remember that the vertical position of the arrow is insignificant.)

Staying with Centre in the Square, the same exercise was performed for data without the direct sound. Please see Figure 8. Revised Theory takes longer to achieve its optimum accuracy and, in this case, it happens only after the diffuse field has been established. Thus, in the absence of the direct sound, part of the hypothesis is true. In the absence of the direct sound, Revised Theory apparently does indeed require to be in the diffuse field. Note also in Figure 8 that the accuracy of linear regression is always better than Revised Theory. Reference to Figure 6 shows, however, that this is not universal. Revised Theory is more accurate in 3 of the 11 cases.

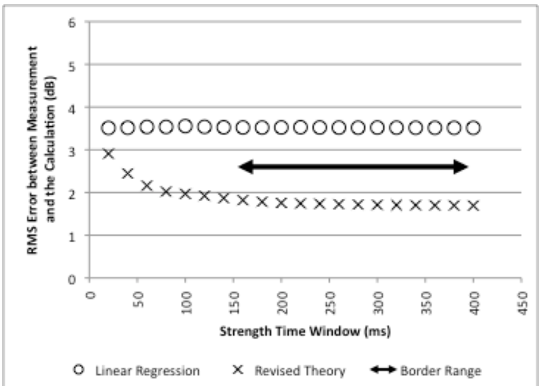


Figure 7 Comparison of rms errors with the Strength Window described in Equation 2. Direct sound *included* in both the measurements and predictions. The arrow indicates the range of Border Times for the 10 measurements performed on stage. Revised Theory reaches its optimum accuracy about the same time that the diffuse field is achieved on stage.

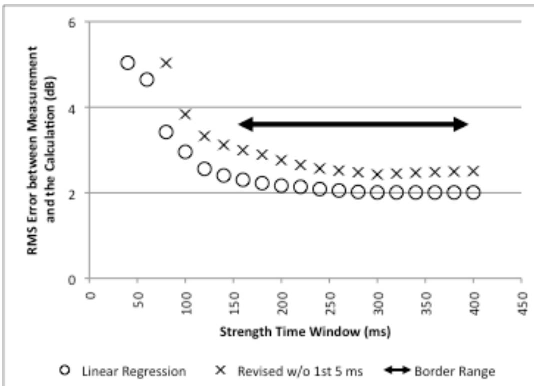


Figure 8 The same experiment as Figure 7 except without the first 5 ms of the measurements and predictions, i.e. *without* the direct sound. Both linear regression and Revised Theory achieve their optimum values after the diffuse field has been established, although it takes Revised Theory a little longer.

## CONCLUSIONS

A new method of counting reflections has been introduced. Measured reflection counts were found to be much smaller than the accepted method of theoretical prediction. The reason for the discrepancy appears to be interference effects, which the theory does not take into account.

A method has also been introduced to define the boundary between the early discrete reflection zone and the late diffuse reflection zone. It is referred to as Border Time. For the set of 12 stages presented here, the Border Times range from just over 100 ms to just under 400 ms.

The hypothesis of this study was that linear regression might be a better predictor than Revised Theory for sound level attenuation on a stage. This has proved correct when the direct sound component is excluded and incorrect when the direct sound is included. When the direct sound is removed, Revised Theory relies on an assumption of a reverberant or diffuse sound field. This study has shown that, in most cases when the direct sound is excluded, Revised Theory does not achieve its optimum accuracy until the diffuse field has been established, i.e. between 100 ms and 400 ms, depending on the signal.

In the preliminary stages of concert hall design, therefore, Revised Theory is, indeed, the appropriate tool to calculate stage levels on the *unoccupied* stage. Dammerud, however, has documented the very significant reduction in sound levels introduced by the prescence of an orchestra on stage[9]. Thus, later in design, scale or computer models should be used to study an occupied stage, notably those locations that do not receive the direct sound component.

## APPENDIX

As mentioned in the Introduction, the data used in this study was collected over a period of 20 years. The sound source for most of the measurements was a dodecahedron with 75 mm drivers. Source height was consistent at 1.1 m. Some later measurements were performed with a larger dodecahedron (with 100 mm drivers) but have not been presented here. Receiver locations were divided up into Self and Other. In each set of measurements there were five Self locations corresponding to Soloist, Violin, Viola, Horn and Bass. The source/receiver pairs for the Other measurements were generated from the ten combinations of the 5 Self measurement locations. For example, a Self measurement would be performed at the Soloist location and the Other measurements between Soloist and Violin, Viola, Horn and Bass respectively. Thus, for each stage there were 15 measurements: 5 for Self and 10 for Other.

Self measurements were originally measured at a 1 m distance from the sound , following Gade's suggestion [7]. Not much later, Self measurements began to be measured at a distance of 0.5 m, following Naylor's practice [8]. Some stages were measured at both Self source/receiver distances but most were performed at 0.5 m. All of the data used in the current study had Self distances of 0.5 m.

Over the course of 20 years, various data acquisition systems were employed but they all had these same things in common: (i) 12 mm Type 1 omni-directional microphones, (ii) MLS excitation signals, (iii) most of the data reduction was performed with computer routines written by the author. All of the data reduction in this study was performed with updated versions of those routines. Although data was studied in the 250 Hz to 4 kHz octave bands, only the 1 kHz octave is reported here.

The dodecahedron was not originally calibrated for Strength (G). To overcome this problem, the Self measurements were used as a form of "monitor microphone", the equivalent of the one used by Dammerud next to a scale model spark source [9]. Dammerud has since elaborated on this procedure for full scale buildings in reference [10]. Calibrated G levels are obtained as follows:

$$G_{other} = 10\log_{10}\left[\frac{E_{Other}\left(0 - t_{window}\right)}{E_{Self}(0 - 5ms)}\right] + K \qquad \text{(A1)}$$

where: $G_{other}$ is the Strength at Other Receivers (dB)
$E_{other}$ is the acoustic energy measured at Other
$E_{self}$ is the acoustic energy measure at Self
$t_{window}$ is the length of the time window (s)
K is a correction factor extrapolating the Self measurement to the standard 10 m free field location. For a Self measurement at 0.5 m, K = 26 dB, at 1.0 m it is 20 dB.

Note also that in the denominator of Equation A1, a floor reflection free integration period of 5 ms has been applied, which is appropriate for a 0.5 source/receiver distance. For a 1 m distance, the floor reflection free integration time is 3 ms. All 5 Self measurements were used for calibration. All the measurements reported here are the remaining 10 Other measurements.

## REFERENCES


1 J. O'Keefe, "On the Sensitivity of Stage Acoustics Measurements" *Proc of the Wallace Clement Sabine Symposium* 223-226 (1994)
2 J. O'Keefe, "Acoustical Measurements on Concert and Proscenium Arch Stages" *Proc of IOA* **17**, Part 1 137-143 (1995). See also http://aercoustics.com/innovations-rd/our-technical-papers.
3 M. Barron, L.J. Lee, 'Energy relations in concert auditoriums. Part I', *J. Acoust. Soc. Am.* **84**, 618-628 (1988)
4 H.D.I. Abarbanel, *Analysis of Observed Chaotic Data* (Springer, New York, 1996)
5 R.H. Bolt, P.E. Doak and P.J. Westervelt, "Pulse statistics analysis of room acoustics" *J.Acoust. Soc. Am.* **22**, 328-340 (1950)
6 V. Grillon, "Auralisation dans les Maquettes: Traitement des Response Impulsionelles", Ph. D. Thesis, Universite du Maine (1995).
7 A.C. Gade, "Subjective Room Acoustics Measurements with Musicians", Ph.D. Thesis, Report No. 32, Technical University of Denmark (1982).
8 G.M. Naylor, 'Musical and Acoustical Influences on the Achievement of Ensemble', Ph.D. Thesis, Herriot Watt University (1987)
9 J.J. Dammerud, "Stage Acoustics for Symphony Orchestras in Concert Halls", Ph.D. Thesis, University of Bath (2009).
10 J.J. Dammerud, "In-situ calibration of the sound source level for measuring G", http://home.no/jjdamm/stageac/docs.html