



Rays or Waves? Understanding the Strengths and Weaknesses of Computational Room Acoustics Modeling Techniques

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ABSTRACT

The pros and cons of wave-based and ray-based room acoustics modeling methods are overviewed. Links between image-source, boundary element and radiance transfer methods are presented. The emphasis is on the main bottlenecks of each method. Accuracy, computational performance and applicability of the output of each method determine for modeling which part of the response they can be used. It is proposed that wave-based methods are used for low frequencies, image source methods for the early part of the room response for mid- and high frequencies, and radiance transfer methods for the rest of the response.

INTRODUCTION

Computational room acoustics modeling software can ease the task of acoustic designers by providing both numerical acoustic data and auralizations which help to evaluate the quality of a room design. However, the usefulness of the computer simulations depends on their accuracy. Experienced acousticians can often detect when the simulations go wrong. Still, ideally the modeling methods should produce reliable results.

It is useful to compare the strengths and weaknesses of different computational room acoustic modeling methods. Especially, specifying the underlying assumptions behind each method helps to determine the physical accuracy of the simulations. On the other hand, the computational efficiency of the methods is another important criterion for choosing the method to use. Such a discussion may help to identify for which purposes each method is best suited. In addition, that could lead to the development of a hybrid method which combines the best qualities of each approach, ideally resulting in an algorithm that is both computationally feasible and physically accurate.

There are two main approaches in room acoustic modeling. The more accurate one is based on solving the actual wave equation numerically. Techniques using this approach are called wave-based. They are all computationally intensive and even so that the workload grows rapidly as a function of the frequency. Thus the wave-based methods are most suitable for low frequencies. These techniques typically discretize either the space or its bounding surfaces to small elements and model interactions between them. For this reason, these techniques are occasionally called as element-based methods.

The other approach is based on geometrical acoustics in which sound is supposed to act as rays and the wavelength of sound is neglected. This means that all the wave-based phenomena, such as diffraction and interference, are missing in those methods whereas in the wave-based methods they are modeled inher-

ently. These methods are called ray-based since they often use some kind of rays or particles that are reflected at the surfaces of the room. The room acoustic rendering equation is a unifying framework that covers all these techniques such that each of them can be considered as a special case of the equation [1]. One major difference to wave-based methods is that they typically compute only sound energies instead of sound pressure or particle velocity used in the wave-based methods.

Acoustic radiance transfer method is one of the newest techniques based on geometrical acoustics [1]. It is kind of a hybrid of both approaches as it is based on the same assumptions than ray-based methods, but uses elements typical in wave-based methods. In acoustic radiance transfer, it is possible to have arbitrary reflection functions. Another interesting feature is that in practice the computational load does not depend on the number of listeners since the solution is computed for the whole space at once similarly than in other element-based methods.

TECHNIQUES

The research literature in room acoustics modeling is extensive and thus it is not purposeful to review all the methods introduced. An interested reader may consult a survey on room acoustics modeling [2]. In the discussion below, three approaches have been reviewed. These are image source methods [3, 4], boundary element methods [5], and radiance transfer methods [1]. Image source methods serve as examples of ray-based acoustics, although other widely-used methods exist, such as ray-tracing [6]. Boundary element methods represent the wave-based methods, which include also finite element methods [7] and finite-difference time-domain methods [8, 9]. Finally, radiance transfer methods include acoustic radiosity methods [10] as special cases where the reflection pattern is limited to Lambertian diffuse reflections.

Complete room acoustics modeling systems often use a combination of the different room acoustics modeling methods [11–

14]. Similarly, auralization systems use different methods for different parts of the modelled responses [15]. These observations reflect the fact that no single room acoustics modeling method can efficiently model the whole impulse response both in full length in the time domain and for full band in the frequency domain. The purpose of the discussion below is to compare the limitations of the different room acoustics modeling methods to determine in which cases and for what part of the response they work best.

Image Source Methods

When a sound field emitted by a source is reflected at an acoustically-hard infinite flat surface, the reflected field is the same as the field produced by a source that is mirrored at that plane. This allows reconstructing the whole field as a sum of the fields of the original source and the mirrored source, i.e. the image source. However, in realistic cases the surfaces are not infinite and flat. Fortunately, the same principle can be utilized in the case of regular rectangular rooms. An image source can be created for each wall and adding their contributions to the field produced by the original source correspond to the field that includes direct and once-reflected sound.

Higher order reflections can be taken into account by creating image sources for the existing image sources. Unfortunately, the number of image sources quickly explodes with increasing reflection order. In the case of a rectangular room, however, an exact solution can be found, which makes this computational model still applicable.

The situation becomes more difficult, when the modelled room has corners with an opening angle that is not an integer fraction of π . The total field is not the same as the sum of the direct field and the fields produced by the image sources. The difference consists of diffraction effects, which have to be accounted for if accurate results are desired.

The diffraction effect is stronger at low frequencies where the wavelength is longer than or comparable to the dimensions of the reflecting objects. But at higher frequencies, the diffraction effects are negligible and the image source model produces plausible results. The errors due to diffraction phenomenon can be corrected by adding a separate diffraction model. [16–21]

Still another issue is the visibility of the source and the image sources and the validity of the image sources. In non-convex rooms the direct and reflected sound paths can be occluded and, to produce correct results, the occlusions must be tested for, most often with the help of ray tracing. In addition, the image sources are not necessarily valid. This is the case when the reflection path constructed using the image source does not actually intersect the surface corresponding to the image source. This requires an additional intersection test. These tests add to the computation burden of image source methods. Figure 1 shows reflections modelled with image sources in a model which is non-convex. The visibility computations can be optimized by using the beam tracing algorithm in which the visibility is precomputed and stored as a beam tree [22, 23].

Boundary Element Methods

In boundary element methods the goal is to numerically solve an appropriate wave equation on the surfaces of the modelled geometry. When the sound field is known on the surfaces it can be calculated at any point in space. The field quantities used are typically pressures and particle velocities. The surfaces are divided into elements on which the acoustic field can be described with the help of some basis functions. Then it is possible to calculate the interactions between the elements according to the

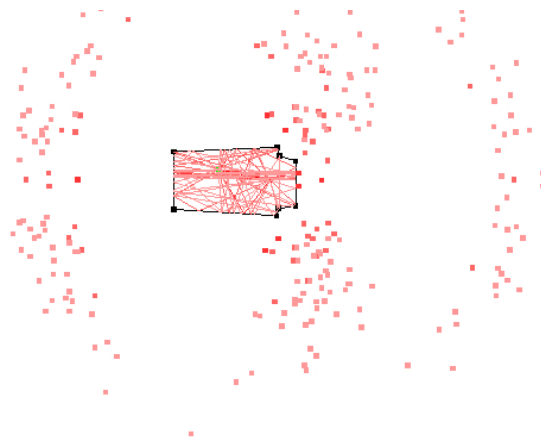


Figure 1: In image source methods, the source radiation reflected from surfaces is represented by image sources which are the sources reflected at the surfaces. The image sources themselves can be further reflected to create higher order image sources. Each reflection path can be traced by beginning from the listener point and tracing towards an image source until a surface is hit. If the surface is the one which was used when creating the image source, the segment of the path is valid. Then the tracing is continued similarly in the direction of the (image) source from which the just-utilized image source was created. The result is a collection of reflection paths.

wave equation and the boundary conditions which are imposed on the surfaces. The interactions between each pair of elements can be collected into a matrix.

Given the initial conditions, the matrix can be used in solving the steady state situation for any frequency. The solution process involves solving a linear system of equations which can be done quite efficiently with appropriate algorithms. However, as the number of elements grows, the computational demands grow rapidly, usually in relation to the third power of that number. On the other hand, the number of elements required depends on the modelled frequency. Typically, 5-10 elements per wavelength are required. Thus, boundary element methods become impractically slow at higher frequencies.

The strength of boundary element methods is that the solution is physically correct even at low frequencies and wave phenomena such as diffraction are modelled. Boundary element methods are usually used in the frequency domain and the solutions are for point frequencies. If a wide-band time-domain solution is desired, one approach is to produce solutions for each frequency separately and then apply the inverse discrete Fourier transform. However, this method is quite cumbersome and involves redundant work.

A time-domain iterative boundary element method has been suggested [24]. The solution is formulated in the time-domain and the solution is calculated one time step at a time. Since the speed of sound is assumed constant, the sound propagates the same distance at every step. Thus, it is possible to determine contributions from which elements reach which elements in a given number of time steps. Theoretically, the time-iterative boundary element method is faster than the frequency domain method. Figure 2 shows one interpretation of the time iterative process. However, in practice, the time-iterative method suffers from numerical instability issues, and the size of the elements must be chosen according to the highest modelled frequency which means futile work at lower frequencies.

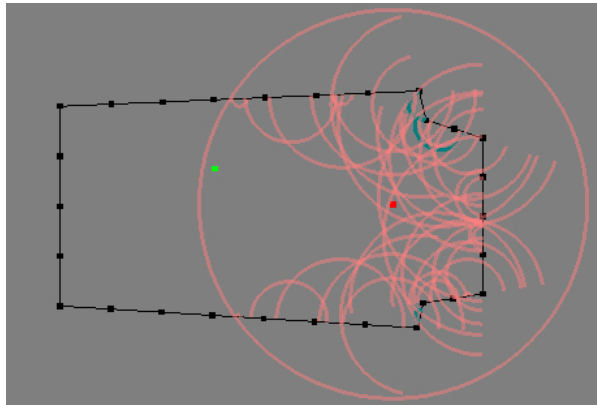


Figure 2: In boundary element methods, the surfaces are divided into elements and interactions between them are used for creating a linear system of equations which yields the solution. The time iterative approach is illustrated, where the elements can be interpreted as secondary sources.

Radiance Transfer Methods

Radiance transfer methods are element-based methods such as the boundary element methods, but the acoustic quantity that is modelled is energy. Thus, phase information is not modelled. The same assumption is made than in the ray-based methods that the wave-based effects are negligible, so the method is most accurate at higher frequencies. If the surface elements are small enough, it is safe to assume that the intensity of the sound does not vary much over the element. Then it is possible to derive same kind of interaction matrix between elements than in the boundary element methods. Now the matrix elements describe what portion of the energy leaving one element reaches another. Specifically, in the method presented in [1], it is possible to account for reflections into which different amounts of energy are distributed in different directions. The directional space is also divided in parts for each element. Thus, one interaction actually represents the energy leaving one patch in certain direction and then gets reflected from another patch to another direction. In the following, this specific method is discussed in more detail.

The actual solution process is time-iterative. Starting with the energy sent from the sound source to the elements, the energy is transferred from element to element by always choosing the element with the highest unpropagated energy and transferring that energy to other elements which are visible. The propagation time is computed and time-dependent energy responses are stored for each outgoing direction at each element. The iterative process is repeated as long as the energy transferred is significant. Eventually, the energy is collected from the elements to a receiver to obtain the time-dependent energy responses of the room for the given source at the receiver position. The responses can be computed for any receiver position without repeating the iterative energy propagation process. Figure 3 illustrates the results of the process.

Obviously, the computational demands do not increase as the response is modelled further in time, since the number of elements is constant throughout the process. Thus, computing the late reverberation is efficient. On the other hand, the size of the elements and the directional resolution affect the accuracy of the early reflections.

Yet, another issue is the memory consumption. Typically, for decent quality responses, hundreds or thousands of elements are required, and for each element dozens of directions must be used to preserve the directional properties at the reflections. Most

importantly time-dependent energy responses require a decent time resolution, meaning thousands of samples. Thus, the memory consumption quickly approaches hundreds of megabytes or even gigabytes.

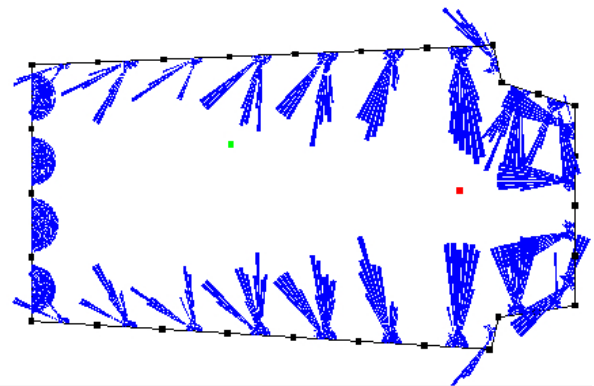


Figure 3: Surface is divided into elements and each element contains information on how much energy has been reflected in each direction. Starting from the source, the energy has been spread at the surfaces and reflected until the process has converged. The figure shows only the total amounts of energy, but in reality time-dependent energy responses are stored at the data structures representing the elements. This way the temporal structure of the acoustic energy is properly modelled.

LINKS BETWEEN TECHNIQUES

Relation of image source method and acoustic radiance transfer

The methods described above have similarities. Both image source methods and the acoustic radiance transfer method are based on the assumptions of geometrical acoustics, i.e. that sound propagation can be described as rays traveling in straight paths. This leads to common limitations such as lack of diffraction and omission of other wave-based effects at lower frequencies. Since the phase information can be incorporated in image source methods, it is possible to include some wave-effects such as interference in it, but since such effects are most significant at low frequencies where the results are inaccurate, the benefits of utilizing phase information are diminished. On the other hand, in the acoustic radiance transfer method, the phase information is totally ignored, since energy does not have a phase. When the sound field is almost diffuse, the interference effects are negligible and the energetic model is sufficient. The result of the acoustic radiance transfer consists of time-dependent impulse responses, instead of impulse responses as in image source methods, and thus a conversion is required if the responses are to be used in auralization purposes. If computation of room acoustical parameters is the goal, the energetic response is useful as such.

Relation of boundary element method and acoustic radiance transfer

Similarities between boundary element methods and the acoustic radiance transfer method are also obvious. In both methods the surface of the modelled room is divided into surface elements and interactions between them are calculated. The resulting linear systems of equations are similar. Thus, the solving algorithms have similar properties. In the case of boundary element methods, a direct solution in the frequency domain is possible by solving the linear system of equations. On the other hand, the original acoustic radiance transfer method is a time-domain algorithm and the time-iterative solution strategy is most fitting. In this respect the acoustic radiance transfer method resembles

the time-iterative boundary element method [24]. However, the numerical instabilities are easier to control in the case of the acoustic radiance transfer method. Since energy is transferred, the energy conservation principle can be enforced by scaling the energy received by other elements to exactly match the energy sent from a source element. If this is not done, even minimal changes in energy levels tend to accumulate quickly and make the process unstable. In the case of the time-iterative boundary element method such an energy conservation condition is difficult to formulate and the process will most likely suffer from numerical instabilities.

Relation of image source method and boundary element method

The link between image source methods and boundary element methods is not as apparent, but it can be seen via wave field synthesis and preceding work by Berkhout [25, 26]. Wave field synthesis is a spatial sound reproduction technique in which each virtual sound source is reproduced by a number of real loudspeakers on a given surface. Starting with equations derivable from the Helmholtz-Kirchhoff integral theorem, driving signals can be derived for a loudspeaker array for representing a source that is behind them. Then it is suggested that reverberation can be artificially simulated by using the loudspeaker array to represent image sources which are computed with an image source method. Now, this idea can be brought further by considering the same setup in a virtual environment. The loudspeakers are replaced with ideal dipole sources. Then an image source method can be used for constructing image sources of those sources and the wave field theory can be used for representing those image sources with the loudspeaker array. This leads to linking every virtual 'loudspeaker' to each other. Incidentally, the resulting equations are the same as in the boundary element methods when using zeroth-order basis functions on the elements. The virtual 'loudspeakers' correspond to the elements.

COMPARISON OF TECHNIQUES

The different techniques are compared in the following from three points of view. The accuracy of the technique is obviously important. On the other hand, the computational performance of a room acoustics modeling method affects its applicability. Another thing that affects the applicability of each technique is the output format of the algorithm.

Accuracy

The accuracy of the geometrical acoustics methods, such as image source methods and the acoustic radiance transfer method, is limited by the assumption that sound travels along straight lines or rays. Thus some wave-based phenomena cannot be modelled. A classical example is lack of diffraction. However, this lack of accuracy is most severe at low frequencies, while at high frequencies, the geometrical acoustics is sufficiently accurate for most purposes. It is worthwhile to note that the limiting frequency between low and high frequencies is determined by how detailed the model is since the assumption of geometrical acoustics is valid only when the wavelength of sound is small when compared to surface dimensions of the model. In practice this means that using a more detailed model won't necessarily improve the result accuracy but on the contrary may degrade the quality of acoustic simulations unless an appropriate diffraction model is applied. Another factor affecting the simulation accuracy in geometrical acoustics is the ability of the technique to find all the specular reflection paths. The image source method is guaranteed to find all of them whereas ray-tracing and some approximate beam-tracing techniques only statistically sample the path space. With infinite amount of rays the results are the same, but the less rays are used the more likely it is that some

reflection paths will be missed.

Boundary element methods and other wave-based methods can be physically very accurate, but only as long as proper boundary conditions can be defined. Also the number of elements per wavelength must be above a certain number, typically 5-10 to yield results which contain the phase information correctly. In theory, it should be sufficient to have only two elements per wavelength, but there is an inherent dispersion error in the wave-based methods that renders the modeling results unusable already below the Nyquist frequency. The dispersion error grows as a function of frequency and the required size of the elements is determined by how large dispersion error is acceptable. However, there is an alternative new technique to the traditional wave-based modeling called adaptive rectangular decomposition (ARD) [27]. In that technique the elements are much larger than in the other wave-based methods. In ARD, the numerical errors are of different type and there is no dispersion error at all. The technique is still under development, but in any case it is a promising new approach.

One essential factor affecting the accuracy in both the wave-based and ray-based techniques is handling of the boundaries. In the first models there were only specular reflections, and if that is accompanied by a diffraction model, it should be sufficient to model all the locally reacting surfaces. However, the assumption of local reaction is not always true, especially at low frequencies, and for this reason a diffuse reflection model is often needed as well. In addition, diffuse reflections provide a way to fake missing diffraction in the models.

At higher frequencies some model for air absorption is needed as well. In geometrical acoustics it is most often easy to introduce (see, e.g. [15]) whereas in the wave-based methods it is more complicated. However, in the frequency range where the wave-based methods are typically used this phenomenon is not that crucial. But, in the future, if those models try to reach even higher in the frequency with increased computing power of modern computers, air absorption should be included in the models.

Computational Performance

Accurate geometrical acoustics techniques, such as the image source methods, find all the specular reflection paths, i.e. image sources, in the model. They can be efficient for modeling early reflections, but for higher-order reflections the number of image source grows exponentially, thus the order of reflections must be limited for practical purposes. With the beam tracing optimization, the situation is a little better, but the problem of exponential growth remains.

Techniques that model energy transfer in the scene can be efficient for the whole response. In the acoustic radiance transfer method, the computational demands remain the same for each moment of time in the response. Ray-tracing and other similar techniques are also efficient at the late part of the response, since the number of rays usually either remains constant or grows conservatively in relation to the reflection order. Thus, these techniques work for modeling the late part of the response.

Diffraction computation in the ray-based models is typically computationally expensive, and if an accurate result is to be obtained most of the time can be easily spent on computing the diffracted components. For this reason, it is typical to use some approximative method for diffraction such that computational cost and accuracy can be balanced such as in the i-Sound real-time auralization system [28].

Wave-based methods typically use a constant number of elements and thus the computational demands stay the same for the whole response. In the typical finite-element and boundary-element techniques computation is performed in the frequency domain such that in one computation the whole response for one point frequency is performed whereas in the finite-difference time-domain techniques the computation is done in time steps such that in one iteration the solution progresses one time step for all the frequencies. Unfortunately, the computational costs are high, since in many cases a huge linear system of equations must be solved. The size of the system is dependent on the frequency since as noted earlier, there must be a certain number of elements per wavelength. The system might require amount of memory that is proportional to the square of the number of elements and the computation time is proportional to the third power of the number of elements. In the FEM the number of elements is much larger than in the BEM, but the resulting matrix to be inverted is sparse in the FEM whereas in BEM it is much more densely populated. In the FDTD simulations, the memory requirements are smaller, but it still suffers from the computational load growing as a function of the fourth power of the frequency. In practice, high frequency modeling is not practical with the wave-based methods.

Applicability of the Output

In image source methods, the output is usually a collection of reflection paths. These can be easily converted into room impulse response or to time-dependent energy response. The former can be beneficial in auralization applications and the latter when computing room acoustic parameters. The reflection paths themselves can also often be directly auralized and some room acoustic parameters require the information of arriving sound direction for which purpose the reflection paths are ideal. The responses are full-band responses.

In the acoustic radiance transfer method as well as in ray-tracing, the output is a time-dependent energy response whose samples are approximately proportional to the squares of the samples of the impulse response. This kind the response is useful for room acoustical parameter calculation, especially since the directions of the arriving sounds are known. On the other hand, to be applicable to auralization, the phase information has to be re-created, which is not possible. The sound intensity in an auralization is likely to be approximately correct, but since the phase information is wrong, at least the early reflections might sound incorrect. Fortunately, the directions of the arriving sounds are usually available when using these techniques, which might perceptually compensate for the incorrect phase.

Boundary element and finite element methods typically work with one frequency at a time. Often the processing is performed in the frequency domain and the result is a steady-state situation. Such results are difficult to use in room acoustic parameter calculation and auralization, but are suitable for modal analysis of the space, for example. Time-domain wave-based methods exist, but even then, one problem is that the direction of the arriving sound is more difficult to approximate than in the ray-based methods. Room acoustic parameters that are dependent on direction are thus more difficult to obtain.

Summary

It is useful to compare different room acoustic modeling approaches to see at which part of the response they work the best. Table 1 compares the performance of the methods at different frequencies and temporal locations in the response. Boundary element methods are best at low frequencies whereas the geometrical methods work better at higher frequencies. Boundary element methods and image source methods can model the

early part of the response accurately, although diffraction effects are missing in image source methods, and boundary element methods are computationally demanding. The acoustic radiance transfer method and boundary element methods work better at the late reflections. Also note that according to Schroeder the late part of the response can be modelled statistically for sufficiently high frequencies when the modelled space is large enough [29]. Figure 4 further illustrates where each of the methods performs best.

The actual frequency in which the approach has to be switched from wave-based to ray-based depends on the complexity and size of the model, such that in small rooms the wave-based methods can be used in the mid-frequency range whereas in large concert halls those models are limited only to the low-frequency region with current computers [30, 31].

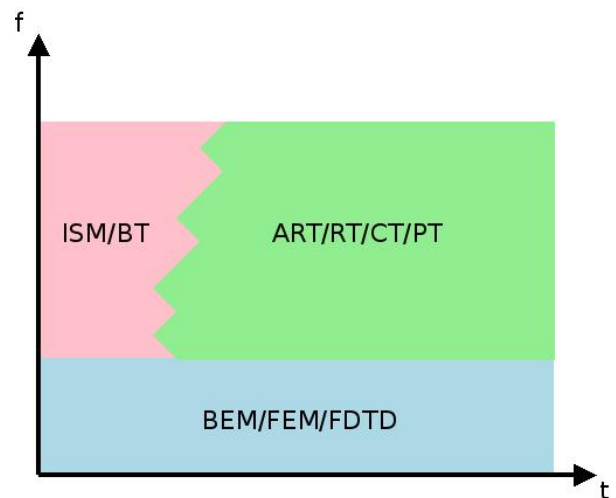


Figure 4: Different methods at different frequencies and temporal parts of the modelled response. Adapted from [32]. Explanation of the abbreviations: ISM = image source method, BT = beam tracing, ART = acoustic radiance transfer, RT = ray tracing, CT = cone tracing, PT = particle tracing, BEM = boundary element method, FEM = finite element method, FDTD = finite difference time domain methods.

CONCLUSIONS

Three approaches to room acoustics modeling were compared: image source methods, acoustics energy transfer methods, and wave-based methods. It was noted that the different approaches had some links between them. In addition, it was noted that each approach had its strengths and weaknesses which determine which part of the room response they could be used best. Image source methods produce results that are widely applicable, although these methods lack diffraction and require too much computational resources for the higher-order reflections. Thus they should be used for the early part of the response. Methods based on acoustic energy transfer lack phase information, but could be efficient at the late part of the response. Wave-based methods are computationally demanding and lack the directional information. Thus they are at their best at low frequencies.

A hybrid method could be implemented where three different methods, one belonging to each of the above mentioned categories, could be combined. The links between the methods could be utilized to combine the results for full room response. That would lead to both efficient and accurate room acoustics modeling.

Table 1: Comparison of room acoustics modeling methods.

	Low f	High f	Early Reflections	Late Reflections	Phase	Reflection Directions
ISM/BT	No Diffraction	Good	Good	Slow	Yes	Yes
BEM/FEM/FDTD	Good	Slow	Good	Good	Yes	No/No/Yes
ART/RT/CT/PT	No Diffraction	Good	Decent	Good	No	Yes

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