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Introducing higher order diffraction into beam tracing based on the uncertainty relation

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ABSTRACT

The lacking simulation of diffraction is still a main problem of ray tracing in room and, even more, in city acoustics. The author´s approach to diffraction is an energetic one based on the uncertainty relation (UR). In many numerical experiments, it has been validated quite well at the single screen and the slit as reference cases, compared with Svensson´s exact wave-theoretical secondary edge source model. To avoid an explosion of computation time, the long-term objective is to combine this diffraction method once with Quantized Pyramidal Beam Tracing (QPBT). For preparation, it has therefore been modified to the more efficient beam tracing technique and with that has been tested for some additional configurations. Some improved by-pass-distance- and angle-dependent diffraction functions have been investigated to also fulfil the reciprocity principle. Recent experiments dealt with possible errors of unintended double diffraction, also with double diffractin at a cascade of two edges. Some new numerical results and comparisons with the reference model will be reported. The further aim is to investigate the general applicability of the model to higher order diffraction. This, unfortunately, has not been reached up to the deadline to submit this paper and therefore will be presented only orally.

INTRODUCTION - THE BASIC IDEAS

In room and urban acoustics respectively noise immission prognosis, ray or beam tracing methods (RT/BT) are well approved. The sound particle method with its detector technique and its statistical evaluation [1] is a version of RT [16]. BT, especially with pyramide shaped beams [17], is an efficient deterministic straight forward implementation of the mirror image source method MISM [15]. However, these methods naturally neglect diffraction.

The aim is an efficient handling of arbitrary diffraction and reflection also for higher orders, but without explosion of the number of rays and computation time. A diffraction module is desired, recursively applied, as an approximation for short, but not very short wavelengths.

The basic idea for solving the 'explosion problem' is a reunification of 'similarly running' rays. This is only possible if rays are spatially extended, i.e. rather beams, in order to exploit their overlap, to interpolate and to re-unify them. For this purpose, Quantized Pyramidal Beam Tracing (QPBT) was developed in 1996 [4,7].

This chance is the reason, why now beam instead of ray diffraction is preferred; the transition from RT to BT is described here. The numerical benefits are described in [2].

A pre-condition for an effective pyramidal beam tracing is a subdivision of the room into convex sub-rooms. Diffraction events at 'inner edges' may be effectively detected on the transparent dividing 'walls'. Furthermore, RT is accelerated considerably. Fig.1 illustrates this vision.

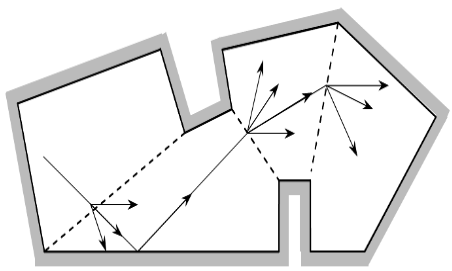


Figure 1: Multiple diffractions in a (2D) room which is subdivided into convex sub-rooms: 'transparent' dividing walls are dashed; a ray is scattered/ diffracted several times on these 'walls' near edges (only one path is drawn)

Recently a new method for the convex sub-division was found even capable of handling 'holes' (buildings on the ground plan of a city), this is decribed in [3].

As a high frequency approach for ray diffraction the UTD exists [11] and was recently utilized by Tsingos et. al. within BT [12]. Svensson developed a secondary edge source model valid even for low frequencies [9] (also only for hard wedges). However, both methods work recursively for higher order diffraction combined with the MISM for higher order reflections, hence the computation time explodes with both.

Basic hypotheses for an introduction of diffraction are here:

- diffraction is mainly an edge effect,

- energetic superposition, hence RT can be used.

But there another problem arises: with RT, rays never hit edges exactly, they pass only near by.

Basic ideas for solving both problems are:

- not all combinations and paths of diffracted/ reflected rays or particles are important, only those where particles pass ***close to*** edges,
- the bending effect on a sound particle – the diffraction probability- should be the stronger the closer the by-pass-distance.

This idea is inspired by Heisenbergs Uncertainty-Relation (UR): the by-pass-distance as an 'uncertainty'. Thereby, the diffraction pattern is the spatial Fourier transform of the transfer function of a slit. Already in 1986, the author made a successful approach for a sound particle diffraction based on the UR [5]. In 1999, Freniere et al. also used an UR based diffraction method in another way successfully in optical RT [13]. In 2006 the author's approach has been generalized, embedded in a full 2D ray tracing program, now also for finite distances [14]. The results have been compared earlier with the Maekawa's 'classical' 'detour-model' [8], later with Svensson's model for the screen. (The impulse responses were Fourier transformed and the transfer functions octave band averaged.) Reference cases were the semi-infinite screen as a 'must' and the slit (two edges) as self-consistency-test. After a long time the UR based sound particle diffraction model has right now been published in-depth with all these extensions and validations in ACUSTICA [6]. Also the faster beam diffraction model has been tested for many additional configurations.

This paper is as a continuation of last year's papers (a previous summary is in [14]). Some recent investigations have been devoted to the checking of the fulfilment of the reciprocity principle. Last year, some discrepancies occurred in some cases. To overcome this, now some improved versions of the two basic functions (described below) have been tested:

- the 'Diffraction angle probability density function' (DAPDF)
- and the 'Edge Diffraction Strength' (EDS).

A new DAPDF could be derived from wave theory. Some other versions have been tested.

Further more, the applicability of the model to double diffraction has been investigated numerically,

- at a slit, but now with finite source and receiver distances,
- unintended double diffraction at two edges respectively the attached 'transparent walls' (as in fig.1)
- at two edges in cascade, forming a 'thick' obstacle.

THE SOUND PARTICLE DIFFRACTION MODEL

There are two basic concepts of implementation: the 'Diffraction angle probability density function' (DAPDF) and the 'Edge Diffraction strength' (EDS). Here, only a very rough outline is given (full description in [6]).

The idea of that DAPDF (with non-split-up particles) emerges from the UR. But it is more efficient (and physically equivalent) to split up the rays into new ones with partial energies according to the DAPDF (fig. 2).

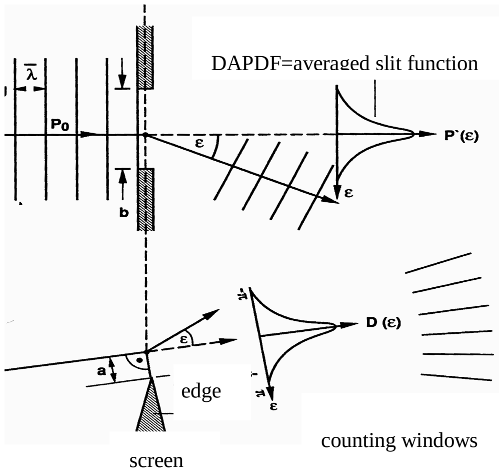


Figure 2: The sound particle diffraction model: Each moment a particle passes an edge of a screen at a distance *a* (below), it 'sees' a slit (above with the DAPDF on the right hand side). According to the uncertainty relation a certain EDS causes the particle to be diffracted according to the DAPDF= $D(\varepsilon)$. Below on the right some angle windows used to count the diffracted particles and to add up their energies to the transmission degrees (acc. eq.5). All the shifted DAPDFs of the different rays add up to the screen transmission function (as e.g. in fig. 6).

The DAPDF

The DAPDF (see fig.2) is derived from the Fraunhofer diffraction at a slit

$$\propto \sin^2 u / u^2 \text{, where } u = \pi \cdot b \cdot \sin\varepsilon \qquad (0),$$

here approximated by $u \to v = \pi \cdot b \cdot \varepsilon$, valid for parallel incident and diffracted rays. The DAPDF, averaged over a wide frequency band (similar as for 'white light') is roughly approximated by

$$D(v) = D_0 / (1 + 2v^2) \text{ with } v = 2 \cdot b \cdot \varepsilon \qquad (1)$$

where *b* is the apparent slit width in wavelengths, ε is the deflection angle and D_0 *is a* normalization factor such that the integral over all deflection angles is 1. The D_0-factor must be computed for each edge by-pass since its value depends on b and the angle limits of the wedge. In the following all distances are expressed in units of wavelengths λ .

The EDS

To develop a modular model which is applicable also to several edges that are passed near-by simultaneously, the 'Edge Diffraction Strength' (EDS(*a*)) is introduced such that the EDS of several edges may be added up to a total TEDS,

$$TEDS = \sum EDS_i \qquad (2)$$

To be used as input for the DAPDF, an 'effective slit width' is then

$$b_{eff} = 1 / TEDS \ . \qquad (3)$$

By self-consistency-considerations (the RT experiment at a slit should re-produce the energy distribution of itself) it turns out that

$$EDS(a) = 1/(6 \cdot a) \quad (4)$$

So, with only one edge, a by-passing particle would 'see' a relative slit-width of $b_{eff}=6a$.

Method of evaluation

For a systematic analysis, 2D -RT and -BT was evaluated for sources S and receivers R at finite distances rs and rr of 1,3,10,30,100 wavelengths λ and 15 angles φ_r (and later also φ_s) -84…+84° in steps of 12°, in total 5*5*15=375 combinations at the screen (fig.3) as well as at the slit of width b between two edges at –b/2 and +b/2 on the y-axis. All distances are measured in units of wavengths.

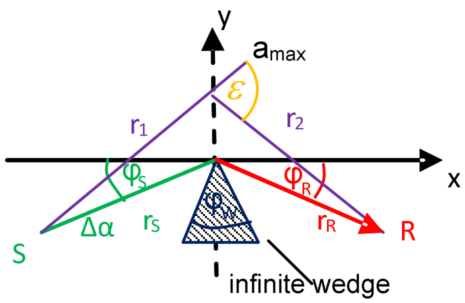


Figure 3: Geometrical definitions at the screen; dotted: the 'transparent wall' (aperture) at which the particles are diffracted

For all these parameters, the transmission degree T was determined. T is defined as the intensity with the diffraction of an obstacle relative to the intensity in free field where 'intensity' in 2D is 'sound power/width' instead of 'power/ surface'. But the proportion of T is the same in 3D. For the slit, T equals directly the DAP, the energy portion for a certain angle range $\Delta\beta$ relative to the energy incident only onto the slit. The results were compared with the known reference functions, evaluating the mean, max, min and the standard deviations over all. Curves as in fig. 6 are plotted.

With RT, many – typically 10…100 – particles are shot closely over the edge, i.e. within the by-pass distance range of 0…7 λ (this means that some thousands primary particles per full range of 2 π have to be emitted). In the first approach [5], their energies were counted in 'angle windows' in infinity on the other side to compute from that the T of the semi-infinite screen. Now, in order to simulate also finite receiver distances, the particles are detected utilizing a grid of quadratic particle detectors [1,7]. For sound particles the immission formula [1] is valid

$$T_{BTSP} = \frac{2 \cdot \pi \cdot R}{M_0 \cdot S_d} \cdot \sum_M \sum_{n=1}^{n_0} w_{M,n} \cdot D'(\beta_{M,n}) \quad (5)$$

where the $0 < D'(\beta_{M,n}) < 1$ are the energy fractions of diffracted rays (integrals over the DAPDF) in the angle range $\Delta\beta$ of the Mth incident ray and for each of the nth diffracted and received ray 'within an imagined beam', R is the direct distance source-receiver, S_d the detector surface for sound particles and w_{Mn} are the inner crossing distances of particles in detectors.

RESULTS OF RAY DIFFRACTION EXPERIMENTS

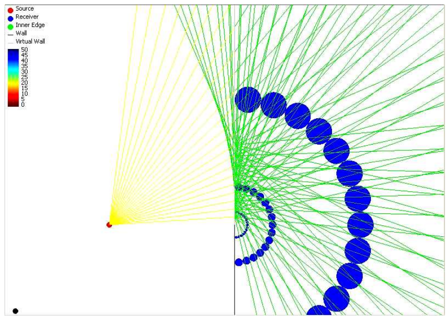


Figure 4: First order diffraction on a single screen. The yellow sound particles emitted from the source are diffracted to secondary, green, sound particles. The detectors are aligned in four radii which fifteen receivers each.

At the first go (without any parameter fitting), the agreements with the reference function (Maekawa) were again very good for almost all cases, now also for finite distances (standard deviation in most cases <1 dB, curves similar as in fig.6). In 1986, this happened even for many cases of the slit. Now, also the comparison with Svensson's result yielded good results (standard deviations of 0.66dB).

FROM RAY TO BEAM DIFFRACTION

To prepare the later implementation of QPBT and to reduce the number of energy carriers (to enhance efficiency), now beam diffraction was tested. For mirror image sources (as represented by beams), there is no stochastic variation and the $1/r^2$ -distance law may be applied to compute the immitted intensities at the receiver points (in 2D a 1/r-law, r_{BM} is the distance bending point -receiver):

$$T_{BT}' = \frac{2 \cdot \pi \cdot R}{M_0 \cdot \Delta\beta} \cdot \sum_M \frac{D(\beta_M)}{r_{BM}} \quad (6)$$

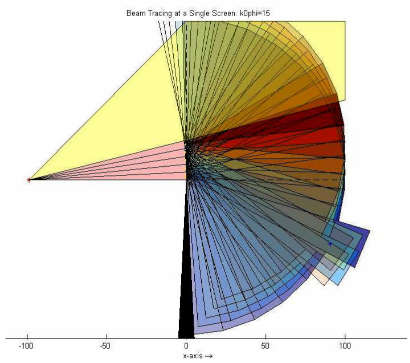


Figure 5: 2D beam diffraction, specialized for the screen (black wedge in the middle): Typically 10…100 beams ('fans' in 2D) (left, pink) arrive within the by-pass distance range of 0…7 λ (here exaggerated). The direct sound passes above (yellow). To reach all receivers, beams are split up into each typically 10…100 secondary beams (here best as many as receivers exist, i.e. 15…31). To the right the diffracted beams: the darker the colour the higher the intensity –and this mainly in straight forward directions; bottom right: the beams relevant for one specific receiver are drawn elongated.

$D(\beta_M)$ is the same as $D'(\beta_{M,n})$ in eq. 5 for the Mth incident beam which belongs to the Mth relevant incident and diffracted beam (elongated in fig.5). So, for one receiver, only one loop over all beams (M=1…M_0) is necessary, not a secondary loop over each time an additional number of secondary particles (n=1…n_0). So, RT can be equivalently be replaced by BT being much more effective. The valid by-pass distance of a beam is the middle ray's distance within the beam.

A mathematical analysis [2] shows that, in order to reach a certain numerical accuracy, one needs, as a thumb rule, at least 10 times more particles and detector crossings than beams with respectively smaller computation time.

Results of beam diffraction at a screen

- The agreements RT /BT were very good (standard deviation of only 0.67dB);

- The direct comparison between BT and the Maekawa screen transmission functions yielded a standard deviation of 0.74dB,

- the comparison with Svenssons's exact coherent secondary edge source model as analytical reference model yielded only 0.39dB (see fig. 6).

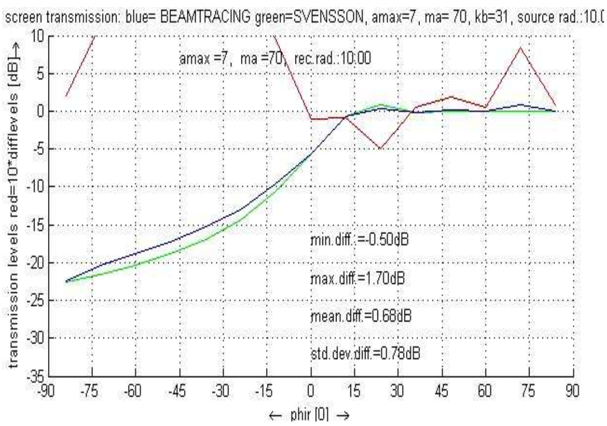


Figure 6: Example of a comparison between beam tracing (green) and Svensson's reference method (blue, falling to the left). The transmission degree in dB is given as function of the receiver angle, to the left the 'shadow' region; red curve, rising to the left: deviation* 10 (70 incident * 31 diffracted beams within $a_{max}=7\lambda$, source and receiver distance: 10λ, source at y=0).

Also, **the influence of the inner wedge angle** φ_w (fig.3) was investigated: For smaller inner angles their influence is low, but for the case of 90°, compared with 0°, the differences in the transmission levels are up to 4dB (mean difference are typically 0.4dB). However, in Svensson's reference model, hard flanking walls are assumed whereas in the interaction model based on the UR only the position of the edge is relevant, not any flanking walls.

Parallel beam diffraction at a slit

For this self-consistency-test, both, source and receiver are in infinity, hence, the incident parallel beams carry a fraction of energy according the portion of the slit width, the diffracted beams carry energy according their angle width. Fig.7 shows the experiment similar as explained in fig.5, drawn by the program with exaggerated beam widths.

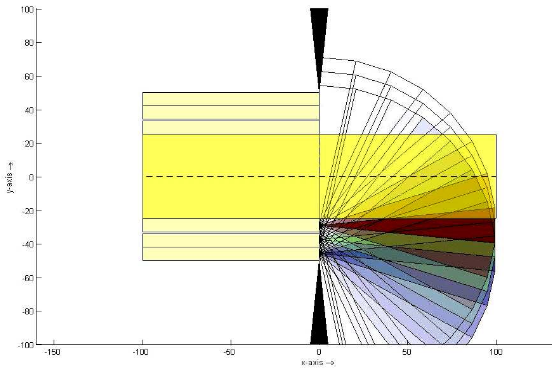


Figure 7: Only the beams near the lower edge are evaluated by reasons of symmetry, the yellow beam in the middle carry the undiffracted energy (outside certain maximum bypass-distances.) The EDSs of the two edges were added (Eqs. 2-4).

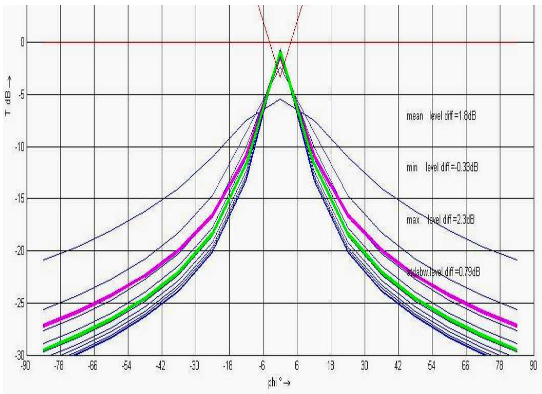


Figure 8: Transmission curves as a function of diffraction angle at a 10 λ wide slit (upper violet curve) as the sum over all DAPDFs = lower blue curves. Green: reference function.

The standard deviation for all cases is only 0.75 dB, but there are up to 3 dB too high levels at high angles ('deep in the shadow') compared with the slit function itself (green curve). (Without the a_{max}-limitation, even deviations up to 5dB, with the EDSE much better, see below). This result depends hardly on the number of beams.

FROM BEAM DIFFRACTION TO INTEGRATION

Now, to exclude any numerical error with future optimizations due to the finite number of beams (M_0), from the beam summation formula (6) a beam integration (BI) formula was derived. With $\Delta\alpha = 2\pi / M_0 \rightarrow d\alpha$ and $D(\beta_M)/\Delta\beta \rightarrow d(\beta(\alpha))$ (the DAPDF) equ. 6 converges to

$$T_{BI} = R \cdot \int_{\alpha \min}^{\alpha \max} \frac{d(\beta(\alpha))}{r_{BM}(\alpha)} d\alpha \qquad (7)$$

where $\alpha_{\min/\max}$ are the min and max incident angles, $\alpha_{\min} = -\varphi_s$. The difference in comparing the results of BT and BI for the screen was only 0.38dB standard deviation.

A first attempt for an approvement of the DAPDF

Already in the early investigations [5] a better approxiamated DAPDF (instead of eq.1) was used:

$$D(v) = D_0 \cdot \begin{cases} 1 - v^2 & \text{für} \quad |v| \le v_0 \\ \dfrac{1/2}{\sqrt{2} - 1 + v^2} & \text{für} \quad |v| > v_0 \end{cases} \qquad (1b)$$

$$\text{with} \quad v_0 = \sqrt{1 - 1/\sqrt{2}} \approx 0.5412$$

This DAPDF2 has a wider top as the former, better approaching the averaged slit-diffraction function $\sin^2(u)/u^2$.

But, as it turned out astonishingly now: its use does not pay: The standard deviation at the screen became even slightly higher than with before (0.9dB). With the slit there is hardly any improvement.

In [13] is proposed a gaussian distribution; but this is inconsequent, as the transfer function (and hence its Fourier transform) of a slit is not gaussian.

A first improvement of the EDS

As it turned out, at least for the slit, the edge diffraction strength for wider by-pass-distances is to high. Therefore another EDS was tested again [5] with an exponentially decreasing strength and a limitation to 7 λ :

$$EDSE(a) = \frac{1}{3 \cdot a + e^a} \quad \text{for } 0 < a < 7, \text{ else } 0 \qquad (4b)$$

With this (instead of the EDS of eq. 4) at the slit the agreements become much better: maximum deviation 1dB, standard deviation 0.5dB. (With the single screen, they become slightly worse, especially at short distances, std. dev. 0.8dB).

In [13] is also proposed to evaluate only distance to the nearest edge. Then the total EDS should (by self-consistency) be defined as

$$TEDS(a_1, a_2) = 1/(4 \cdot \min(a_1, a_2)). \qquad (4c)$$

But, the result is much worse than with the EDSE: max. deviations were up to 5dB, std.dev. 1.4dB.

THE PROBLEM WITH THE RECIPROCITY

Do the same diffraction levels result with a permutation of source and receiver? This does not follow evidently from the application of the UR, resp. eqs. 1-4 or eq. 7. Hence, if the reciprocity were fulfilled, this would be an important indication of the correctness of the model. Earlier simulations revealed: the reciprocity principle is fulfilled (max. dev. 0.49dB, std. dev. 0.21dB) if only r_s and r_r are interchanged, assuming only the ***total*** diffraction angle $\varphi_s + \varphi_r \approx \varepsilon$ were relevant regardless of the position of the integration area (the 'transparent wall' in fig.3. or restricted for $\varphi_s = 0$. If, however, also φ_s and φ_r are interchanged, severe deviations (mean deviations up to -10dB) occurred in cases of high negative values of φ_s. The reason is: Equ. 7 is ***not*** symmetric with respect to an interchange of source and receiver.

Some geometrical transformations lead to the alternative integral over the by-pass-distance a:

$$T = R \cdot \int_0^{a_{\max}} \frac{d(\varepsilon(a), b_{eff}(a)) \cdot \cos(\varphi_s)}{r_1(a) \cdot r_2(a) \cdot \cos(\Delta\alpha(a))} da \qquad (8)$$

where ***d*** is the DAPDF involving the EDS $\boldsymbol{b_{eff}(a)}$, ***R*** is the direct source-receiver distance, $\boldsymbol{r_{1,2}}$ are the radii to source and receiver from the bending point, $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$, where $\alpha_{\max}$ corresponds to $a_{\max}$, and $\Delta\alpha$ is the angle at the source (see fig. 3). So, equ. 8 should be made symmetric by introducing a $\cos(\varphi_r)$ factor in the nominator. This might be justified as according Lambert's law no radiation is possible stronger than accoding a cosine-law expecially at small angles relative to the diffracting 'transparent wall'. Or the DAPDF (being up to now a function strinctly depending only on a total diffraction angle) should be completed approximately by a $\cos(\varepsilon)$ factor in the nominator of equ. 1 with $u = \pi \cdot b \cdot \sin\varepsilon$ from now on. But a pure empirical approach is not satisfying, so an analytical 'derivation' of the missing cos–factor was aimed at.

Attempts at optimizations of the DAPDF

The classical textbook derivation of the Fraunhofer formula (0) is only an approximation for small angles and for a plane perpendicular incident wave [10]. A more thorough derivation starts with the Kirchhoff-Helmholtz-Integral (KHI) for the aperture of a slit assuming parallel incident and emerging waves but with angles φ_s onto resp. φ_r from the slit. The differentiations with the KHI delivers the previously missing typical cosine projection factors, together the factor

$$f = [\cos(\varphi_s) + \cos(\varphi_r)]/2 \qquad (9)$$

The pressure at the receiver is then

$$p \propto f \cdot b \cdot \left(\frac{\sin(u)}{u}\right) \cdot e^{ik(r_s + r_r)} / r_r \qquad (10).$$

where $\sin(u)/u$ is the commonly known slit function with $u = \pi \cdot b \cdot \sin\varepsilon$. To get the energetic transmission ***T***, ***f*** has to be squared. Physically (as the 'transparent wall' is just a fiction), only the total angle $\varphi_s + \varphi_r = \varepsilon$ may be relevant such that $\varphi_s = \varphi_r = \varepsilon/2$ and the characteristic factor $f^2 = \cos^2(\varepsilon/2) = (1 + \cos\varepsilon)/2$ occurs. With that the reciprocity is better fulfilled. The following DAPDFs were tested again by the described sound particle diffraction simulations at the screen compared with the Svensson reference model over all the 375 combinations for 3 cases:

- for the screen with the (more decreasing) EDSE,
- for the screen with the old EDS,
- for the slit with the EDSE (e.g. for the typical case of b=10 λ slit width)

Some summarited results (standard deviations) are:

Tab.1.	comp.ref. screen EDSE	EDS	slit [dB]
1) $D1(b_{eff}, \varepsilon) = D(u)$	1.3	0.7	0.5
2) $D2(b_{eff}, \varepsilon) = D(u) \cdot \cos\varepsilon$	3.0	1.8	0.7
3) $D3(b_{eff}, \varepsilon) = D(u) \cdot (1 + \cos\varepsilon)/2$	1.8	0.8	0.5

Discussion

Over all, $\boldsymbol{D_3}$, seems to be the optimum DAPDF. This is also the consequence of the above considerations. But the result is not really satisfying: while for the screen, the old EDS seems better, for the slit the EDSE with the exp-term is better. A reason might be: To introduce any function of only the total diffraction angle ε into the integrand of equ. 8 does not solve the problem of unsymmetry of this formula; actually really the factor $\cos(\varphi_r)$ should be introduced – but this is not in accordance with the UR based particle diffraction model which should depend only on the by-pass distance and deliver a DAPD only depending on the *total* angle. On the other side the introduction of 'transparent walls' of quasi arbitrary orientation remains a critical fiction anyway. Figure 9 shows one of the screen results.

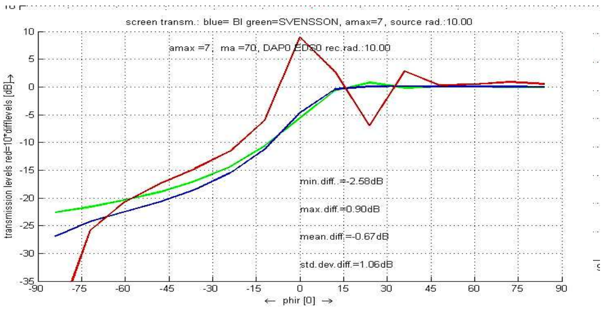
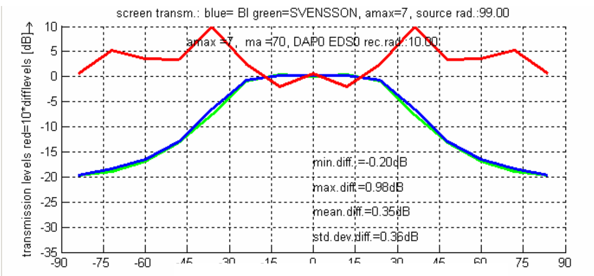


Figure 9: Comparison between ray tracing (blue) with the DAPDF D3 (1+cos(ε)) and Svensson's reference method (green); transmission degree in dB given as function of the receiver angle, to the left the 'shadow' region; red curve: deviation*10; a_{max}=7 λ, source and receiver distance: 10 λ, $\varphi_s = 0$.

NON PARALLEL RAY DIFFRACTION AT A SLIT FOR FINITE SOURCE DISTANCES

It is not self-understanding that, applying the EDS-model, the simultaneous diffraction at the two edges of a slit for *finite* source-edge distances also results in good agreements with wave theoretical models. Recently, using the improved EDS-function with the exp-term (equ. 4b), this has been also verified (again with a ray diffraction model). Fig. 10 and 11 show the result of one of many examples; the standard deviations of all the 375 angle-radii-combinations is lower than 1dB.



← diffraction angle →

Figure 10: Transmission function of a slit of width 10 λ for a source distance 99 λ and receiver distance 10 λ. UR based ray diffraction with the EDS of equ. 4b (blue) in comparision with Svenssons secondary source diffraction model (green). Red: difference*10, standard deviation here only 0.36dB.

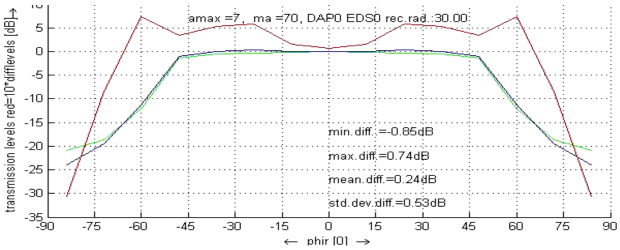


Figure 11: Same kind of comparison as with fig.10, but with the DAPDF D3 and for a wider slit of width b=30 λ, source and receiver distance: 30 λ

SOME EXPERIMENTS WITH UNINTENDED DOUBLE DIFFRACTION

The necessary procedure of convex sub-division (see fig. 1) may often – by quasi random effects – deliver two 'transparent walls' instead of one, for ex. if in a rectangular room (as in fig.4) two edges are found to be connected with a third edge instead of just one. Physically, however, the situation is the same and the diffraction result should be the same– one of some paradox cases of unintentional formal (not real) double-diffraction to be handled sufficiently. Fig. 12 shows the case of two split-up 'transparent walls' forming with the screen an Y. The errors (standard deviations) compared to the non-split-up case (its respective reference functions for the case single diffraction in the middle) were astonishingly small as indicated by the functions in fig. 13 only 0.91dB.

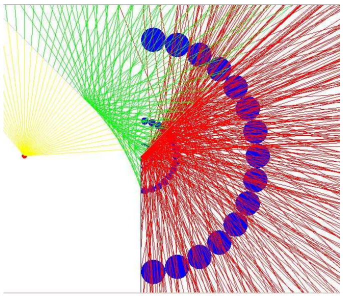
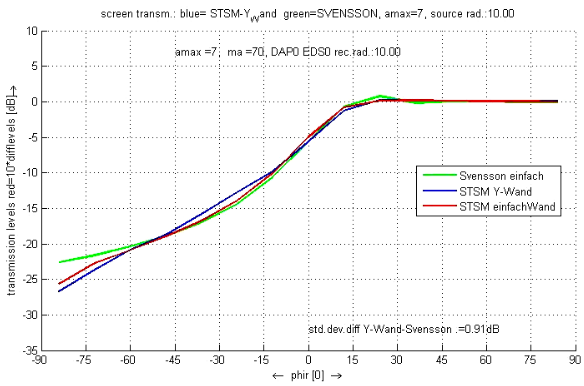


Figure 12: Double diffraction at 2 'transparent walls' forming with the screen (bottom in the middle) an Y (see the colour borders reaching the upper left and right corners in 45° direction) ; green: rays 1.order, red: 2. order diffracted. Blue circles: particle detectors at the receivers



← diffraction angle →

Figure 13: Transmission function of unintentionally 2 transparent Y walls (blue curve fig. 12) compared with one (red) and Svensson's reference function for one (green); transmission degrees in dB as function of the receiver angle, ex. for source-edge and edge-receiver distance of 10 λ

Another case of often double diffraction where physically rather a single may diffraction be assumed is that of two edges very close to each other, i.e. smaller than a wavelength, typically as real (not infinitely thin) walls of buildings. It least it should be clear that these cases should be handled as if there were only one diffraction event - the idea is in further simulations to 'switch off' the diffraction strengths of those closely following edges. To investigate this case, in the middle, instead of one, two edges in a distance of 0.1 λ were created. Astonishingly, comparing the total transmissions-functions of the double with the single diffraction (fig. 14), again, in typical situations, reveals standard deviations of only in the order of 1dB. By the way: to 'switch off' or attenuate the second diffraction in this close case did not yield better result.

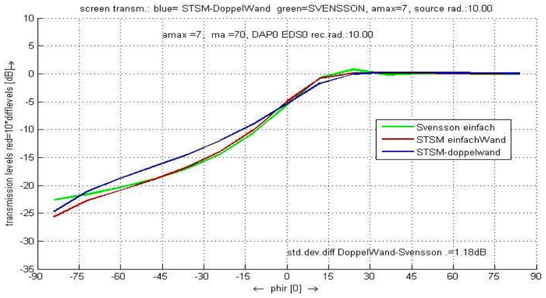


Figure 14: Transmission functions of two close (0.1 λ) transparent walls in the middle (blue line) compared with one (same kind of diagram as fig. 13); standard deviation for source-edge and edge-receiver distances of 10 λ: 1.18dB

DOUBLE DIFFRACTION AT A CASCADE OF TWO EDGES

Finally, first time by ray diffraction experiments, the double diffraction after each other at two edges (fig. 15) has been investigated and compared with wave theoretical reference functions for this case by Svensson. The new parameter is the distance d between the two edges (in wavelengths).

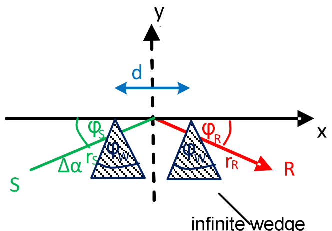


Figure 15: Geometry for the double diffraction at two edges after each other (cascade)

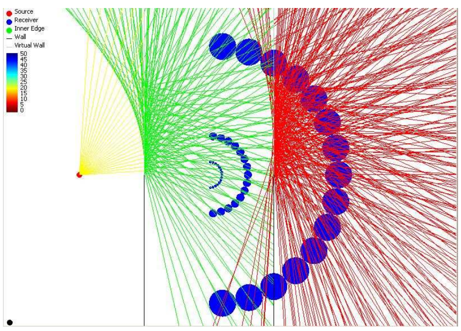


Figure 16: Double diffracted rays at the 'transparent walls' above these two edges (of fig. 15);
green: rays 1.order, red: 2. order diffracted.
blue circles: particle detectors at the receivers

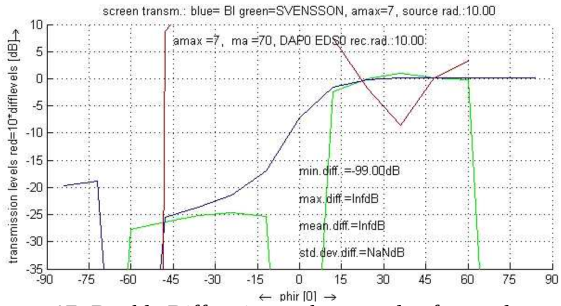


Figure 17: Double Diffraction at the cascade of two edges of fig.15 in distance 10 λ : transmission functions for the exampel of a source and receiver distance of 10 λ to the middle. UR based ray diffraction with the EDSE of equ. 4b (blue) in comparison with Svenssons secondary source diffraction model (green). Red: difference*10.

Discussion

As can be seen in fig. 17, the results (here only one example for source and receiver distance of 10 λ) is almost unusable. The Svensson reference function yields zero transmission (minus infinite levels) for angles beyond +-60° this is plausible as for the chosen distances of 10 λ also for the edge distance then nor reasonable source and receiver positions exist; also, the model was not usable for source-receiver positions at 0° in line with the two edges (special case of double diffraction); for an diffraction angle of -45° (into the shadow) there is an agreement recognizable (also for positive angles in the visibility range). Similar errors occurred in other cases. Up to now, no better comparisons succeeded. Another difficulty should be mentioned: in wave theory, boundary conditions have to be fulfilled; so, there is a difference whether flanking surfaces are 'hard' or 'soft', or whether they exist or not, for ex. a 'roof' connecting the two edges of fig. 15; the uncertainty based sound particle diffraction model is not sensitive to these things – only to the vicinity of edges.

Optimum numerical parameters

Numerically, it is useful that a maximum by-pass distance of a_{max}= 7 λ (see fig.2) may be established; beyond that, direct transmission may be performed (figs. 5+7). In the case of the slit (or several edges), a_{max} even ***must*** be defined to reduce the effect of the EDS (if not the EDSE is used): the level deviations to the reference functions at the screen were without a_{max}: max 3.47dB, std.dev. 0.91dB ; with a_{max}=5 only max 0.94dB, std.dev. 0.4 dB. The maximum deviations increase with decreasing minimum by-pass distances, it is almost possible to take 1 λ . This cannot be improved with more particles. With RT, a decisive quantity is the number of incident particles within a close by-pass distance a_{min}. That should be maximum 0.1 λ . As a technical improvement for BT, one incident beam onto the range near the edge is sufficient, a group of diffraction points within 0… a_{max} may then be established from which several beams are emitted. The number of secondary beams should be in the order of the number of relevant targets or receivers on the other side.

Fortunately, also the orientation of the 'diffracting surface' 'above' the screen (dashed lines in fig. 1) has only a weak influence (at +-45° less than 1dB). This is important for the practical implementation of the model in sub-divided rooms.

CONCLUSIONS

As was found with the first investigations already a long time ago, for the classical cases of screen and slit, the agreements between the UR based sound particle diffraction model and the wave theoretical reference functions were in most cases very good, the standard deviations were mostly better than 1dB; as now emerged, even for some cases of double diffraction. First time also the case of diffraction at the edges of a slit from finite distances has been compared with reference functions and revealed good results. So, generally, it seems like the uncertainty relation may be applied also to acoustics and sound may be handled as particles even with diffraction.

The old DAPDF combined with the attenuating EDSE were affirmed to be a good combination of diffraction functions for screen and slit.

An improved DAPDF intended to better fulfill the reciprocity principle was derived from the Kirchhoff-Helmholtz- Theorem, but the standard deviations were not much lower than before, with that the beam integration formula is not yet strictly symmetric.

Some crucial cases of unintentional double diffraction events revealed to be not harmful. However, the case of double diffraction at two edges in cascade could not yet be investigated sufficiently.

For many comparisons the faster beam diffraction method was used. For faster and safe validation (avoiding numerical errors due to a finite number of beams) an integral formulation was found.

The more efficient beam diffraction method delivered good results. This is important, as the re-unification technique by QPBT to avoid computation time explosion for higher order diffraction, is based on beams rather than on rays.

OUTLOOK

First of all, other more general events of multiple diffractions will have to be investigated, preferably also by beam instead of ray diffraction. One of the questions in this context is the limiting distance between edges for ‘independent’ subsequent diffractions and other crucial cases of physically not plausible multiple diffractions, e.g. passing of long surfaces with slight curvatures. Also still better DAPDFs – probably to be combined with modified EDS functions will have to be found to fulfil the reciprocity postulation.

The strong frequency dependence of diffraction (influencing the question what are ‘near’ edges and what is the best a_{max}) remains a fundamental problem. In final simulations, each beam should carry energies of several octave bands.

The next big step is the extension of the diffracting procedured to three dimensions. In principle, this should not be a problem, as there is actually not added a degree of freedom; edge diffraction happens mainly in the area perpendicular to the edge; it is basically a 2D effect; if, however, the edge is finite, there is also a diffraction component along the edge direction. Thus, much more secondary rays or beams are necessary – and the more important re-unification algorithms.

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REFERENCES

1 U. M. Stephenson, „Eine Schallteilchen-Computer-Simulation zur Berechnung der für die Hörsamkeit in Konzertsälen maßgebenden Parameter“, *ACUSTICA 59 (1985), p. 1-20*

2 A. Pohl and U. M. Stephenson, “From ray to beam tracing and diffraction – an analytical prognosis formula for the trade-off between accuracy and computation time”, *Proceedings of DAGA 2010 (Berlin)*

3 A. Pohl and U. M. Stephenson, “Efficient simulation of sound propagation including multiple diffractions in urban geometries by convex sub-division”, *Proceedings of Internoise 2010 Lisbon*

4 U. M. Stephenson, “Quantized Pyramidal Beam Tracing - a new algorithm for room acoustics and noise immission prognosis”, *ACUSTICA united with acta acustica* **82**, (1996)

5 U. M. Stephenson and F.P. Mechel, “Wie werden Schallteilchen gebeugt?”, *Proceedings of DAGA 1986* (Oldenburg)

6 U. M. Stephenson, “An Energetic Approach for the Simulation of Diffraction within ray Tracing based on the Uncertainty Relation”, *ACUSTICA united with acta acustica*, 96 (3) (2010) p. 516-535

7 U. M. Stephenson, “Quantized Pyramidal BeamTracing or a Sound-Particle-Radiosity-Algorithm? - New solutions for the simulation of diffraction without explosion of computation time”, *Proceedings of research symposium (Inst. of Acoustics, Salford, 2003)*

8 Z. Maekawa, “Noise reduction by screens”, *Applied Acoustics* **1**, (1968)

9 U. P. Svensson and R.I. Fred, “An analytic secondary source model of edge diffraction impulse responses”, *J. Acoust. Soc. Am.* **106**, (1999)

10 A. D. Pierce, *Acoustics- an Introduction to its Physical Principles and Applications*, Acoust. Soc. of. Am., 2nd print, 1991

11 R.G. Kouyoumjian and P.H. Pathak, “A Uniform Geometrical Theory of Diffraction for an Edge in a Perfectly Conduction Surface”, *Proceedings of IEEE **62**, (1974)*

12 N. Tsingos, T. Funkhouser, A. Ngan and I. Carlbom, “Modeling acoustics in virtual environments using the uniform theory of diffraction”, *Proceedings of ACM Computer Graphics Siggraph, (2001)*

13 E.R. Freniere, G.G. Gregory and R.A. Hassler, “Edge diffraction in Monte Carlo ray tracing”, *Optical Design and Analysis Software, Proceedings of SPIE, Denver, (1999)*

14 U.M. Stephenson, “Can also diffracted sound be handled as flow of particles? - Some new results of a beam tracing approach based on the uncertainty principle”, *Proceedings of Acoustics joint SFA, EAA and ASA conference, Paris, (2008)*

15 J. Borish, “Extension of the image model to arbitrary polyhedral”, *J. Acoust. Soc. Am.* **75**, (1984)

16 M. Vorländer, “Simulation of the transient and steady-state sound propagation in rooms using a new combined ray-tracing image source algorithm”, *J. Acoust. Soc. Am.* **86**, (1989)

17 A. Farina, “a New Pyramid Tracer for Medium and Large Scale Acoustic Problems”, *Proceedings of Euronoise (1995, Senlis, France*)