



# Identification of admittance parameters in arbitrarily shaped interiors based on sound pressure measurements using a 3d-FEM-based inverse algorithm

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## ABSTRACT

Knowledge about the dynamics of walls surrounding arbitrarily shaped spaces is important to determine, predict and optimize the acoustics within. The dynamics may be quantified in terms of the acoustical boundary impedance or boundary admittance, respectively. The authors aim to approximate a discrete distribution of admittance values for the entire boundary. An inverse, FEM-based algorithm shall utilize sound pressure data obtained from a set of microphone placements to estimate those system parameters. The algorithm requires phase related knowledge of the excitation through sound sources and structural vibration on the boundary. Computationally, it leads to a generally ill-conditioned, non-linear least-squares problem whose operation depends mainly on the relation of the numbers of unknowns and microphones, their locations and noise. Results obtained on three-dimensional numerical models will be discussed.

## INTRODUCTION

Simulating sound in spaces connected to any vibrating structure should usually require the solution of a fully coupled system. If the acoustical medium is light enough, a one-sided coupling of structure and fluid may be assumed. Still, the structure itself is generally elastic and features mass and internal damping which couples the vibro-acoustics along a structural path. Starting with the acoustics of the fluid while considering a one-sided coupling it is possible to account locally for these dynamic properties of the structural wall by introducing a mixed boundary condition. This condition includes a local parameter, the so called boundary admittance. With this boundary condition a purely acoustical formulation may not include any dynamics along the boundary. The authors' contribution to the ICA 2010 in Sydney [1] discusses the meaning and difference of the local admittance boundary condition to a fully coupled structure fluid system in detail.

The current presentation deals with the reconstruction of these local admittance or impedance parameters. With impedance tube measurements reliable results can be obtained for perpendicular wave impact on plane boundaries [2]. There are few more attempts to consider different environments, e.g. [3]. However, within rooms of complex geometry such local methods can not record the influence of any discontinuities on the walls onto the sound fields.

In contrast this research deals with the reconstruction of entire admittance value distributions in rooms of arbitrary geometry. Besides analysing the spatial construction of the room, measurement of the sound sources and the sound pressure using microphone arrays is required.

Presumably, the easiest possible way to handle sound sources would be utilising small loudspeakers. Their radiation characteristics could easily be quantified by multipoles. And using sound pressure data — being the answer to the excitation — to recon-

struct those system parameters leads to an inverse problem. It is based on a description of the interior acoustics at harmonic excitation. The related boundary value problem consists of the homogeneous Helmholtz equation and the Robin boundary condition

$$\Delta p + k^2 p = 0 \quad \vec{x} \in \Omega_f \quad (1)$$

$$v_f - v_s = Y p \quad \vec{x} \in \Gamma, \quad (2)$$

where  $k$  denotes the wave number. Within this work the fluid is excited by structural vibration, numerically expressed by the normal structural velocity  $v_s$ . The normal fluid velocity  $v_f$  is related to the normal pressure derivative on the boundary by means of the Euler equation

$$v_f = \frac{1}{i \rho_0 \omega} \frac{\partial p}{\partial n} \quad (3)$$

with  $\omega = 2\pi f = ck$ , frequency  $f$ , average density  $\rho_0$  and speed of sound  $c$ .

In order to solve this inverse problem of global reconstruction the authors discretise the acoustical boundary value problem by means of the finite element method. The resulting matrix equation denotes

$$(\mathbf{K} - k^2 \mathbf{M} - ik\mathbf{D})\mathbf{p} = i\rho_0 ck \mathbf{F} \mathbf{v}_s. \quad (4)$$

The unknown admittance parameters  $\mathbf{Y}$  reside in the damping matrix  $\mathbf{D}$ .

## 3D - ADMITTANCE RECONSTRUCTION

As best described in [4], the FEM-based inverse problem leads to a nonlinear optimisation problem. Its objective function denotes the least squares over all sound pressure measurements  $\check{\mathbf{p}}_m$

$$\mathcal{F}(\vec{y}) = \|\check{\mathbf{p}}_m - \mathbf{p}_m(\vec{y})\|^2 \longrightarrow \min! \quad (5)$$

Its solution is an approximation for the spatially piecewise constant admittance parameter.

The quality of that solution depends on several aspects. Firstly, there is the ratio between the numbers of unknown complex parameters and the number of microphone measurements. The number of surfaces on the boundary with one parameter each can be chosen with respect to the expected characteristic wave length and the probable significant change of the admittance over the boundary. The number of microphone locations and a sensible distribution of those is just a matter of measurement expense.

A Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm solves the nonlinear system of equations. At each iteration step the algorithm requires the gradient of the objective function. The algorithm itself estimates the Hessian matrix. A randomly-oriented feature of the BFGS allows for a global minimum search with vague initial values. The first gradient is provided by making use of the adjungated operator so that higher numbers of unknowns requires the same computational effort as smaller numbers do.

In order to keep computational time and memory for systems with high numbers of degrees of freedom in limits, the system matrices are stored in the compressed sparse row format and solution of the linear system of equations is done by a GMRES algorithm preconditioned by a incomplete LU decomposition.

The author will also explain a simple method that preconditions the generally noise impaired sound pressure data.

In order to test the algorithm a numerical model of a car interior is established. The three-dimensional acoustical domain is discretised with Lagrange second order finite elements. The measurement data is simulated using a different FE mesh than the mesh used for the inverse procedure whereas the location of the microphones do not depend on nodal coordinates. Convergence of the FE forward solution and the inverse solution is ensured up to the highest investigated frequencies. Additionally, artificial noise is added to the data of  $p_m$  to make it more realistic. Thus, in this simulation so called “inverse crime” is prevented in the first place.

## SUMMARY

Unlike most inverse acoustical problems, where sound sources are reconstructed, the method presented serves as system parameter identification. The parameters to be identified are a set of complex admittance values globally distributed over the entire boundary of a closed acoustical space. The algorithm is based on a finite element discretisation that handles arbitrary geometries. The complexity of the boundary geometry enclosing the fluid and the segmentation of the boundary into surfaces with single admittance parameters can be adapted and downsized with respect to the expected wave length.

The sensitivities of different aspects of the algorithm are investigated on the model of the car interior and are included in the presentation in order to illustrate the operational reliability of the global admittance reconstruction algorithm applied to three-dimensional problems of complex geometry.

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