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Room Acoustics Modeling with Acoustic Radiance Transfer

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ABSTRACT

Acoustic radiance transfer is a surface-element-based computational room acoustics modeling method. It is based on the room acoustic rendering equation, and it enables modeling of arbitrary reflection functions. In this paper, we review both the time-domain and frequency-domain formulations of the technique. As acoustic radiance transfer is based on geometrical acoustics it lacks diffraction modeling, but there are several ways to incorporate diffraction into it, as discussed. The main novelty of this work is the use of non-uniform sampling in response representation allowing the method to invest more samples to high energy parts of the response thus optimizing resource allocation. The proposed hybrid modeling brings further benefits by combining the advantages of the beam tracing and acoustic radiance transfer methods.

INTRODUCTION

The acoustic radiance transfer method [1] can model acoustic energy propagation in complex room models. However, the computational model sets some limitations to the physical accuracy of the results. In addition, the computation can consume resources and time more than is desirable in some cases. The goal of this paper is to discuss improvements on the basic method bringing benefits in both accuracy and efficiency.

An obvious short-coming of the acoustic radiance transfer method is that the simulations lack phase information such that wave-based phenomena cannot be modelled. However, adding diffraction modeling could still yield more realistic energy distribution in models with diffracting edges. More efficient use of computational resources, without compromising the quality of the simulations, requires knowledge on which parts of the modelled responses are the most important. The goal is that the resource consumption is in proportion to the significance of the modelled feature. Two other related topics are the use of the acoustic radiance transfer method in connection with an auralization system, and in hybrid modeling systems combining the best properties of the acoustic radiance transfer method and other methods.

The paper is organized as follows. First the work related to radiance-based room acoustics modeling is reviewed. Then the acoustic radiance transfer is revisited and the aforementioned improvements to the basic technique are described. Finally, some conclusions are drawn.

RELATED WORK

The research literature in room acoustics modeling is extensive and thus is not purposeful to review all the methods introduced. An interested reader may consult a survey on room acoustics modeling [2]. There are two categories of room acoustics modeling methods: geometrical acoustics and wave-based methods. Some commonly used geometrical room acoustics modeling methods are image source methods [3, 4], ray-tracing methods [5], and acoustic radiosity [6]. The room acoustic rendering equation is a unifying framework that covers all the geometrical modeling techniques [1]. Wave-based methods include boundary element methods [7], finite element methods [8], and finite-difference time-domain methods [9, 10]. There exists some other methods as well, but they usually resemble the aforementioned methods. For example, beam tracing [11] is a visibility-optimized image source method, and cone tracing and different particle tracing schemes fall into same theoretical framework with the ray-tracing methods.

The acoustic radiance transfer method is most closely related to the acoustic radiosity methods [6, 12–14]. The main difference is that while acoustic radiosity assumes Lambertian diffuse reflections, the acoustic radiance transfer allows any directional diffuse reflections. The Lambertian diffuse reflection is not physically-based, but a simplifying assumption that yields approximately correct results only in highly diffusive environments. Adding the directional diffuse reflection is a step towards more realistic reflection models, while preserving the benefits of the element-based techniques, i.e. constant number of modelled elements throughout the computation, which allows modeling the late part of the room response.

Complete room acoustics modeling systems often use a combination of the different room acoustics modeling methods [14–17]. Similarly, auralization systems use different methods for different parts of the modelled responses [18]. These observations reflect the fact that no single room acoustics modeling method can efficiently model the whole impulse response both in full length in the time domain and for full bandwidth in the frequency domain. In this paper, we attempt to make the acoustic radiance transfer method a general purpose room acoustics modeling technique. Also a new hybrid method is suggested.

ACOUSTIC RADIANCE TRANSFER METHOD

The acoustic radiance transfer method is an element-based method such as the boundary element methods, but the acoustic quantity that is modelled is energy [1]. Thus, phase information is not modelled. This same assumption of negligible wave-based phenomena is shared by all the methods based on geometrical acoustics, so that all of them are most accurate at higher frequencies. If the surface elements are small enough, it is safe to assume that the intensity of sound does not vary much over one element. Then it is possible to derive same kind of interaction matrix between elements than in the boundary element methods. Now the matrix elements describe what portion of the energy leaving one element reaches another. It is possible to account for reflections in which different amounts of energy are distributed in different directions. The directional space is also divided in parts for each element. Thus, the interactions actually represent the energy leaving one patch in one direction then reflected from another patch to another direction.

The solution process is time-iterative. Starting with the energy sent from the sound source to the elements, the energy is transferred from element to element by always choosing the element with the highest unpropagated energy and transfering that energy to other elements which are visible. The propagation time is computed and time-dependent energy responses are stored for each outgoing direction at each element. The iterative process is repeated as long as the energy transferred is significant. Eventually, the energy is collected from the elements to a receiver to obtain the time-dependent energy responses of the room for the given source at the receiver position. The responses can be computed for any receiver position without repeating the iterative energy propagation process.

An intuitive representation of the radiance transfer is presented in Fig. 1. There the acoustic energy transfer is presented first from the source to the elements

Obviously, the computational demands do not increase as the response is modelled further in time, since the number of elements is constant throughout the process. Thus, computing the late reverberation is efficient. On the other hand, the size of the elements and the directional resolution affect the accuracy of the early reflections.

Yet, another issue is the memory consumption. Typically, for decent quality responses, hundreds or thousands of elements are required. Then, for each element dozens of directions must be used to preserve the directional properties at the reflections. And, what is most important, time-dependent energy responses require a decent time resolution, meaning thousands of samples. Thus, the memory consumption quickly approaches hundreds of megabytes or even gigabytes.

IMPROVING ACOUSTIC RADIANCE TRANSFER METHOD

The radiance transfer method is a relatively new addition to geometric room acoustics modeling methods and thus there is still room for improvement. The computation time is still quite long and the memory consumption is excessive in complex scenarios. Diffraction effects are not accounted for.

In the following discussion, the theoretical background of the acoustic radiance transfer method is briefly revisited. Then, transforming the time-domain method into a frequency-domain method is discussed. Some thoughts on applying diffraction modeling in the radiance transfer method are presented. Finally, optimizing the response representation for decreasing memory consumption is discussed.

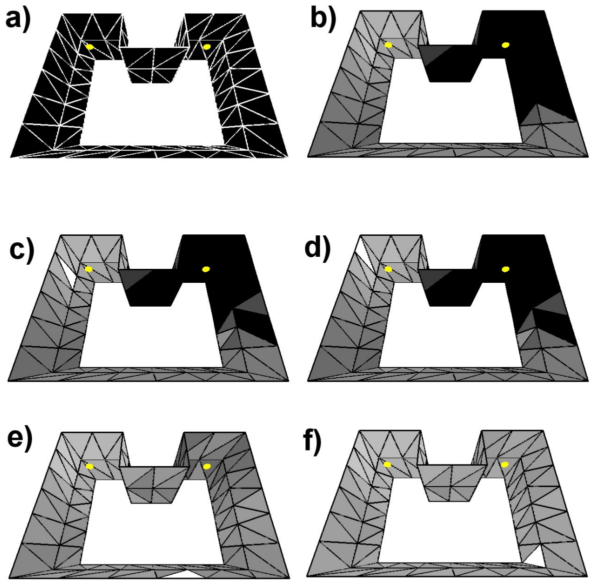

Figure 1: The progressive radiance transfer solution. (a) The model without floor and ceiling before the initial shooting, where the source is the light dot on the left and the receiver is the dot on the right. (b) The model after the initial shooting, where the lightness of the element is proportional to the amount of energy it has received. (c) The element with the highest energy is emphasized, and the energy reflected from it has been added to the elements visible to that element. This element now no longer has the highest energy. (d) The next element with the highest unshot energy is shown, and the energy is propagated similarly to (d). (e) The situation several steps later, and (f) the energy distribution in the model when the solution has converged. The energy from elements visible to the receiver is collected into it in the final gathering phase.

Introduction to Acoustic Radiance Transfer

The acoustic radiance transfer method can be derived from the room acoustic rendering equation [1] which presents the relationship of acoustic radiance at different surface points:

$$L(\vec{y},\Omega_e) = L_0(\vec{y},\Omega_e) + \oint_G R(\vec{x},\vec{y},\Omega_e) L(\vec{x},-\Omega_i)\mathrm{d}\vec{x}, \quad (1)$$

where $L(\vec{y},\Omega_e)$ and $L(\vec{x},-\Omega_i)$ are the total radiance from surface points $\vec{y}$ and $\vec{x}$ in direction Ω_e and $-\Omega_i$, respectively, and $L_0(\vec{y},\Omega_e)$ is the radiance irradiated by the surface itself at point $\vec{y}$ in direction Ω_e. $R(\vec{x},\vec{y},\Omega_e)$ is the reflection kernel which corresponds to the portion of acoustic radiance arriving from point $\vec{x}$ reflected at point $\vec{y}$ in direction Ω_e. The details can be found in [1]. The integral notation refers to integrating over whole surface area by using some two-dimensional parametrization.

A Neumann series solution to this equation can be written as

$$\begin{aligned} L_{n+1}(\vec{y},\Omega_e) &= \oint_G R(\vec{x},\vec{y},\Omega_e) L_n(\vec{x},-\Omega_e)\mathrm{d}\vec{x} \\ L(\vec{y},\Omega_e) &= \sum_{n=0}^{\infty} L_n(\vec{y},\Omega_e). \end{aligned} \quad (2)$$

This formulation can be seen as a reflection-iterative solution to the room acoustic rendering equation.

The room acoustic rendering equation can be discretized to derive an element-based modeling algorithm called acoustic radiance transfer technique. The surface of the geometric model is divided into N elements. Then the Neumann series terms can be written as a sum of integrals over the elements:

$$L_{n+1}(\vec{y},\Omega_e) = \sum_{i=1}^{N} \int_{A_i} R(\vec{x},\vec{y},\Omega_e) L_n(\vec{x},-\Omega_e)\mathrm{d}\vec{x}. \quad (3)$$

The left side of the equation can also be expressed for elements by using the average reflected radiance,

$$L_{n,j}(\Omega_e) = \frac{1}{A_j}\int_{A_j} L_n(\vec{y},\Omega_y)\mathrm{d}\vec{y}, \quad (4)$$

which yields an approximation where the radiation is assumed invariant over an element

$$L_{n+1,j}(\Omega_e) = \frac{1}{A_j}\sum_{i=1}^{N}\int_{A_j}\int_{A_i} R(\vec{x},\vec{y},\Omega_e) L_{n,i}(-\Omega_e)\mathrm{d}\vec{x}\mathrm{d}\vec{y}, \quad (5)$$

where x is on element i and y is on element j. The direction can also be discretized

$$L_{n,k,j} = \frac{1}{\int_{\Phi_k}\mathrm{d}\Omega}\int_{\Phi_k} L_{n,j}\mathrm{d}\Omega, \quad (6)$$

where Φ_k is the solid angle covered by the directional segment k. Then an approximation where the radiation is assumed invariant over the directional segment is

$$L_{n+1,j,k} = \frac{1}{A_j\int_{\Phi_k}\mathrm{d}\Omega}\sum_{i=1}^{N}\int_{\Phi_k}\int_{A_j}\int_{A_i} R(\vec{x},\vec{y},\Omega_e) L_{n,i,\Gamma_i(-\Omega_e)}\mathrm{d}\vec{x}\mathrm{d}\vec{y}\mathrm{d}\vec{\Omega}_e, \quad (7)$$

where operator $\Gamma_i(-\Omega_e)$ maps the direction $-\Omega_e$ on element i into a directional segment index. This can be written in a clearer form by introducing the discretized reflection kernel

$$R_{i,j,k} = \frac{\int_{\Phi_k}\int_{A_j}\int_{A_i} R(\vec{x},\vec{y},\Omega_e)\mathrm{d}\vec{x}\mathrm{d}\vec{y}\mathrm{d}\vec{\Omega}_e}{A_j\int_{\Phi_k}\mathrm{d}\Omega}, \quad (8)$$

which gives

$$L_{n+1,j,k} = \sum_{i=1}^{N}\sum_{l\in\Psi} R_{i,j,k} L_{n,i,l}, \quad (9)$$

where indices $l \in \Psi$ correspond to a set of directional segment indices produced by the operator $\Gamma_i(-\Omega_e)$ over the surface integrals. This operator is piecewise constant over the elements, and since the integration is a linear operator, the surface integrals can thus be expressed as a sum of constant values over the regions where the integrand is constant.

This reflection iterative formulation allows the whole solution to be written as

$$L_{j,k} = L_{0,j,k} + \sum_{n=0}^{\infty}\sum_{i=1}^{N}\sum_{l\in\Psi} R_{i,j,k} L_{n,i,l}. \quad (10)$$

The time dependence is implicitly modelled with the time delay operator in the reflection kernel.

The acoustic radiance transfer method evaluates this sum directly. The $n = 0$ values are determined by the direct radiance from the sound source, which is reflected at the elements. Then the propagation of the radiation is iterated for increasing values of n until the transferred radiance has attenuated below a desired threshold. Finally, the radiance $L_{j,k}$ can be collected from the elements to a listener.

Frequency-Domain Processing

Since the transformation from the time domain to the frequency domain is linear, the operations of the acoustic radiance transfer can be performed in the frequency domain [19]. The processing as such is not faster in the frequency domain, but for some applications such as auralization, it is often useful to convolve the resulting response with a sound stream. That can be done efficiently in the frequency domain as an element-wise multiplication. Thus, it is useful to have the results directly in the frequency domain. This is illustrated in Fig. 2.

Diffraction Modeling

Adding diffraction modeling to the acoustic radiance transfer method is challenging since energy is used in the computation. Many diffraction models, such as the Biot-Tolstoy-Medwin model [20, 21], work with pressures, and incorporate phase information in the computation. The method using time-dependent energy responses cannot be combined with the pressure-based diffraction model directly, although there has been an attempt to do that [22]. The problem is that, at least in the line integral formulation [23], it is necessary to sum contributions from individual points along the diffracting edges. Figure 3 shows diffraction paths via points on the edge. Since the energy is typically proportional to the square of the pressures, summing the squares of the pressure contributions does not give the same result as summing the pressure contributions and then squaring the result. However, it is probably not entirely impossible to utilize the pressure-based diffraction model, since the edge diffraction contribution is strongest via the so-called apex point at the edge, and considering only that contribution could be possible. The error in such cases is quite small at least from perceptual point of view.

Another approach is to use an energy-based diffraction model in the first place. The unified theory of diffraction has been successfully used in connection with other room acoustics modeling methods [24], so it is likely that it can be used with the acoustic radiance transfer methods as well.

Response Representations

Storing full energy responses with a constant resolution for each element and directional slot takes a large amount of memory. Taking into account the structure of the impulse responses in typical scenarios allows more efficient response representation. Typically, the direct sound, if not occluded, is perceptually most important. Then come the early reflections and first-order diffraction. Single peaks in the are still important. Finally, in the late reverberation the reflections merge together so that it is impossible to tell where the single reflections are coming from. In the late part of the response, the general trend of the energy decay is more important than single reflections.

Since the early part of the response is perceptually more important, it is reasonable to use higher resolution in the early part of the response and lower resolution in the latter part as long as the energy is decaying properly. This leads to one possible improvement to the memory usage of the responses based of exponential energy decay model.

Exponential energy decay model

It could be reasonable to make the resolution depend on the energy at each instant of time. However, if the energy could be known beforehand, modeling the energy response would be futile, since it is already known. Thus, an assumption must be made concerning the time-dependent power levels in the response. In rooms, the acoustic power is often assumed to

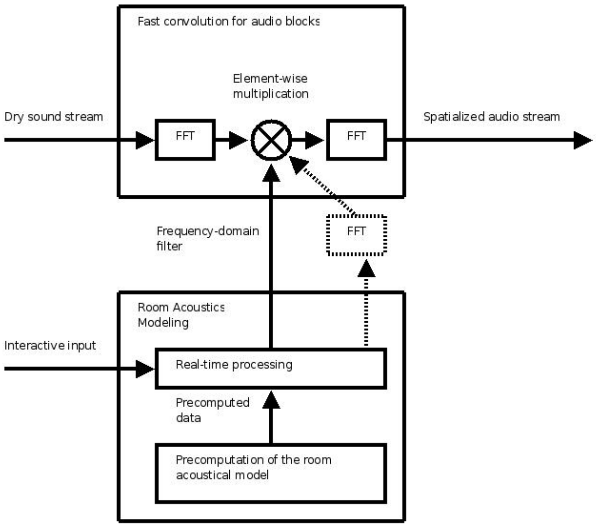

Figure 2: Performing the room acoustics modeling in the frequency domain simplifies the computation in the auralization pipeline. The dry sound stream is convolved with the room impulse response, which yields spazialized audio as output. If the room acoustics modeling were performed in the time domain, an additional fast Fourier transform (FFT) would be required as shown as a dotted line in the illustration. To be able to instantly respond to the interactive input given by a user, most of the responses on the elements can be precomputed. Then the precomputed responses are only collected to the listener position in the real time.

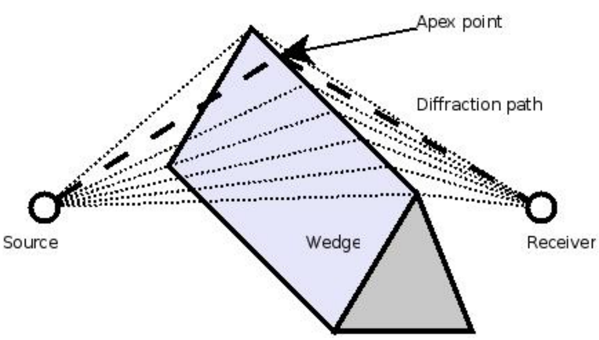

Figure 3: Diffraction path via points on an edge. The source is behind a wedge in the room geometry so that the receiver cannot directly see the source. In the geometrical room acoustics modeling, the contribution from that source to the receiver would be zero. However, in reality, the source affects the observed response at the receiver position. The sounds "bends" behind the wedge so there is actually a path between the source and receiver through which the sound is transmitted. This can be modelled by considering the points on the diffracting edge as secondary sources and summing the contributions from the secondary sources together. For accurate results a large number of points is required, but the contribution is strongest via the apex point, which is the shortest path from the source to the receiver.

decay exponentially, i.e.

$$P(t) = \frac{\mathrm{d}E}{\mathrm{d}t}(t) = P_0 e^{-\alpha t}, \tag{11}$$

where P_0 is the power of the source and α an attenuation factor. With such a decay, the resolution can be adjusted so that each sample contains the same amount of energy. Let the instants of time limiting the samples be $t_0, t_1, t_2, \ldots$ so that the first sample represents energy received between 0 seconds and t_0 seconds, the second sample energy received between t_0 seconds and t_1 seconds etc. Figure 4 illustrates this sampling approach. The energy in any sample is then

$$\begin{aligned} E &= \int_{t_n}^{t_{n+1}} P_0 e^{-\alpha t} \mathrm{d}t \\ &= \frac{P_0}{\alpha}(e^{-\alpha t_n} - e^{-\alpha t_{n+1}}). \end{aligned} \tag{12}$$

Further,

$$\begin{aligned} \frac{\alpha E}{P_0} &= e^{-\alpha t_n} - e^{-\alpha t_{n+1}} \\ e^{-\alpha t_{n+1}} &= e^{-\alpha t_n} - \frac{\alpha E}{P_0}. \end{aligned} \tag{13}$$

Let us denote $C = E/P_0$ and write a recursion rule

$$t_{n+1} = -\frac{1}{\alpha}\ln(e^{-\alpha t_n} - \alpha C). \tag{14}$$

A closed form solution can also be found

$$\begin{aligned} t_{n+1} &= -\frac{1}{\alpha}\ln(e^{-\alpha t_n} - \alpha C) \\ &= -\frac{1}{\alpha}\ln(e^{-\alpha t_{n-1}} - 2\alpha C) \\ &= -\frac{1}{\alpha}\ln(e^{-\alpha t_{n-2}} - 3\alpha C) \\ &\ldots \\ &= -\frac{1}{\alpha}\ln(e^{-\alpha t_0} - (n+1)\alpha C) \end{aligned} \tag{15}$$

or

$$t_n = -\frac{1}{\alpha}\ln(e^{-\alpha \Delta t} - n\alpha C), \tag{16}$$

where $\Delta t = t_0$.

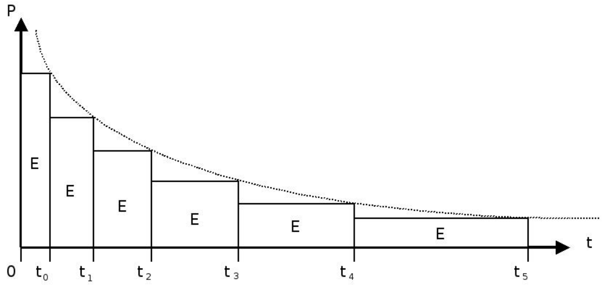

Figure 4: Energy decays exponentially. Each sample contains the same amount of energy. The instants of time defining the samples are denoted t_0, t_1, t_2, etc. The vertical axis is acoustic power, P, and the horizontal axis is time, t. Energy $E = \int_{t_n-1}^{t_n} P(t)\mathrm{d}t \approx P(t_n)(t_n - t_{n-1})$ is constant.

Constant C can be computed from the first sample if its temporal length Δt is fixed

$$\begin{aligned} E &= \int_0^{\Delta t} P_0 e^{-\alpha t}\mathrm{d}t \\ E &= \frac{P_0}{\alpha}(1 - e^{-\alpha\Delta t}) \\ C &= \frac{1}{\alpha}(1 - e^{-\alpha\Delta t}). \end{aligned} \tag{17}$$

In an optimal case the attenuation factor should be close to the real attenuation factor. But, again it is usually difficult to estimate the exact value accurately beforehand. An alternative approach is to optimize the attenuation factor to the length of the response. For that purpose it is assumed that modeling the decay of 60 dB is sufficient. Thus attenuation factor α can be computed as follows

$$\begin{aligned} e^{-\alpha T} &= \varepsilon \\ \alpha &= -\frac{1}{T}\ln(\varepsilon), \end{aligned} \tag{18}$$

where T is the temporal length of the response and $\varepsilon = 10^{-6}$. T should be chosen so that it is proportional to the reverberation time of the modelled room.

The number of samples in the response can be derived from Eq. (16) by substituting $t_n = T$ and $n = N+1$:

$$T = -\frac{1}{\alpha}\ln(e^{-\alpha\Delta t} - (N+1)\alpha C), \tag{19}$$

then utilizing Eqs. (23) and (22) the equation becomes

$$\begin{aligned} \ln(\varepsilon) &= \ln(e^{\frac{\Delta t}{T}\ln(\varepsilon)} - (N+1)(1 - e^{\frac{\Delta t}{T}\ln(\varepsilon)})) \\ \varepsilon &= e^{\frac{\Delta t}{T}\ln(\varepsilon)} - (N+1)(1 - e^{\frac{\Delta t}{T}\ln(\varepsilon)}) \\ N+1 &= \frac{e^{\frac{\Delta t - \varepsilon}{T}\ln(\varepsilon)}}{1 - e^{\frac{\Delta t}{T}\ln(\varepsilon)}} = \frac{\varepsilon^{\frac{\Delta t}{T}} - \varepsilon}{1 - \varepsilon^{\frac{\Delta t}{T}}} = \frac{1 - \varepsilon}{1 - \varepsilon^{\frac{\Delta t}{T}}} + 1 \\ N &= \frac{1 - \varepsilon}{1 - \varepsilon^{\frac{\Delta t}{T}}}. \end{aligned} \tag{20}$$

If the number of samples, N, is fixed, the temporal length of the response can be computed

$$T = \frac{-\Delta t\ln(\varepsilon)}{\ln\left(\frac{N+1-\varepsilon}{N}\right)}, \tag{21}$$

as well as the attenuation factor

$$\alpha = \frac{\ln\left(\frac{N+1-\varepsilon}{N}\right)}{\Delta t} \tag{22}$$

and constant

$$C = \frac{\Delta t(\varepsilon - 1)}{N\ln\left(\frac{N+1-\varepsilon}{N}\right)}. \tag{23}$$

These equations are used when implementing the exponential sampling for responses. Equation (21) ties together the number of samples, N, the response length in time, T, and the initial sample length, Δt. If two of these variables are fixed, then the third one can be computed. It depends on the application which two variables are chosen. Then α and C can be computed using Eqs. (22) and (23), respectively. Finally, Eq. (16) is used for computing the points in time, t_n, which separate the samples. These times are sufficient information for response processing.

Hybrid model

Since strong, nearly-specular early reflections typically have the most energy and they are perceptually important, modeling them separately makes sense. The acoustic radiance transfer could be used for modeling only the late part of the response. The early reflections could be efficiently modelled with the beam tracing method which produces accurate reflection paths and can incorporate phase information [11, 25]. This hybrid technique could be utilize the advantages of the different approaches and thus be both efficient and accurate. This idea has been introduced before, but only with less sophisticated beam tracing approach and the Lambertian radiance transfer model [14].

The implementation requires changes to the acoustic radiance transfer method. The bidirectional-reflectance distribution functions (BRDFs) could be modified so that the specular part is omitted. That means that the acoustic radiance transfer models only the directional diffuse part of the energy transfer.

The beam tracing method then models the early reflections up to a fixed depth. The results of the beam tracing are collected separately and then finally combined with the results of the acoustic radiance transfer method. It could be possible to combine the beam tracing method with diffraction modeling for more accurate modeling.

However, some error is caused by the fact that the beam-tracing is cut abruptly at a certain depth and the acoustic radiance transfer method does not compensate for the missing higher-order specular reflections. Thus, the missing energy transfer must be modelled. This is done by creating one level of beams beyond the highest-order beams in the beam tree. Reflection paths are computed from the source to the polygons corresponding to those beams. The portion of energy reaching the polygon is computed by using the reflection coefficients as usual in the beam tracing. The beam is then mapped to the BRDF of that polygon and the energy is distributed to the directional slots used in the acoustic radiance transfer method according to the portion that the beam covers of the directional slots. Obviously, the beam tracing algorithm is run first, and the acoustic radiance transfer method begins with elements prefilled with the beam tracing method. Figure 5 illustrates the modeling scheme.

This modeling approach does not allow chains of specular and diffuse reflections where specular reflections occur after diffuse reflections. However, the results are still likely better than with either of the methods alone. Lower resolution is probably sufficient to achieve similar quality in the acoustic radiance transfer method when using the hybrid approach compared to the mere acoustic radiance transfer method. This can decrease the memory usage and speed up the computation.

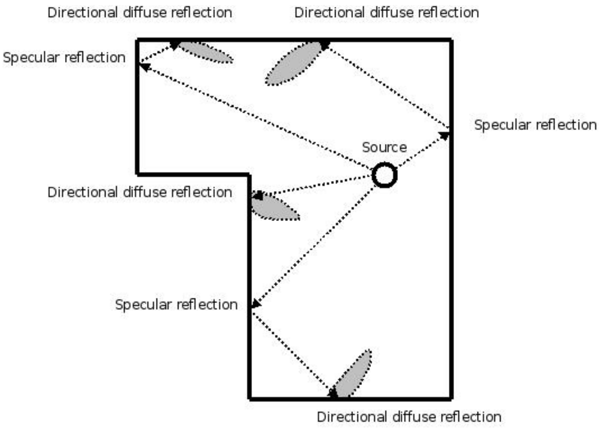

Figure 5: In the hybrid model, early reflections are modelled as specular reflection with a beam tracing method and later reflections are modelled as directional diffuse reflections with the acoustic radiance transfer method. Also pure directional diffuse reflection paths are included as well as low order pure specular reflections.

CONCLUSIONS

The accuracy and efficiency of the acoustic radiance transfer technique can be improved by considering extensions to it. Frequency-domain modeling allows efficient computation in auralization systems. Diffraction modeling can improve the accuracy of the results by taking into account this wave-based phenomenon. The time-dependent energy responses can be presented more efficiently by taking benefit from the exponential energy decay that is typical in room responses. A hybrid model with the beam tracing technique could further improve the quality of the results.

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