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NON-LINEAR ASPECTS OF HUMAN BODY'S MOTION IN CAR CRASH

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Abstract

Using the model developed by the authors in a previous paper, this article purposes a study of the human body's motion in car crashes. We realized numerical simulations for the main parameters of the human body system using realistic values. The most important diagrams were captured and discussed. Finally we presented a few conclusions.

1. INTRODUCTION

The mechanical model we shall use in our study is presented in figure 1.

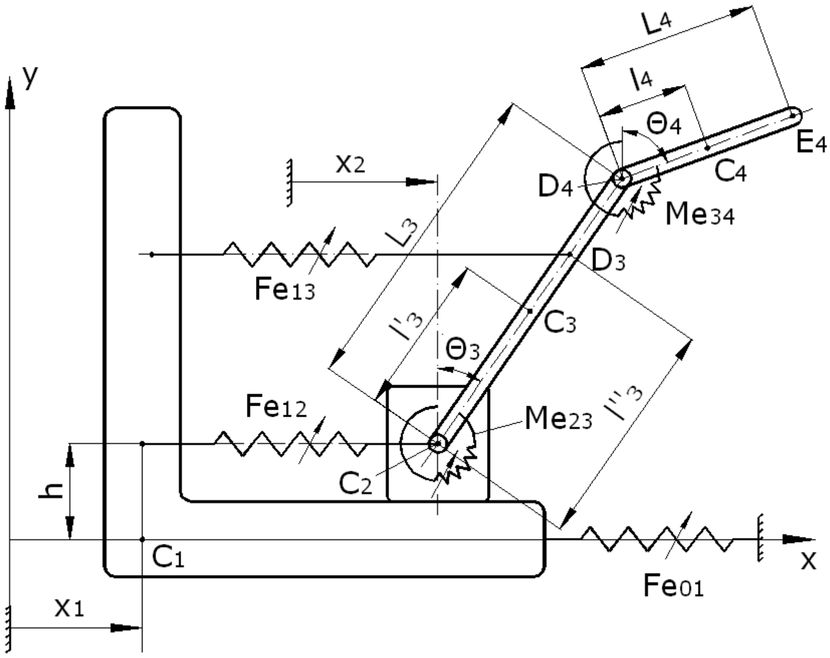

Figure 1. Mechanical model.

Our model was presented in a previous work [9], it has our degrees of freedom, and the equations of motion read

$$A_{11}\ddot{x}_1 + A_{12}\ddot{x}_2 + A_{13}\ddot{\theta}_3 + A_{14}\ddot{\theta}_4 = B_1;\; A_{21}\ddot{x}_1 + A_{22}\ddot{x}_2 + A_{23}\ddot{\theta}_3 + A_{24}\ddot{\theta}_4 = B_2;$$
$$A_{31}\ddot{x}_1 + A_{32}\ddot{x}_2 + A_{33}\ddot{\theta}_3 + A_{34}\ddot{\theta}_4 = B_3;\; A_{41}\ddot{x}_1 + A_{42}\ddot{x}_2 + A_{43}\ddot{\theta}_3 + A_{44}\ddot{\theta}_4 = B_4, \tag{1}$$

where

$$A_{11} = m_4 l_4 \cos\theta_4;\; A_{12} = m_4 l_4 \cos\theta_4;$$
$$A_{13} = m_4 L_3 \cos\theta_3 l_4 \cos\theta_4 + m_4 L_3 \sin\theta_3 l_4 \sin\theta_4 = m_4 L_3 l_4 \cos(\theta_4 - \theta_3); \tag{2}$$
$$A_{14} = J_4 + m_4 l_4^2 \cos^2\theta_4 + m_4 l_4^2 \sin^2\theta_4 = J_4 + m_4 l_4^2,$$

$$A_{21} = (m_3 + m_4) l_3' \cos\theta_3 + m_4 (L_3 - l_3')\cos\theta_3;$$
$$A_{22} = (m_3 + m_4) l_3' \cos\theta_3 + m_4 (L_3 - l_3')\cos\theta_3;\; A_{23} = J_3 + m_3 (l_3')^2 + m_4 L_3^2; \tag{3}$$
$$A_{24} = m_4 l_4 l_3' \cos(\theta_4 - \theta_3) + m_4 l_4 (L_3 - l_3')\cos(\theta_4 - \theta_3),$$

$$A_{31} = m_3 + m_4;\; A_{32} = m_2 + m_3 + m_4;$$
$$A_{33} = m_3 l_3' \cos\theta_3 + m_4 L_3 \cos\theta_3 - \mu m_3 l_3' \sin\theta_3 - \mu m_4 L_3 \sin\theta_3; \tag{4}$$
$$A_{34} = m_4 l_4 \cos\theta_4 - \mu m_4 l_4 \sin\theta_4,$$

$$A_{41} = m_1 + m_3 + m_4;\; A_{42} = m_2 + m_3 + m_4;\; A_{43} = m_3 l_3' \cos\theta_3 + m_4 L_3 \cos\theta_3;$$
$$A_{44} = m_4 l_4 \cos\theta_4, \tag{5}$$

$$B_1 = m_4 L_3 l_4 \dot{\theta}_3^2 \sin(\theta_3 - \theta_4) + m_4 g l_4 \sin\theta_4 - M_{e34}(\theta_3 - \theta_4);$$
$$B_2 = m_4 l_4 L_3 \dot{\theta}_4^2 \sin(\theta_4 - \theta_3) - F_{e13}(x_2 + l_3'' \sin\theta_3) l_3'' \cos\theta_3 +$$
$$+ m_3 g l_3' \sin\theta_3 + m_4 g L_3 \sin\theta_3 + M_{e34}(\theta_4 - \theta_3) - M_{e23}(\theta_3);$$
$$B_3 = m_3 l_3' \dot{\theta}_3^2 (\sin\theta_3 - \mu\cos\theta_3) + m_4 L_3 \dot{\theta}_3^2 (\sin\theta_3 + \mu\cos\theta_3) + \tag{6}$$
$$+ m_4 l_4 \dot{\theta}_4^2 (\sin\theta_4 + \mu\cos\theta_4) - F_{e13}(x_2 + l_3'' \sin\theta_3) -$$
$$- \mu(m_2 + m_3 + m_4) g - F_{e12}(x_2);$$
$$B_4 = m_3 l_3' \dot{\theta}_3^2 \sin\theta_3 + m_4 L_3 \dot{\theta}_3^2 \sin\theta_3 + m_4 l_4 \dot{\theta}_4^2 \sin\theta_4 - F_{01}(x_1);$$

hence $A_{ij} = A_{ji}(x_1, x_2, \theta_3, \theta_4, \dot{x}_1, \dot{x}_2, \dot{\theta}_3, \dot{\theta}_4)$, $B_i = B_i(x_1, x_2, \theta_3, \theta_4, \dot{x}_1, \dot{x}_2, \dot{\theta}_3, \dot{\theta}_4)$, $i = \overline{1,4}$, $j = \overline{1,4}$.

At this relations we add another one, name it: if $x_2 < x_1$ then $x_2 = x_1$ and $\dot{x}_2 = \dot{x}_1$.

2. PARTICULAR CASES

First case is characterized by $\mu = 0$ (no friction). We modify

$$A_{33} = m_3 l_3' \cos\theta_3 + m_4 L_3 \cos\theta_3;\; A_{34} = m_4 l_4 \cos\theta_4;$$
$$B_3 = m_3 l_3' \dot{\theta}_3^2 \sin\theta_3 + m_4 L_3 \dot{\theta}_3^2 \sin\theta_3 + m_4 l_4 \dot{\theta}_4^2 \sin\theta_4 - F_{e13}(x_2 + l_3'' \sin\theta_3) - F_{e12}(x_2) \tag{7}$$

The second case is characterized by $l'_3 = l''_3$ it means that $F_{e_{13}}$ is applied in the weight center of the third body. The following parameters modify

$$\begin{aligned} B_2 &= m_4 l_4 L_3 \dot{\theta}_4^2 \sin(\theta_4 - \theta_3) - F_{e_{13}}(x_2 + l'_3 \sin\theta_3) l'_3 \cos\theta_3 + \\ &+ m_3 g l'_3 \sin\theta_3 + m_4 g L_3 \sin\theta_3 + M_{e_{34}}(\theta_4 - \theta_3) - M_{e_{23}}(\theta_3); \\ B_3 &= m_3 l'_3 \dot{\theta}_3^2 (\sin\theta_3 + \mu\cos\theta_3) + m_4 L_3 \dot{\theta}_3^2 (\sin\theta_3 + \mu\cos\theta_3) + \\ &+ m_4 l_4 \dot{\theta}_4^2 (\sin\theta_4 + \mu\cos\theta_4) - F_{e_{13}}(x_2 + l'_3 \sin\theta_3) - \mu(m_2 + m_3 + m_4) g - F_{e_{12}}(x_2). \end{aligned} \tag{8}$$

The elastic forces and moments are linear. Now we can write

$$\begin{gathered} F_{e_{12}}(x_2) = k_{12} x_2 ;\; F_{e_{13}}(x_2 + l''_3 \sin\theta_3) = k_{13}(x_2 + l''_3 \sin\theta_3); \\ M_{e_{34}}(\theta_4 - \theta_3) = k_{34}(\theta_4 - \theta_3);\; M_{e_{23}}(\theta_3) = k_{23}(\theta_3). \end{gathered} \tag{9}$$

The forces and the moments are linear on domains. We can write

$$\begin{gathered} F_{e_{12}} = \begin{cases} k_{12} x_2 \text{ for } x_2 \geq 0 \\ 0 \text{ otherwise} \end{cases};\; F_{e_{13}} = \begin{cases} k_{13}(x_2 + l''_3 \sin\theta_3) \text{ for } x_2 + l''_3 \sin\theta_3 \geq 0 \\ 0 \text{ otherwise} \end{cases}; \\ M_{e_{34}} = \begin{cases} k_{34}^{(1)}(\theta_4 - \theta_3) \text{ for } \theta_4 - \theta_3 \geq 0 \\ -k_{34}^{(2)}(\theta_3 - \theta_4) \text{ otherwise} \end{cases};\; M_{e_{23}}(\theta_3) = \begin{cases} k_{23}^{(1)}\theta_3 \text{ for } \theta_3 \geq 0 \\ k_{23}^{(2)}\theta_3 \text{ otherwise} \end{cases}. \end{gathered} \tag{10}$$

all k_{ij} being non-negative.

The elastic forces and moments are non-linear on domains. We have

$$\begin{gathered} F_{e_{12}} = \begin{cases} k_{12}^{(1)} x_2 + k_{12}^{(3)} x_2^3 \text{ for } x_2 \geq 0 \\ 0 \text{ otherwise} \end{cases}; \\ F_{e_{13}} = \begin{cases} k_{13}^{(1)}(x_2 + l''_3 \sin\theta_3) + k_{13}^{(3)}(x_2 + l''_3 \sin\theta_3)^3 \text{ for } x_2 + l''_3 \sin\theta_3 \geq 0 \\ 0 \text{ otherwise} \end{cases}; \\ M_{e_{34}} = \begin{cases} k_{34}^{(11)}(\theta_4 - \theta_3) + k_{34}^{(13)}(\theta_4 - \theta_3)^3 \text{ for } \theta_4 - \theta_3 \geq 0 \\ -k_{34}^{(21)}(\theta_3 - \theta_4) - k_{34}^{(23)}(\theta_3 - \theta_4)^3 \text{ otherwise} \end{cases}; \\ M_{e_{23}}(\theta_3) = \begin{cases} k_{23}^{(11)}\theta_3 + k_{23}^{(13)}\theta_3^3 \text{ for } \theta_3 \geq 0 \\ k_{23}^{(21)}\theta_3 + k_{23}^{(23)}\theta_3^3 \text{ otherwise} \end{cases}, \end{gathered} \tag{11}$$

in which $k_{12}^{(1)}$, $k_{13}^{(1)}$, $k_{34}^{(11)}$, $k_{34}^{(21)}$, $k_{23}^{(11)}$ and $k_{23}^{(21)}$ are strict positive, and $k_{34}^{(13)}$, $k_{34}^{(23)}$, $k_{23}^{(13)}$, and $k_{23}^{(23)}$ can be positive if we have a hard characteristic or negative for a soft one.

Everywhere $F_{e_{01}} = k_{01} x_1$.

3. NUMERICAL SIMULATION

We selected the following values:

$$\begin{gathered} m_1 = 1000[\mathrm{kg}];\ m_2 = 26.68[\mathrm{kg}];\ m_3 = 46.06[\mathrm{kg}];\ m_4 = 5.52[\mathrm{kg}];\ \mu = 0.7; \\ L_3 = 0.427[\mathrm{m}];\ L_4 = 0.24[\mathrm{m}];\ k_{34}^{(11)} = 180\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{34}^{(13)} = 0\left[\mathrm{Nm}/\mathrm{rad}^3\right]; \\ k_{34}^{(21)} = 300\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{34}^{(23)} = 0\left[\mathrm{Nm}/\mathrm{rad}^3\right];\ k_{23}^{(11)} = 350\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{23}^{(13)} = 0\left[\mathrm{Nm}/\mathrm{rad}^3\right]; \\ k_{23}^{(21)} = 1000\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{23}^{(23)} = 350\left[\mathrm{Nm}/\mathrm{rad}^3\right];\ k_{13}^{(1)} = 600000\left[\mathrm{N}/\mathrm{m}\right];\ k_{13}^{(3)} = 0\left[\mathrm{N}/\mathrm{m}^3\right]; \\ k_{12}^{(1)} = 600000\left[\mathrm{N}/\mathrm{m}\right];\ k_{12}^{(3)} = 0\left[\mathrm{N}/\mathrm{m}^3\right];\ k_{01} = 800000\left[\mathrm{N}/\mathrm{m}\right], \end{gathered} \tag{12}$$

the bodies are homogeneous and the elastic force $F_{e_{13}}$ is applied in the mass center of the third body for the linear case, and the same assumptions but the values

$$\begin{gathered} m_1 = 1000[\mathrm{kg}];\ m_2 = 26.68[\mathrm{kg}];\ m_3 = 46.06[\mathrm{kg}];\ m_4 = 5.52[\mathrm{kg}];\ \mu = 0.7; \\ L_3 = 0.427[\mathrm{m}];\ L_4 = 0.24[\mathrm{m}];\ k_{34}^{(11)} = 180\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{34}^{(13)} = 5\left[\mathrm{Nm}/\mathrm{rad}^3\right]; \\ k_{34}^{(21)} = 300\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{34}^{(23)} = 7\left[\mathrm{Nm}/\mathrm{rad}^3\right];\ k_{23}^{(11)} = 350\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{23}^{(13)} = 20\left[\mathrm{Nm}/\mathrm{rad}^3\right]; \\ k_{23}^{(21)} = 1000\left[\mathrm{Nm}/\mathrm{rad}\right];\ k_{23}^{(23)} = 50\left[\mathrm{Nm}/\mathrm{rad}^3\right];\ k_{13}^{(1)} = 600000\left[\mathrm{N}/\mathrm{m}\right];\ k_{13}^{(1)} = 70\left[\mathrm{N}/\mathrm{m}^3\right]; \\ k_{12}^{(1)} = 600000\left[\mathrm{N}/\mathrm{m}\right];\ k_{12}^{(3)} = 70\left[\mathrm{N}/\mathrm{m}^3\right];\ k_{01} = 800000\left[\mathrm{N}/\mathrm{m}\right], \end{gathered} \tag{13}$$

for the non-linear case.

The step time is $\Delta t = 0.001[\mathrm{s}]$, and the initial values are

$$\begin{gathered} x_1^0 = 0[\mathrm{m}];\ x_2^0 = 0[\mathrm{m}];\ \theta_3^0 = -\frac{\pi}{10}[\mathrm{rad}];\ \theta_4^0 = \frac{11\pi}{180}[\mathrm{rad}];\ \dot{x}_1 = 15\left[\mathrm{m}/\mathrm{s}\right]; \\ \dot{x}_1 = 0\left[\mathrm{m}/\mathrm{s}\right];\ \dot{\theta}_3^0 = 0\left[\mathrm{rad}/\mathrm{s}\right];\ \dot{\theta}_4^0 = 0\left[\mathrm{rad}/\mathrm{s}\right]. \end{gathered} \tag{14}$$

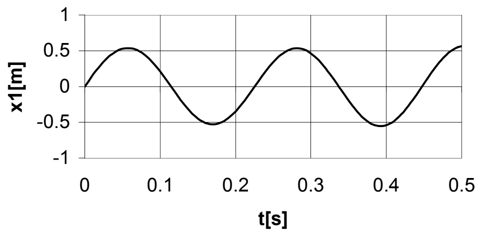

Figure 2. Time history for x_1 in the linear case.

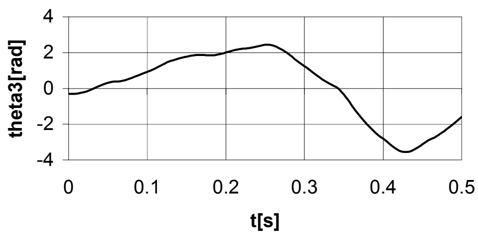

Figure 3. Time history for x_3 in the linear case.

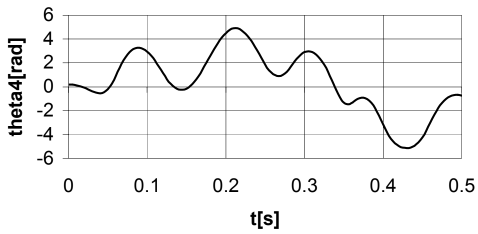

Figure 4. Time history for x_4 in the linear case.

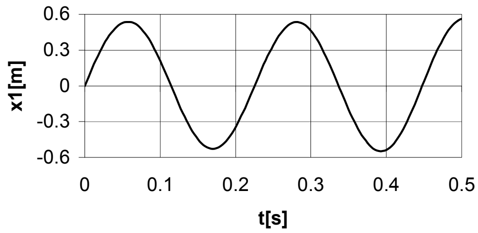

Figure 5. Time history for x_1 in the non-linear case.

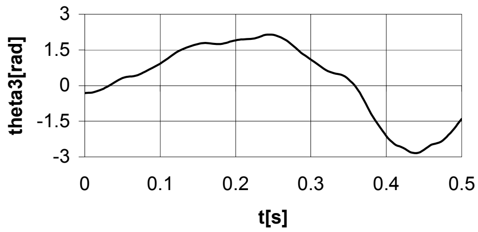

Figure 6. Time history for x_3 in the non-linear case.

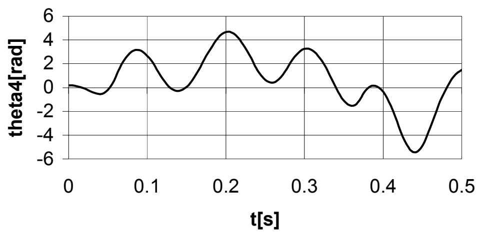

Figure 7. Time history for x_4 in the non-linear case.

The graphics were plotted in the figures 2, 3, 4, 5, 6, and 7.

4. CONCLUSIONS

Based on a model presented in a previous paper, we realized realistic simulation for the behavior of a human body in a car crash. The elastic forces and moments are considered both in a linear case an also in a non-linear case. It is easy to observe tat the non-linear case leads to a diminution in amplitude for the motions of the bodies. In fact, the security belts are almost non-linear so the non-linear model is closer to the reality.

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