ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

ROBUST STABILITY IN FEEDBACK ACTIVE NOISE CONTROL

Damien Leclercq, Mike Kidner and Colin Hansen

School of Mechanical Engineering, The University of Adelaide
Adelaide, SA5005, Australia
damien.leclercq@adelaide.edu.au

Abstract

A major hurdle in the design of a feedback system for active noise control applications is the difficulty in guaranteeing robust stability while maintaining adequate noise reduction performance. This requirement is particularly limiting when variations in the control plant response are large, as is the case with active headrests, where head movements can drastically modify the acoustic propagation between the control sources and error sensors, which affects both the magnitude and phase of the plant transfer function. It is possible, to some degree, to design a robustly stable controller that can accommodate plant variations within a range that is determined during preliminary laboratory measurements of all expected operational configurations. The robustly stable control filter is then designed by a computationally expensive optimisation procedure based on the assumed disturbance spectrum, the nominal plant response, and its expected maximum variation. However, plant variations are in some cases so large that it is not feasible to design a controller around a nominal plant response with a pre-determined uncertainty bound, and the nominal plant response needs to be characterised at various times during controller operation. This mode of operation is made possible by the appearance of control prototyping platforms constructed around powerful DSPs and equipped with considerable memory space. Although this does not allow the expensive optimisation to be carried out by the DSP, there is great potential to make robustly stable plants that can accommodate major variations in the system and primary disturbance. This paper presents the development of a feedback controller based on the internal model control architecture and evaluates the performance of various strategies to design an adaptive, feedback control filter that is robustly stable to variations in primary disturbance spectrum and plant transfer function.

1. INTRODUCTION

Feedback control is in some situations the only practical strategy for effective active noise control, as, for example, in the passenger compartment of a high-speed vehicle where broad band noise is to be controlled. In such cases, the noise sources are distributed throughout the cabin, and over the external wall, as for example in ships and aircraft where boundary layer and propulsion noises combine to generate a diffuse field around the passengers. In this context, it is not feasible to account for all of the energy in the acoustic field surrounding the passenger with reference sensors, and feedforward noise control strategies are not suitable.

This has led some researchers to investigate various feedback control systems for such applications, such as the active headset and headrest systems. Active headsets are by far the most successful applications, with a great variety of systems available as consumer products. Active headrests, on the other hand, are yet to find a large market application, despite promising results [1,2]. Anyone taking up research in feedback active control will be struck by the apparent simplicity of the strategy, and subsequently daunted by the scope of work to ensure robust stability of the system whilst guaranteeing a minimum level of control performance. This complexity arises in part from the fact that there is no longer one optimal solution to the control problem, as it becomes necessary to find the best compromise between efficient control and robust stability, which, for practical applications, leads to a number of design choices that depend on the situation at hand as well as the person designing the controller.
After a brief discussion of robust stability in feedback controllers, a representative test case is presented based on the characteristics of an actual plant, and an idealised primary disturbance spectrum is used. A model of the plant uncertainty is also given. Three different techniques aimed at ensuring that the adaptive controller remains robustly stable are then briefly described and implemented in the computer-based model developed in the Matlab/Simulink simulation environment. Controller performance using these three techniques is assessed and conclusions are drawn regarding the suitability of each method for the case at hand.

1. FEEDBACK CONTROL AND ROBUST STABILITY

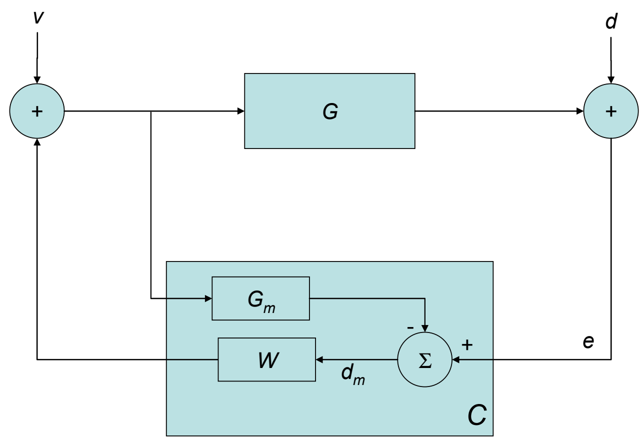

Figure 1 Feedback control block diagram (with Internal Model Control)

Consider the feedback controller illustrated in Figure 1, where G and C denote the plant and compensator transfer functions respectively. The residual error e is the sum of the plant output and the primary disturbance signal d. The control filter is in this case based on the Internal Model Control (IMC) strategy, where a primary disturbance estimate d_m is calculated from the measured error signal and the control signal filtered by a plant of the model G_m. The sensitivity function of this feedback controller is the transfer function between the disturbance and the error:

$$S = 1/(1-GC) \tag{1.}$$

The IMC formulation of this equation is:

$$S = (1+G_mW) / (1-(G-G_m)W) \tag{2.}$$

where W is the control filter transfer function, so that the condition for stability is:

$$|(G-G_m)W| < 1 \qquad (3.)$$

at all frequencies. Assuming that G_m is an accurate model of the nominal plant transfer function G_0, and that the actual plant G differs from G_0 by ΔG, then the robust stability constraint is rewritten:

$$|G_0BW| < 1 \qquad (4.)$$

where B is the upper bound of the normalised plant uncertainty $\Delta G/G_0$. Robust stability is of course critical for any application outside of the research laboratory, and there are different ways of ensuring that a feedback controller satisfies the robust stability criterion. The plant uncertainty factor B is determined by a series of measurements combined with a good understanding of the plant under consideration and the various environmental parameters that are likely to affect its response. The simplest approach to ensure the robust stability of non adaptive feedback controllers is to design the compensator and subsequently adjust the gain until phase and gain margins are larger than a value determined by the type of application. Ensuring the robust stability of adaptive feedback controllers is however more complex, due to the adaptation process itself. Various methods can be used to enforce the robust stability constraint on the adaptive process, and three of these methods are described and evaluated on a practical application in this paper.

2. PLANT AND UNCERTAINTY MODELS

The plant under consideration consists of an actuator and an error microphone located in the vicinity of a curved wall, and the plant transfer function measured between the signal fed to the power amplifier and that produced by the microphone is plotted in Figure 2.

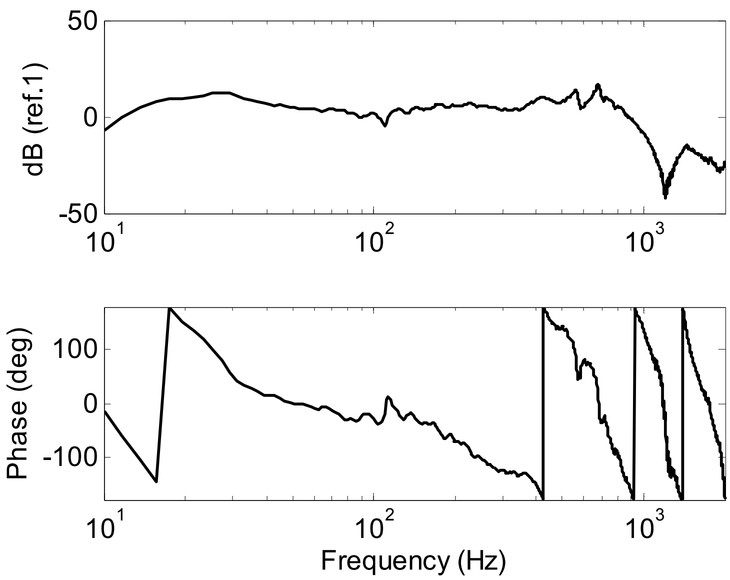

Figure 2 Measured plant transfer function G

This plant was designed to ensure sufficient output at low frequencies, since the intended application is to control broadband noise between 50 and 200 Hz, and its transfer function shows good potential for this application. However, the steep phase shift at frequencies below 40 Hz is associated with large group delays in excess of 5 ms, which is a significant issue in terms of performance and stability when implementing feedback control on broadband noise. This measured transfer function was used to evaluate the control algorithm in the Matlab/Simulink environment, where the primary disturbance is modelled as a white noise filtered by a second order Butterworth band pass filter with cut-off frequencies of 40 Hz and 80Hz. It can be anticipated that control will be affected by the high group delay at the low end of this spectrum, which will provide a good test for robust stability. The normalised plant uncertainty model shown in Figure 3 was derived from measurements in various configurations and is a very conservative envelope of the obtained data where control is either not necessary or unwanted.

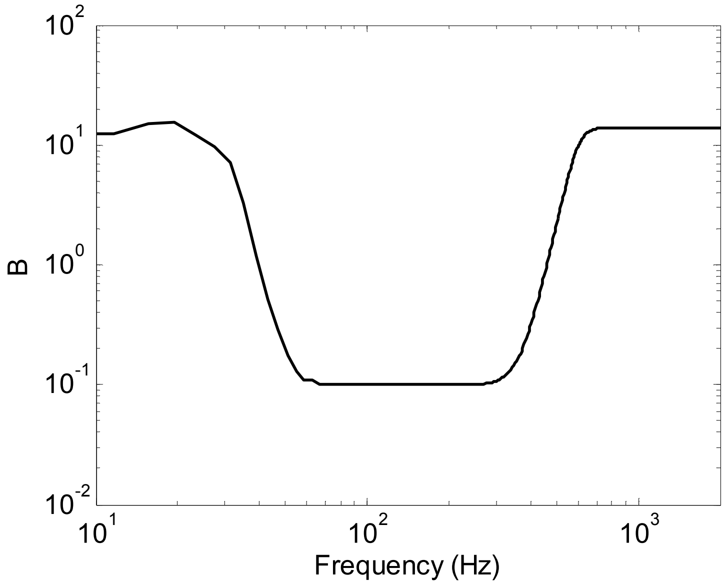

Figure 3 Model of the normalised plant uncertainty function *B*

3. ADAPTIVE FEEDBACK CONTROLLER THREE STRATEGIES FOR ROBUST STABILITY

The feedback control algorithm is developed in the Matlab/Simulink environment for subsequent application on a rapid prototyping platform. The actual plant is modelled using the transfer function shown in Figure 2, and a 12 bit quantisation on the input and output, as well as additional electrical noise on the sensor to ensure that a controller validated through simulations performs as expected when implemented on an experimental set-up. The controller algorithm is a standard implementation of the IMC shown in Figure 1, where the control filter W is adapted using the filtered reference block LMS algorithm after an initial plant identification phase. Other supervisory processes monitor the state of the controller to detect the likelihood of the controller going unstable, or to analyse non stationary noise events in order to determine, for example, whether instability is occurring or an impulsive noise is overloading the inputs. The block LMS update equation is [3]:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n)+\mu\,\mathrm{IFFT}\{(X^{*}(k)E(k)\}_{+} \qquad (5.)$$

where $\boldsymbol{w}$ is the vector of filter taps, X and E are the FFT of the blocks of reference and error signals, respectively. The convergence coefficient μ is automatically reduced until the

adaptation process is stable. Several options are available to maintain the robust stability condition during the control filter adaptation, and three of them are investigated for the present application.
Rafaely and Elliott [4] propose various equations for filter update with constraints on the filter, which are implemented as convex penalty cost functions with a weighting factor σ_1 that sets the emphasis on the additional cost of constraint violation. The robust stability constraint corresponding to Eq.4 is written as:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n)+\mu\,\mathrm{IFFT}\{(X^*(k)E(k)+4\sigma_1 N[|WGB|^2\text{-}1]_z|GB|^2W\}_+ \tag{6.}$$

where W is the FFT of $\boldsymbol{w}$, N is the number of filter coefficients, and σ_1 is the weight given to the penalty associated with the violation of the robust stability constraint. The $[]_z$ operator produces the value of its argument when it is positive, and zero otherwise. Rafaely and Elliott successfully applied this approach to active sound equalisation, and demonstrated its superiority compared with the implementation of a frequency independent leakage factor. However, it is also possible to adjust the leakage factor to ensure that the updated filter fulfils robustness stability constraints, which is similar to the approach suggested by Elliott [5]. This second method is implemented by setting the reference value of the leakage coefficient at ν_0, and deriving the frequency-dependent leakage coefficient $\nu(f)$ from the robust stability constraint at every step of the adaptive algorithm:

$$\begin{aligned} W(n+1) &= \nu(n)\, W(n)+\mu\{(X^*(k)E(k)\}_+ \\ \nu(n+1) &= \nu_0(n)\, \exp(-\sigma_2[|W(n)GB|^2\text{-}1]_z) \end{aligned} \tag{7.}$$

The leakage factor is thus reduced at frequencies where the robust stability constraint is not satisfied, which effectively reduces the control filter gain at these frequencies.
A third alternative [6] is to optimise the control filter based on a cost function that is the sum of the mean square error and the magnitude of the $|GWB|$ weighted by a factor σ_3. This is equivalent to adding to the disturbance estimate a white noise of variance σ_3 run through a shaping filter of response B. The cost function that the filter adaptation process minimises is in this case [7]:

$$\int S_{ee}+\sigma_3|GWB|^2\, \mathrm{d}\omega \tag{8.}$$

Unlike the constrained optimisation of Rafaely and Elliott and the frequency dependent leakage adjustment, this method does not provide a "hard" boundary to guarantee that the adapted control filter satisfies the robust stability condition. However, due to the sensor noise analogy this approach may prove useful in some situations where a time domain LMS algorithm is used.
These three methods were implemented within the controller developed in Simulink. Figure 4 shows the evolution of the overall gain at the error sensor in a reference case without a requirement for robust stability, as well as with the three robust stability constraints methods. The initial convergence coefficient was set at μ_0=0.01, and the following weighting factors were determined after a series of trials: σ_1=0.001, σ_2=1 and σ_3=0.1. After the initial increase in noise level at the error sensor corresponding to the plant identification phase, the controller adaptation starts and the error decreases. Sudden jumps in primary disturbance attenuation occur every time the LMS algorithm is close to becoming unstable, in which case the control filter is reset, the convergence coefficient is reduced, and plant adaptation resumes. Noise

reduction performance varies greatly between the different implementations of the robust stability constraint. The overall reduction lies between a minimum of 2.3 dB for the sensor noise approach (method 3) and a maximum of 8.2 dB for the adjusted leakage coefficient (method 2). The constrained optimisation (method 1) provides a reduction of 4.3 dB, while an attenuation of 5.7 dB is reached when the requirement for robust stability is disregarded. The adaptation process itself behaves differently between the different methods, and the most notable feature is the fact that the LMS algorithm remains stable for relatively high values of the convergence coefficient using Eq.(7), while all other simulations show that the convergence coefficient needs to be reduced several times to keep the adaptation stable. This can be expected since the purpose of leakage is to keep the LMS algorithm stable, but is also explained by comparing the error spectra obtained from each simulation, which are plotted in Figure 5

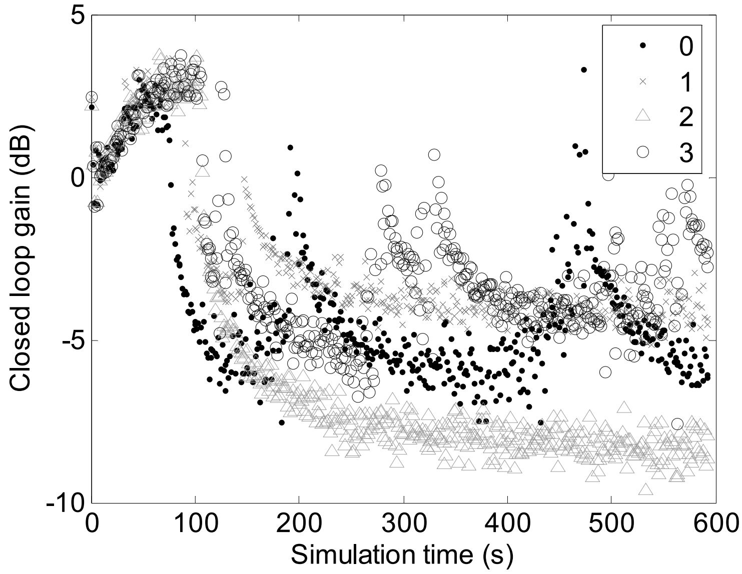

Figure 4 Overall gain during control. 0: no robust stability constraint;

1: constrained adaptation; 2: frequency dependent leakage adjustment; 3: sensor noise.

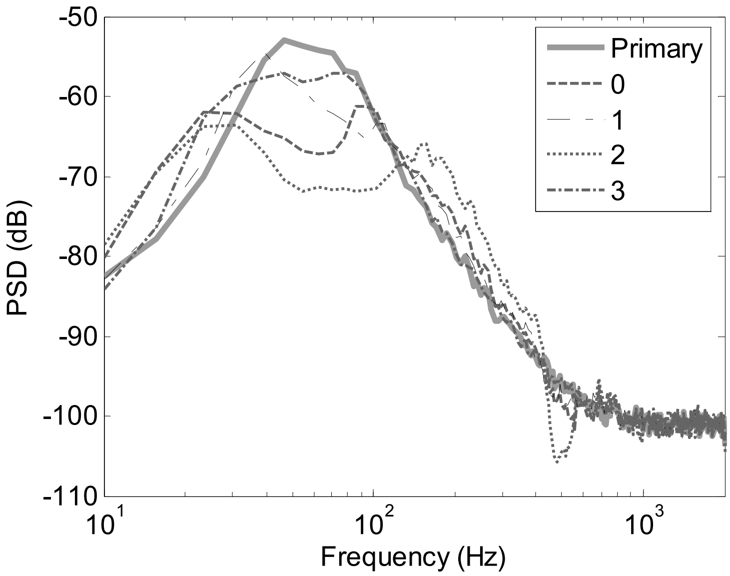

Figure 5 Primary disturbance and error spectra.

The high group delay at low frequencies tends to make the feedback controller unstable below 50 Hz. However, due to the high value of *B* at low frequencies, the controller gain remains small when Eq.7 is used. In Figure 6 the value of the robust stability criterion |GWB| obtained with the three proposed methods is compared to that obtained without the constraint.

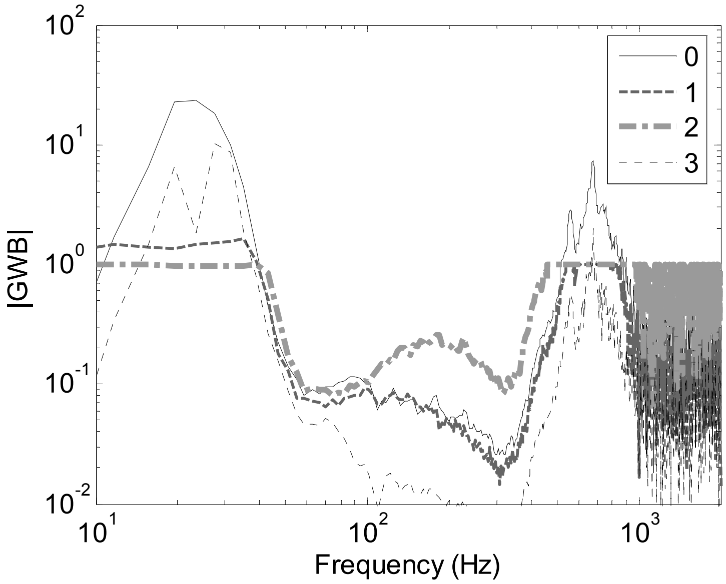

Figure 6 Robust stability criterion at the end of the 600 s simulation period.

A value of $|GWB|$ below 1 indicates that the controller is robustly stable.

Eq.6 and Eq.**7** adequately enforce robust stability, while the sensor noise method merely improves robust stability, but by no means guarantees it. The value of σ_1 could be increased to maintain a value of less than unity at low frequencies, but it was found to have an excessively adverse effect on the control performance, and a better option would be to introduce some leakage in the update equation. However, this would have made the comparison difficult and was not done for this paper, although it certainly is a practical option for the actual implementation of the controller. Finally, the magnitude of the adapted control filter transfer function is plotted and compared with the H_2 optimal filter [6].

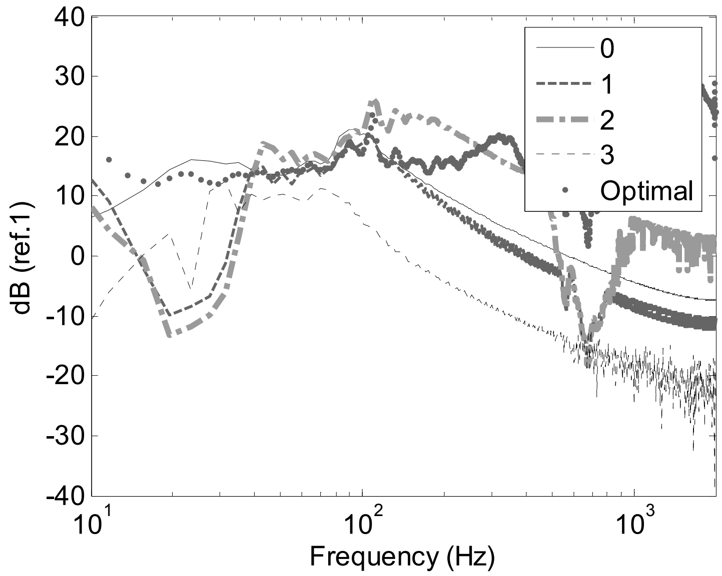

Figure 7 Control filters obtained after adaptation, compared with the H_2 optimal controller.

This figure shows that the LMS adaptation converges to the H_2 optimal filter in the frequency range where the primary disturbance spectrum has a high energy, except for the sensor noise method, for which the level of noise required to enforce robust stability is such that the coherence between the reference signal and the error signal becomes excessively low.

CONCLUSIONS

Very powerful optimisation techniques can be used to develop a control filter that satisfies the requirements for robust stability and good noise reduction. However, they are too computationally expensive to be implemented directly by the controller in real time using current commercially available technology. This paper explores three strategies that can easily be integrated in the controller algorithm in order to provide a robustly stable controller using internal model control. Although useful for time domain LMS adaptation, the sensor noise method drastically diminishes control performance and does not guarantee a robustly stable controller. In cases where filter adaptation is based on a frequency domain LMS algorithm, the constrained adaptation equation proposed by Rafaely and Elliott [4] can guarantee a robustly stable controller and adequate performance. The frequency-dependent leakage adjustment was found to have the additional advantage of stabilising the adaptation at frequencies where the plant is difficult to control because of a large and sharp phase shift in its frequency response. These two methods seem well adapted to the example used for the work presented in this paper, and will be tested on the physical plant. It is likely that the final controller design, to be tested in the near future on a physical platform in a realistic environment, will implement a combination of these two methods.

ACKNOWLEDGEMENTS

Financial support from The Australian Research Council is gratefully acknowledged.

---

[1] Rafaely B. and Elliott S. 1999. H_2/H_∞ active control of sound in a headrest: design and implementation. IEEE Transaction on Control Systems Technology 7(1) pp.79-84.
[2] Dignan, M., et.al. 1994. The Active Control of Airborne Noise in a High Speed Patrol Craft, Noise-Con 94, Ft. Lauderdale, Florida, May 1994
[3] S.M. Kuo and D.R. Morgan 1996. Active noise control systems: algorithms and DSP implementations. Wiley series in telecommunications and signal processing. John Wiley & Sons, Inc. 1996. Chapter 8.
[4] Rafaely B. and Elliott, S. 2000. A computationally efficient frequency domain LMS algorithm with constraint on the adaptive filter. IEEE Transactions on Signal Processing 48(6) 1649-1655.
[5] Elliott S. 2001. Signal Processing for active noise control. Academic Press, London 2001. Chapter 7.
[6] Elliott S. 2001. Signal Processing for active noise control. Academic Press, London 2001. Chapter 6.
[7] Elliott S. and Sutton T. 1996. Performance of feedforward and feedback systems for active control. IEEE Transactions on Speech and Audio Processing 4(3) 214-223