ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

A MODEL FOR CALCULATING VIBRATION FROM A RAILWAY TUNNEL BURIED IN A FULL-SPACE INCLUDING RIGID BEDROCK

L. Rikse[1 *], H.E.M Hunt[2], M.F.M Hussein[3], G. Degrande[1], S. Gupta[1]

[1]K.U.Leuven, Department of Civil Engineering, Kasteelpark Arenberg 40, B-3001 Leuven, Belgium

[2]Engineering Department, University of Cambridge, Trumpington Street, Cambridge, CB2 1PZ, UK

[3]School of Civil Engineering, University of Nottingham, Coates Building University Park, Nottingham, NG7 2RD, UK

[*]lars.rikse@student.kuleuven.be

Abstract

Subway induced vibration has become an important environmental problem. Ground borne noise and vibration have a significant impact on the comfort of inhabitants of buildings near railway tunnels. Therefore, legislation has become strict and the design of vibration countermeasures is of increasing importance. Engineers need accurate and fast running models. Over the last 10 years a computationally efficient model, known as the Pipe-in-Pipe (PiP) model, has been developed. In its basic formulation, the PiP model accounts for a tunnel in a full space. Two methods are presented in this paper to account for a rigid bedrock underneath the tunnel.

The first is the mirror-image method and takes the bedrock into account by mirroring the tunnel and the applied load around the soil-bedrock interface. Vibration generated by the mirrored load represents the reflected wave field. The mirror image method is very efficient but it is only an approximation of the real boundary condition at the soil-bedrock interface.

The second method is a new development of the PiP model and is based on the assumption that the near field displacements are not influenced by the existence of the rigid bedrock. Green's functions for a full space are used to calculate the internal source in a full space that would produce the same displacements at the tunnel-soil interface as calculated by the original PiP model (e.g. a tunnel in a full-space). This internal source is then used to calculate the far field displacement by using Green's functions for a half-space with a fixed surface.

The two different methods are compared with an alternative Finite Element-Boundary Element (FE-BE) model, from which it is concluded that the modified PiP model produces more accurate results then the mirror-image method.

1. INTRODUCTION

Vibration in buildings near underground tunnels is generated due to irregularities of wheels and tracks. Vibration propagates into the surrounding soil and finally reaches the nearby buildings. This causes the structure to vibrate which is perceptible in the frequency range from 1 to 80 Hz. Vibration in buildings causes disruption of activities and discomfort due to rumbling noise and shaking of objects. It may also cause structural damage, especially in old masonry buildings. In the wide frequency range from about 30 to 200 Hz, ground vibration may excite bending resonances in the floors and walls of buildings which radiate an unpleasant rumbling noise directly into the rooms.

Vibration countermeasures are used to reduce the vibration levels which can be achieved by isolating the building. Steel springs and rubber bearings are used as elements for base isolation. Isolating the source, however, is more effective because it has an effect on multiple buildings. Ballast mats, soft railpads and floating-slab tracks are widely used as vibration countermeasures for underground railway tracks. Implementing these countermeasures is very expensive and therefore railway engineers need theoretical models to predict the level of vibration generated by running trains and to assess the efficiency of vibration countermeasures. From an engineering point of view, these models have two substantial requirements: accuracy, for obvious reasons, and computational efficiency.

Two models have been developed to calculate vibration from railway tunnels. The first is the coupled Finite Element-Boundary Element (FE-BE) model [1]. This is a three-dimensional numerical model. The dynamic tunnel-soil interaction is solved with a subdomain formulation, using a finite element formulation for the tunnel and a boundary element method for the surrounding soil. The periodicity of the geometry in the direction of the tunnel is exploited using the Floquet transform which significantly reduces the discretization effort. The model is accurate but attention should be paid to the mesh discretization. The main advantage of this model is its flexibility as it can account for a free surface, rigid bedrock, layering and non-circular tunnel walls. Although improvements have been made to the model, it requires high performance computational resources and takes a long time to run. The FE-BE model is useful for research purposes but is computationally too expensive to be used as an engineering tool.

The second is the Pipe-in-Pipe (PiP) model [2] which comprises two concentric pipes. The first pipe represents the tunnel. The second pipe has an infinite outer radius and represents an infinite soil with a cylindrical cavity. Both pipes are modelled with the elastic continuum theory. The PiP model has the advantage of being computationally efficient. The model has recently been developed to account for a half-space [3, 4], and is based on the assumption that the near field displacements are not influenced by the presence of the free surface.

This paper presents two new methods to account for a rigid bedrock underneath the tunnel. An outline of the problem is presented first, followed by a detailed description of both methods: the mirror image method and the modified PiP model. The results and computational efficiency of these two methods are compared with results obtained with the Finite Element-Boundary Element (FE-BE) model.

2. PROBLEM OUTLINE

In this paper, the vertical displacements in the soil due to a load applied at the tunnel invert are calculated. The soil is modelled as a half-space with a fixed surface at a distance D underneath the tunnel center (Figure 1). There is no free surface. The load takes the form $F = \tilde{F}e^{i(\omega t+\xi x)}$, with ξ the wavenumber along the x-axis and ω the angular frequency.

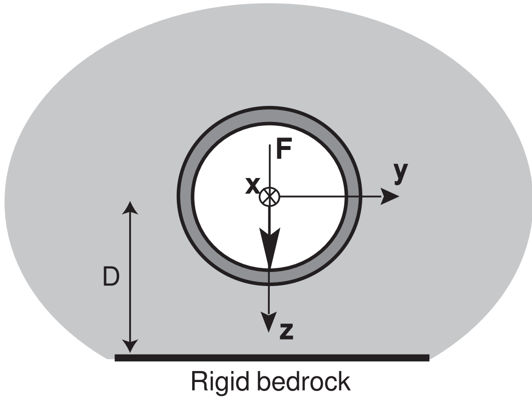


Figure 1. A tunnel in an elastic half-space with a fixed surface, representing the rigid bedrock underneath the tunnel.

3. THE MIRROR-IMAGE METHOD

To account for the bedrock, the tunnel and the applied load are mirrored around the soil-bedrock interface, as illustrated in Figure 2(a). Displacements in the soil are now caused by the direct field $\tilde{F}$, as well as by the reflected field $\tilde{F}_m$ represented by the mirrored tunnel.

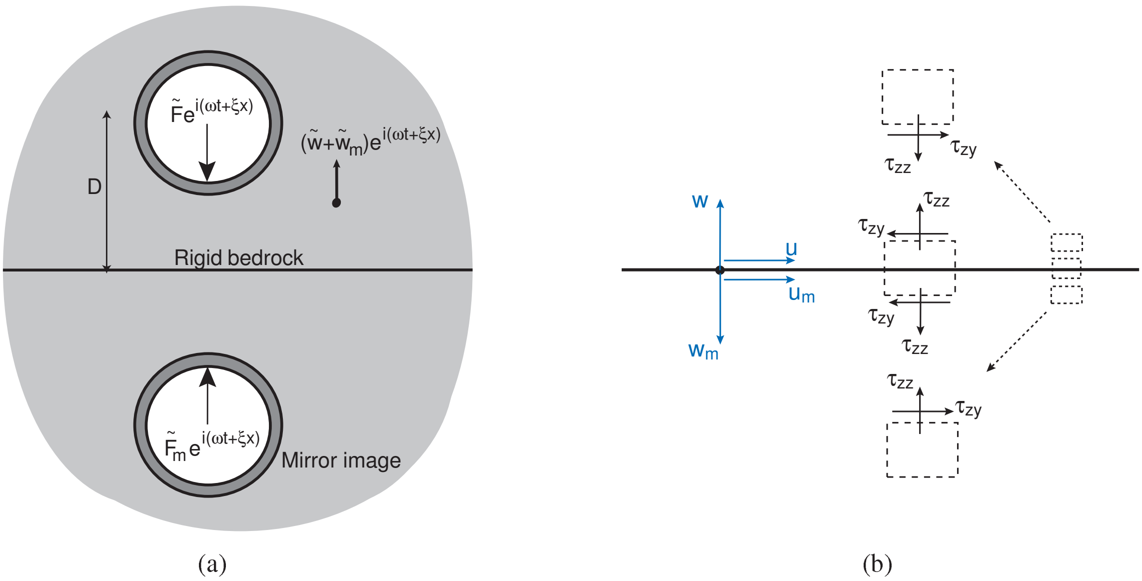


Figure 2. The mirror image method to account for rigid bedrock: (a) a cross sectional view (x=0) and (b) boundary conditions at the soil-bedrock interface.

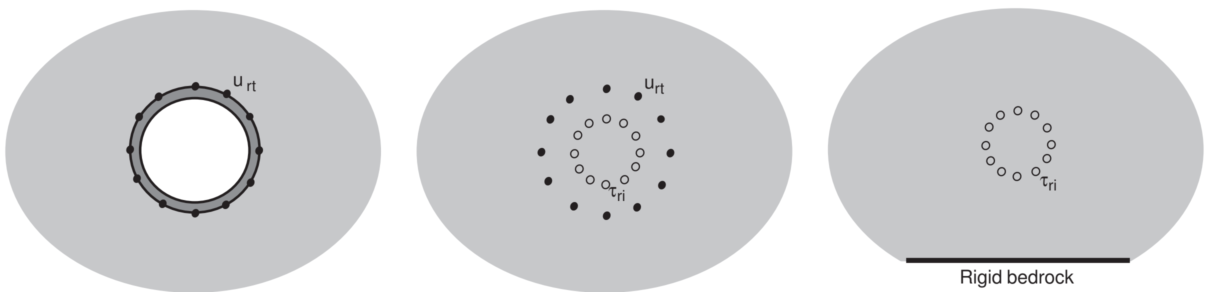
Rigid bedrock implies that all displacements are zero at the soil-bedrock interface. From Figure 2(b) it can be seen that, when using the mirror image method, only vertical displacements are zero. As there are no external forces applied at the boundary, it is also clear that horizontal stresses are zero at the boundary, from which we can conclude that the mirror image method represents a sliding boundary at the bedrock, rather than a fixed boundary. The horizontal displacements in the soil, due to a vertical force at the tunnel invert, are rather small compared to vertical displacements and therefore the mirror image method can be expected to be a good approximation for the real boundary condition. The mirror-image method also has the advantage to be very computationally efficient. The original PiP model [2] just has to be used twice, once for the direct field and once for the reflected field.

4. THE MODIFIED PIP MODEL

The modified PiP model is based on the assumption that the displacements at the tunnel-soil interface are not influenced by the presence of the rigid bedrock. A similar method has been developed to include a free surface [4].

The introduced method consists of 3 important steps (Figure 3) which will be discussed in detail in the following subsections.

1. Calculate displacements at the tunnel-soil interface, using the original PiP model (e.g. a tunnel in a full-space).
2. Use Green's functions for a two-and-a-half-dimensional elastodynamic full-space to calculate the equivalent internal source in a full-space, which produces the same displacements at the tunnel-soil interface as calculated in step 1.
3. Use this internal source in a half space with a fixed surface, to calculate the far-field displacements.



(a) (b) (c)

Figure 3. Methodology of the modified PiP model: (a) calculating the displacements at the tunnel-soil interface using the original PiP model, (b) calculating the equivalent internal source in a full-space and (c) using this internal source in a half space with a fixed surface.

4.1. The displacements at the tunnel-soil interface

The displacements $\boldsymbol{u}_{\boldsymbol{r}_t}$ $\boldsymbol{u}_{r_t}$ at the tunnel-soil interface, due to a load $F = \tilde{F}e^{i(\omega t+\xi x)}$ at the tunnel invert, can be calculated by using the original PiP model [2]. The vector $\boldsymbol{u}_{\boldsymbol{r}_t}$ gives the longitudinal, horizontal and vertical displacements, which can be expressed as $\boldsymbol{u}_{\boldsymbol{r}_t} = \sum \tilde{\boldsymbol{u}}^{\boldsymbol{n}}_{\boldsymbol{r}_t} e^{i(\omega t+\xi x)}$. Note that $\tilde{\boldsymbol{u}}^{\boldsymbol{n}}_{\boldsymbol{r}_t}$ is written in the wavenumber-frequency domain and that the sum accounts for the circumferential modes ($n = 0, 1, 2, \ldots N$).

4.2. The internal source

In this step, the internal source is calculated at a radius less than the outer radius of the tunnel wall, that produces the same displacements $\boldsymbol{u}_{\boldsymbol{r}_t}$ in an elastodynamic full-space. One way to calculate this internal source is to use the two-and-a-half dimensional Green's functions for a full space calculated by Tadeu and Kausel [5] along with the known displacements at the tunnel-soil interface. An alternative is to model a full-space that comprises 2 submodels: (1) a cylinder with an infinite outer radius and an inner radius $r_i < r_t$, representing a full space with a cylindrical cavity and (2) a solid cylinder with outer radius r_i and inner radius equal to zero. This formulation is used to calculate the applied stress at a virtual cylinder with radius r_i (i.e. the interface between the two submodels) which will produce displacements $\boldsymbol{u}_{\boldsymbol{r}_t}$ at a radius r_t in the full-space. This methodology in cylindrical coordinates, described by Hussein et al. [4], is similar to the one used for the original PiP model and is therefore much more efficient to calculate the internal source $\boldsymbol{\tau}_{\boldsymbol{r}_i} = \sum \tilde{\boldsymbol{\tau}}^{\boldsymbol{n}}_{\boldsymbol{r}_i} e^{i(\omega t+\xi x)}$. Note that $\boldsymbol{\tau}_{\boldsymbol{r}_i}$, as well as $\boldsymbol{u}_{\boldsymbol{r}_t}$, is a function of the cylindrical coordinate θ. From these stresses, M line loads at the virtual cylinder can be calculated. Each load has three components and can be expressed as $\boldsymbol{f}_j = \tilde{\boldsymbol{f}}_j e^{i(\omega t+\xi x)}$ $(\boldsymbol{j} = 0, 1, \ldots M)$.

4.3. The far-field displacements

In this last step, the internal source is applied in a half-space with fixed surface. Green's functions are required to express the relationship between displacements and applied forces. The direct stiffness formulation will be used here. The method is based on the transfer matrix formulation [6] and modified to a stiffness matrix formulation [7]. The building blocks of the method are the half space and the layer element of which the dynamic stiffness matrices are assembled in the wavenumber-frequency domain. A MATLAB toolbox (ElastoDynamics Toolbox 2.0), based on this theory, has been developed to model seismic wave propagation [8, 9]. This toolbox can be used to model a half-space with a fixed surface. The Green's functions are used to compute the far-field vertical displacements from the following relationship:

$$\tilde{w} = \left[\tilde{G}_{z1} \ \tilde{G}_{z2} \ \ldots \ \ldots \ \tilde{G}_{zM}\right] \left[\tilde{f}_1 \ \tilde{f}_2 \ \ldots \ \ldots \ \tilde{f}_M\right]^T \tag{1}$$

where $\tilde{G}_{zk}$ is a 3×1 matrix whose elements give the vertical displacement at a level z in the soil, due to a longitudinal, horizontal and vertical unit force at the point where $\tilde{f}_k$ is applied.

5. RESULTS

5.1. Parameters

A unit concentrated force is applied at the tunnel invert at $x = 0$. The tunnel has an outer radius $r_t = 3\ m$, a thickness $h = 0.25\,\text{m}$, a modulus of elasticity $E_t = 50\,\text{GPa}$, a Poisson's ratio $\nu_t = 0.3$, a density $\rho_t = 2500\,\text{kg/m}^3$, and a loss factor $\eta_t = 0.3$ associated with both Lamé's constants. The soil has a compression wave speed $c_p = 944\,\text{m/s}$, a shear wave speed $c_s = 309\,\text{m/s}$, a density $\rho_s = 2000\,\text{kg/m}^3$ and a loss factor $\eta_s = 0.06$ associated with both Lamé's constants. The results in Figures 4, 5 and 6 are calculated with a value $M = 21$. It should be mentioned that, in the FE-BE model, the full space is modelled as a very deep tunnel (depth = 100 m) and that the bedrock is modelled as a soil with very high values of c_p and c_s.

5.2. Comparison of the results

Figures 4, 5 and 6 show the vertical displacements at $\{0, 20, -20\}$ and $\{0, 25, -7, 5\}$ with a rigid bedrock at respectively 20, 15 and 10 m below the tunnel center.

It can be seen from Figure 4 that, if the distance from the tunnel to the bedrock is large enough ($D = 20\,\mathrm{m}$), the results from the modifed PiP model agree well with the results from the mirror-image method. The agreement is better for the point with a lateral distance of 20 m from the tunnel center (Figure 4(a)) because the other measuring point (Figure 4(b)) is further away from the tunnel in the horizontal direction. Consequently the horizontal boundary condition at the bedrock, which is different for the two methods, is more important and therefore another interference pattern arises. The modified PiP model agrees well with the results from the FE-BE model.

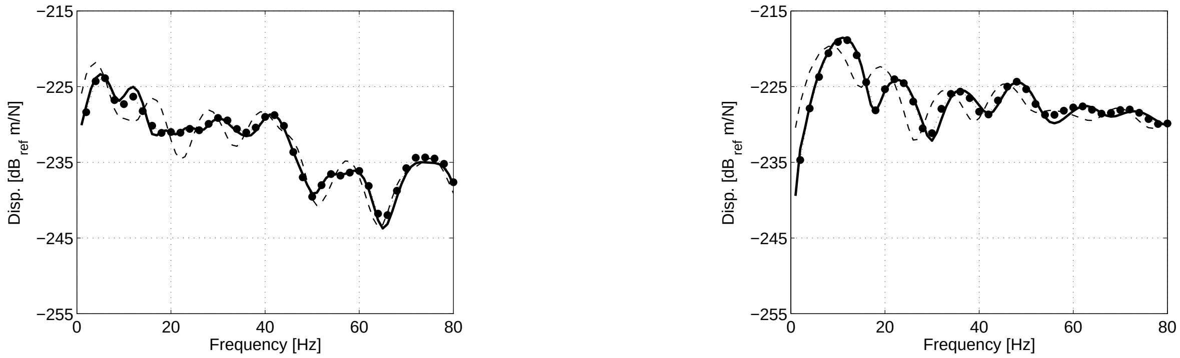
From Figure 5 it can be seen that, if the bedrock comes closer to the tunnel ($D = 15\,\mathrm{m}$), results from the mirror-image method do not match well with results from the modified PiP model. The FE-BE model still agrees with the modified PiP model. It is clear that the boundary condition becomes more important here.

If the bedrock comes even closer to the tunnel ($D = 10\,\mathrm{m}$), the difference between the mirror-image method and the modifed PiP model increases (Figure 6). The agreement between the FE-BE and the modifed PiP model is good but some small differences can be seen. Further research is in progress to investigate wether this is because of the approximation of the bedrock in the FE-BE model or due to the approximation in the modified PiP model.

The running time for the mirror image method and the modifed PiP model to produce the results is approximately 90 seconds on a PC with 1GB RAM and 2.0GHz processor. The same results are calculated by the coupled FE-BE model in approximately 26 hrs on a similar computer.

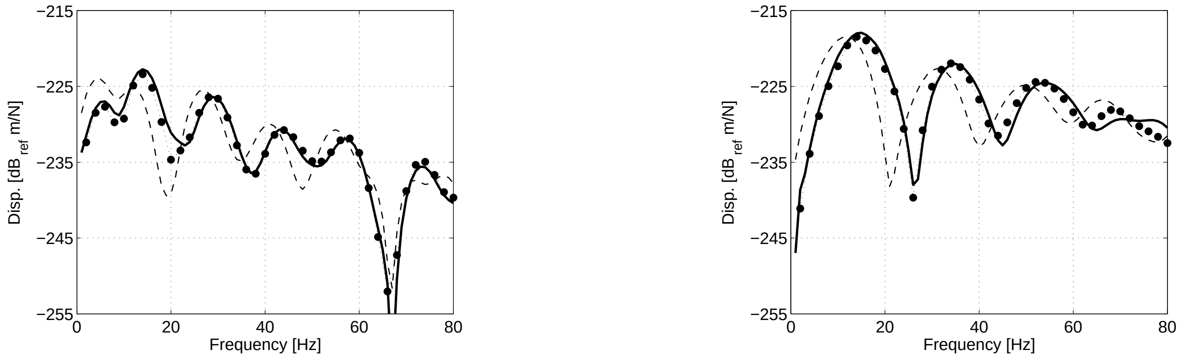
6. CONCLUSIONS

Two different models for calculating vibration from a railway tunnel buried in a half space with a fixed surface have been presented in this paper. It is clear that the approximation made by the mirror image method is not acceptable if the bedrock is close to the tunnel. The PiP model, however, agrees very well with the FE-BE model. Work is in progress to determine the limitations of the modified PiP model, such as the maximum distance between the tunnel and the bedrock at which the bedrock has significant effect on the near-field displacements. It is clear that this modified PiP model is accurate and computationally efficient and therefore it could be used as an engineering tool to predict levels of vibration due to underground railway trains. The methodology of the modified PiP model has been recently developed so that it can be used for a layered half-space as well [10].



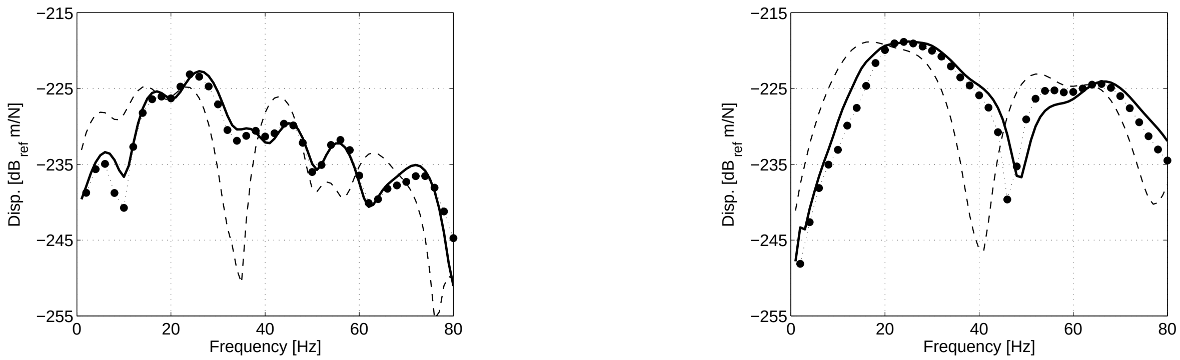
(a) (b)

Figure 4. Vertical displacement at the point (a) $\{0, 20, -20\}$ and (b) $\{0, 25, -7.5\}$ in a homogeneous soil with bedrock at 20 m below the tunnel center, calculated with the mirror-image method (– –), the PiP model (—) and the FE-BE model (●).



(a) (b)

Figure 5. Vertical displacement at the point (a) $\{0, 20, -20\}$ and (b) $\{0, 25, -7.5\}$ in a homogeneous soil with bedrock at 15 m below the tunnel center, calculated with the mirror-image method (– –), the PiP model (—) and the FE-BE model (●).



(a) (b)

Figure 6. Vertical displacement at the point (a) $\{0, 20, -20\}$ and (b) $\{0, 25, -7.5\}$ in a homogeneous soil with bedrock at 10 m below the tunnel center, calculated with the mirror-image method (– –), the PiP model (—) and the FE-BE model (●).

7. ACKNOWLEDGEMENT

Lars Rikse would like to thank the Erasmus fund for the financial support, which made it possible to carry out this research at the University of Cambridge.

REFERENCES

[1] G. Degrande, D. Clouteau, R. Othman, M. Arnst, H. Chebli, R. Klein, P. Chatterjee, B. Janssens, "A numerical model for ground-borne vibrations from underground railway traffic based on periodic finite element-boundary element formulation", *Journal of Sound and Vibration*, **293**, 645-666(2006).

[2] J.A. Forrest, H.E.M. Hunt, "A three-dimensional tunnel model for calculating of train-induced ground vibration", *Journal of Sound and Vibration*, **294**, 678-705(2006).

[3] M.F.M. Hussein, S. Gupta, H.E.M. Hunt, G. Degrande, J.P. Talbot, "An efficient model for calulating vibration from a railway tunnel burried in a half space", *Proceedings of The Thirtheenth International Congress on Sound and Vibration (ICSV13)*, 2-6 July 2006, Vienna, Austria.

[4] M.F.M. Hussein, S. Gupta, H.E.M. Hunt, G. Degrande, J.P. Talbot, "A computationally efficient model for calulating vibration from a railway tunnel burried in a half space", *International Journal for Numerical Methods in Engineering*, submitted for publication.

[5] A.J.B. Tadeu, E. Kausel, "Green's Functions for two-and-a-half-dimensional elastodynamic problems", *Journal of Engineering Mechanics*, **126**,10, 1093-1097(2000).

[6] W.T. Thomson, "Transmission of elastic waves through a stratified solid medium", *Journal of Applied Physcis*, **21**, 89-93(1950).

[7] E. Kausel, J.M. Roësset, "Stiffness matrices for layered soils", *Bulletin of the seismological Society of America*, **73**,6, 1743-1761(1981).

[8] G. Degrande, Seismic wave propagation, Department of Civil Engineering, K.U.Leuven, 2006.

[9] M. Schevenels, "The impact of uncertain dynamic soil characteristics on the prediction of ground vibrations",PhD at Department of Civil Engineering, K.U.Leuven, 2007.

[10] M.F.M. Hussein, L. Rikse, S. Gupta, H.E.M. Hunt, G. Degrande, J.P. Talbot, S. François, M. Schevenels, "Using the PiP model for fast calculation of vibration from a railway tunnel in a multi-layered half-space", *In: The 9th International Workshop on Railway Noise and Vibration (IWRNV9)*, 4-8 September 2007, München, Germany.