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BOUNDARY ELEMENT MODELS OF HORN LOADED LOUDSPEAKERS

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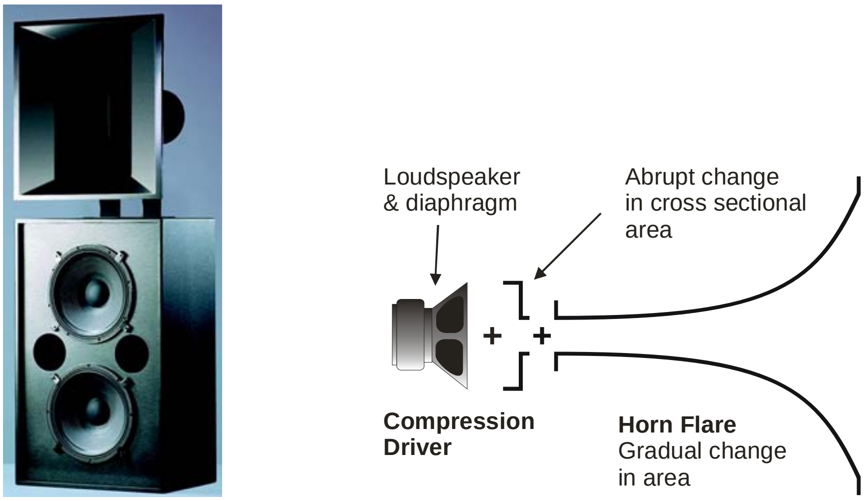
Abstract

Horn loaded loudspeakers are a type of loudspeaker used to efficiently radiate sound in a directional manner. Traditional horn theory takes a lumped parameter approach and these models are found to be inadequate at mid to high frequencies, as well as only being able to predict the sound pressure on axis. No directional information is provided with these models, and alternative approaches such as finite or boundary element methods are sought to overcome these limitations. This paper compares results obtained using two alternative boundary element based techniques, a traditional direct boundary element method and a new source superposition technique. Two representative horn loaded loudspeakers are modelled, and the results are compared to experiment.

1. INTRODUCTION

Horn loaded loudspeakers are often used as components in cinema sound systems. Figure 1 (a) shows a horn loaded loudspeaker mounted on top of a low frequency direct radiator loudspeaker. This system is located behind the cinema screen.

A horn loaded loudspeaker consists of two main components: a compression driver and a horn flare. A typical arrangement for a horn loaded loudspeaker is shown in Figure 1 (b). The source, or compression driver, consists of a small (usually titanium) diaphragm driven by a conventional electro-magnetic drive (voice-coil and magnet) into an abrupt change in cross sectional area. The flare changes the cross sectional area gradually from the throat through to the mouth of the horn, increasing the efficiency of the sound radiation. This means that less amplifier power is required for a given acoustic output, and is the traditional reason for the use of horns in audio.



(a) (b)

Figure 1. (a) Commercially available cinema loudspeaker system. (b) Schematic of a horn loaded loudspeaker system.

Horn flares are also used to control the spatial distribution of sound emanating from the horn mouth (the beamwidth). For the case of cinema audio, it is critical to the listening experience that the sound can be broadcast evenly onto the audience at all frequencies (frequency independent beamwidth) with no variation in volume with frequency (smooth frequency response). Horn design methods published in the last 30 years have often emphasised control over beamwidth rather than frequency response [1, 2]. This is because control over beamwidth can be gained at the expense of smooth frequency response by introducing internal reflections in the horn, and the resulting poor frequency response can be compensated by using a large amplifier and level equalisation. Thus modern horn design is typically a compromise, and good sound quality is often achieved by trial and error because the physical mechanisms that control sound quality are poorly understood.

A review of the horn literature reveals that an approximate equation by Webster [3] can be used to estimate the performance characteristics of horns, provided there is a smooth variation in cross sectional area with distance along the horn axis. Models that use this equation generally tend to model acoustic impedance to a reasonable degree of accuracy, at least for low frequencies, but most acoustic horn models do not accurately model the far field acoustic pressure either on or off axis. This leads to the conclusion that while these simple models may be suitable for optimisation to produce a smooth frequency response, they would not be suitable for optimisation of the beamwidth.

Alternative approaches to modelling acoustic horns such as Finite Element Analysis (FEA) or the Boundary Element Method (BEM) have been found in the literature [4, 5]. However, while these methods can eliminate problems associated with the approximate equation of Webster, it has been found that fully 3-D FEA is intractable for large horn models and the high frequencies of interest for cinema applications, and unsuitable for application to optimisation techniques [6]. There is also evidence that fully 3-D direct BEM is similarly unsuitable for the mid to high frequencies needed for cinema applications [7].

This paper investigates the application of a fully 3-D direct BEM [8], as well as a relatively new source superposition BEM [9], to the modelling of two representative horn loaded loudspeakers. First, experiments to measure the beamwidth of the two small horns are described and results presented. Procedures using both the direct BEM and the source superposition method are given, and the results of calculations compared to experiments. Finally conclusions are drawn as to the utility of numerical modelling of horn loaded loudspeakers using boundary element methods.

2. EXPERIMENTS

Two simple axi-symmetric horns have been manufactured for experimental testing. These horns, shown in Figure 2, both have a 2 inch (50 mm) diameter throat; an 11 inch (280 mm) diameter mouth with a 1 inch (25 mm) flange; and 9.25 inches (235 mm) in length. One horn has an exponential variation in area between the throat and the mouth, and the other is a two step conical horn.

(a) (b)

Figure 2. Representative small horns (a) Exponential (b) Two Step Conical.

The horns were placed unbaffled on an indexed rotating platform on an elevated tower inside a large open space. The pressure frequency response of each horn was measured at a distance of 3 m from the centre of the mouth of the horn in 5º intervals ranging from on-axis (0º) to 90º off-axis. At each frequency of interest, a polar plot of the magnitude of the pressure measured, normalised by the maximum pressure, was produced. Figure 3 shows the sound field of the conical horn at three different frequencies: 550 Hz, which is a low frequency for this size horn and shows a wide beam of sound; 2000 Hz, which shows a narrowing of the sound field; and 4600 Hz, which shows a beam pattern with an “on axis null”, and is evidence that a velocity distribution other than that corresponding to the plane wave mode exists at the horn mouth. These experimental results give impetus for the development of accurate numerical models of horn loaded loudspeakers.

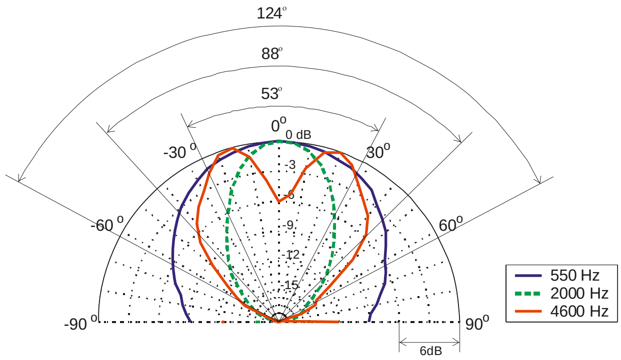


Figure 3. Polar plot of the magnitude of the measured pressure, normalised by the maximum pressure, for a conical horn at three different frequencies.

The beamwidth is defined as the angle formed by the -6 dB points, with reference to the maximum sound pressure value and the source centre [10, 11], and is a measure of the distribution of sound in the specified plane. Figure 3 shows the measured beamwidth for the three different frequencies, and Figure 4 shows a plot of the beamwidth verses frequency for both experimental horns.

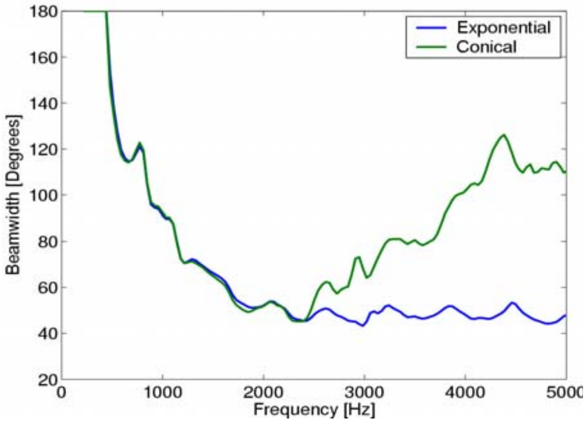


Figure 4. Experimental measurements of the variation of beamwidth with frequency for the exponential and two step conical horns shown in Figure 2.

3. THEORY

The governing equation of time harmonic linear acoustics is the scalar Helmholtz equation [8, 9, 12, 13]:

$$\nabla^2 p(x) + k^2 p(x) = 0 \tag{1}$$

where $p(x)$ is the pressure and $k = \omega / c$ is the wavenumber, $\omega = 2\pi f$ is the circular frequency, f is the frequency and c is the speed of sound in the medium, in this case air. This equation is derived from the linearised equations of conservation of momentum and mass. Equation (1) requires appropriate boundary conditions. The velocity at the interface between a solid and a fluid can be related to the gradient of pressure as:

$$\frac{\partial p(x)}{\partial n} = i\rho\omega v_n(x) \tag{2}$$

where n is the normal direction, ρ is the density of the fluid and $v_n(x)$ is the normal velocity. For external problems, where the sound radiates away from the structure to infinity, another boundary condition called the Sommerfield radiation condition is needed.

One approach to solving Equation (1) would be to discretise it directly and solve for the pressure at every point in the field. This is the approach that FEA takes, but there are limitations when solving problems in an infinite domain that must be truncated in order to solve the problem. The Sommerfield radiation condition must be enforced, otherwise reflections from the boundary can affect the result. The development of appropriate boundary conditions and their incorporation into a finite element analysis is a topic of ongoing research [14].

Another approach is to replace the solid surface that is being modelled with a distribution of fundamental solutions to Equation (1). A monopole is a fundamental solution that can be derived from the linearised equations of conservation of momentum and mass with the addition of a localised volume velocity injection. This represents the sound field due to a point source, and is called the “free space” Green’s function:

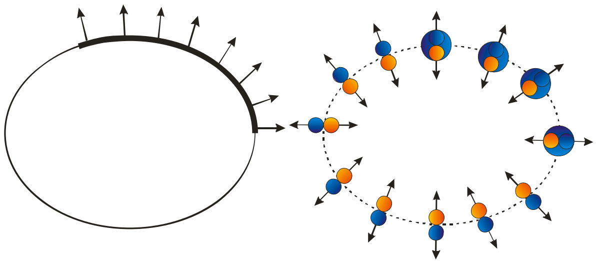
$$g(x_s \mid x) = \frac{e^{ikR}}{4\pi R} \tag{3}$$

where R is the distance between x_s, the position of the source and x the position of the field point. Note that Equation (3) is singular when the source and field point coincide.

A dipole is also a fundamental solution of Equation (1), derived from the linearised equations of conservation of momentum and mass with the addition of a localised force. It represents the sound field of two monopoles in close proximity operating 180º out of phase and is the directional derivative of Equation (3):

$$\frac{\partial g(x_s \mid x)}{\partial n} \tag{4}$$

Conceptually, any solid surface can be replaced by a distribution of monopoles and dipoles. The effect of the surface is replaced by the action of a distribution of forces aligned normal to the boundary, and the imposed velocity is replaced with the injection of volume velocity. Figure 5 shows a representation of this effect.



(a) (b)

Figure 5. A solid surface with an imposed velocity over part of the surface, (a), can be replaced by a suitable distribution of monopoles and dipoles, (b).

4.1 Direct BEM

The Kirchoff-Helmholtz (K-H) integral equation [8, 9, 12, 13]:

$$c(x)p(x) = -\int_s i\rho\omega v_n(x_s)g(x_s \mid x) + p(x_s)\frac{\partial g(x_s \mid x)}{\partial n}\, ds \tag{5}$$

where $c(x)$ is a position dependent constant, can be derived from either physical arguments using monopoles and dipoles [15] or from vector calculus and Green's theorem [9,13]. This is the fundamental equation of direct BEM, and shows that the pressure at any point can be represented by the surface integral of a combination of monopoles and dipoles. In this equation, the dipole source strength is weighted by the surface pressure. Given a distribution of surface normal velocity, once the surface pressure is found, any pressure field can be calculated.

The direct BEM finds the surface pressure by discretising Equation (5) with n_n nodes and n_e elements similar to those used in FEA. If the field point is positioned at each surface node (or "collocated") then a series of n_n equations for the n_n surface pressures can be found for a given velocity distribution. The equations are generated by numerical integration over each element, and the integration technique used must be capable of dealing with the singularities found at the locations of the monopoles and dipoles. The equations can be formed into a matrix and inverted using standard linear algebra techniques. Once the matrix is inverted, and the surface pressures known, the field pressures can be calculated.

There are a number of disadvantages to the direct BEM approach. The K-H integral equation represents the sound field on the exterior of a finite volume. At the natural frequencies of the interior of the finite volume, the exterior problem breaks down and the matrix becomes ill-conditioned. This is well documented [16] and many solutions have been attempted [17, 18].

Another problem occurs when the two surfaces of interest are brought close together, resulting in “thin-shape breakdown” [19]. This means that although an acoustic horn is probably best represented with a thin surface, a direct BEM simulation will have to assume the horn is contained in an enclosing volume to avoid thin shape breakdown.

The direct BEM code used in this research is HELM 3D [8], a Fortran 77 implementation using linear elements. The CHIEF method is used to overcome the interior natural frequency problem. For this application the code was modified to accept quarter symmetric models. The horns simulated are quarter symmetric and this modification was necessary to reduce overall run time.

4.1 Source superposition

The source superposition technique of Koopmann and Fahnline [9] does not solve the K-H equation directly. Instead, it uses an expansion of the pressure at a field point in terms of a series of monopoles and dipoles, each placed at the centroid of each element of the discretised surface:

$$p(x) = \sum_{m=1}^{n_e} s_m \left\{ \alpha_m g(x_m \mid x) + \beta_m \frac{\partial g(x_m \mid x)}{\partial n_m} \right\} \tag{6}$$

where n_e is the number of elements, s_m is the source strength, α_m and β_m are constants depending on whether the source is a monopole, dipole or combination of the two (tripole). Monopoles are used to represent sources on the surface of an infinite baffle, dipoles are used to represent thin surfaces and tripoles are used to represent the surface of the exterior of a finite volume. The use of tripoles eliminates the interior natural frequency problem of the direct BEM and this technique is capable of modelling thin surfaces directly.

The normal velocity can be found using Equations (2) and (6):

$$v_n(x) = \frac{1}{ik\rho c} \sum_{m=1}^{n} s_m \left\{ \alpha_m \frac{\partial g(x_m \mid x)}{\partial n} + \beta_m \frac{\partial^2 g(x_m \mid x)}{\partial n \partial n_m} \right\} \tag{7}$$

and the volume velocity over element μ of the boundary surface can be found by integrating Equation (7) over the element surface,

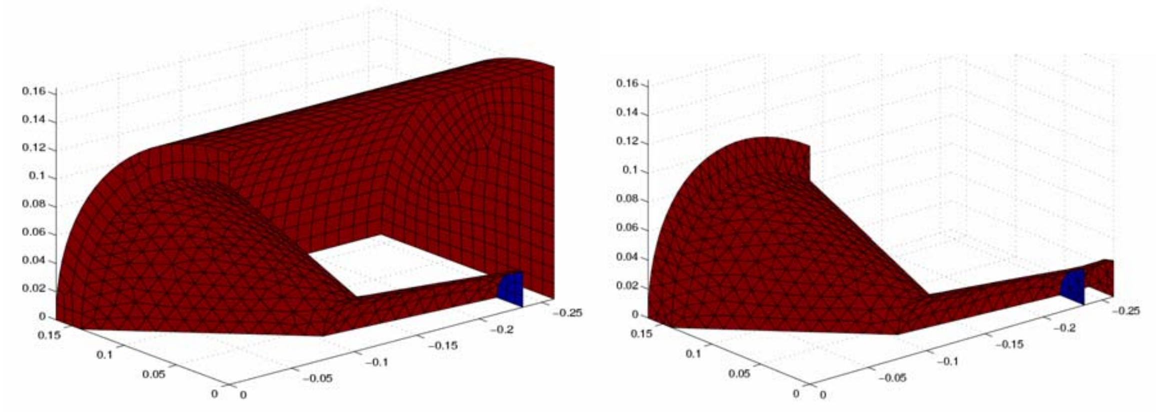
$$\begin{aligned} U_\mu(x) &= \int_s v_n(x) ds \\ &= \int_s \frac{1}{ik\rho c} \sum_{m=1}^{n} s_m \left\{ \alpha_m \frac{\partial g(x_m \mid x)}{\partial n} + \beta_m \frac{\partial^2 g(x_m \mid x)}{\partial n \partial n_m} \right\} ds \end{aligned} \tag{8}$$

for $\mu = 1,..., n_e$. This produces a series of n_e equations with n_e unknown source strengths. The resulting matrix can be inverted to find the source strengths, s_m. Once these strengths are found, the sound field can be reconstructed using Equation (6).

The source superposition code used in this research is the Fortran 77 program Power [9]. This program has also been modified for quarter symmetry.

3. RESULTS

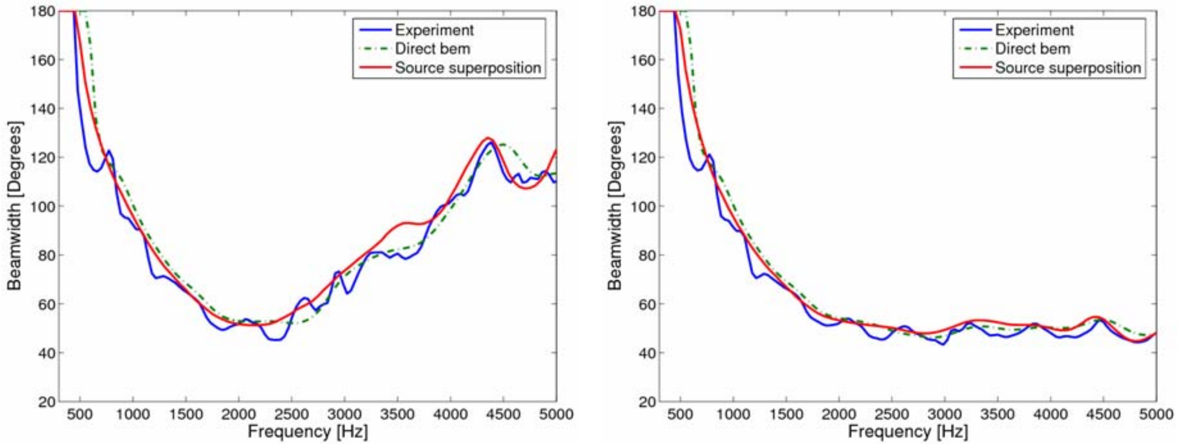
Simulations of both the conical and exponential horn have been undertaken for both the direct BEM and source superposition techniques. Figure 6 shows the surface mesh used to discretise the conical horn, at a nominal 6 elements per wavelength. Note the quarter symmetry and the need for the horn to be placed in an artificial volume for the direct BEM. A small volume is placed over the rear of the horn throat in the source superposition mesh to stop sound radiating out from the rear of the horn. A unit velocity was placed at the throat of the horn, represented by the blue area in Figure 6. For the conical horn, the number of variables to be solved for the direct BEM is 1105 compared to 631 for the source superposition technique.



(a) (b)

Figure 6. Surface mesh of the conical horn (a) direct BEM, (b) source superposition.

The beamwidth of the horns was calculated for frequencies from 300 to 5000 Hz at 50 Hz intervals. The upper frequency was chosen to limit the run time required for the direct BEM method. Figure 7 shows a comparison with experimental results for both direct BEM and the source superposition method. The agreement between both methods and experiment is excellent. The source superposition technique was found to produce results ~15-20 times faster than the direct BEM. On an Intel P4 1500 MHz, running Windows XP, the run time is 222 seconds per frequency for the direct BEM and 10 seconds per frequency for the source superposition technique.



(a) (b)

Figure 7. Comparison of measured and calculated beamwidth for (a) conical and (b) exponential horn.

3. CONCLUSIONS

Numerical models of horn loaded loudspeakers have been developed, which accurately model the beamwidth over the frequency range simulated. The source superposition technique is found to give similar results to the direct BEM, but is 15 to 20 times faster. Future work on the source simulation technique is needed to find its limits. Preliminary simulations have found that accurate results for the beamwidth can be achieved with element spacings of 3 per wavelength, less than the often quoted standard of 6 per wavelength [20]. It has also been found that the matrix produced is very diagonally dominant, and an iterative solution technique could potentially speed up the simulation by an order of magnitude, especially for large problems.

REFERENCES

[1] D. B. Keele, “What's so sacred about exponential horns”, preprint 1038(F-3), presented at the *51st Convention of the Audio Engineering Society*, May, (1975).

[2] C. A. Henricksen, and M. S. Ureda, “The Manta-Ray horns”, *Journal of the Audio Engineering Society*, **26**(9), 629-634, (1978).

[3] A. G. Webster, “Acoustical impedance, and the theory of horns and of the phonograph”, *Proceedings of the National Academy of Sciences*, 275-282, (1919. (Reprinted in the *Journal of the Audio Engineering Society*, **25**(1/2), 24-28, (1977).)

[4] C. I. Beltran, “Calculated response of a compression driver using a coupled field finite element analysis”, preprint 4787(G-7), presented at the *105th Convention of the Audio Engineering Society*, September, (1998).

[5] T. H. Hodgson and R. L. Underwood, “BEM computations of a finite length acoustic horn and comparison with experiment”, in Computational Acoustics and Its Environmental Applications, 213-222, (1997).

[6] Morgans, R. C. Optimisation techniques for horn loaded loudspeakers University of Adelaide, 2005

[7] O. von Estorff, *Boundary Elements in Acoustics*, WIT Press, (2000).

[8] T. W. Wu, Boundary Element Acoustics: Fundamentals and Computer Codes, WITPress, (2000).

[9] G. H. Koopmann and J. B. Fahnline, *Designing quiet structures: a sound power minimization approach*, Academic Press, (1997).

[10] D. Davis and C. Davis, *Sound system engineering*, Butterworth-Heinemann, (1997).

[11] M. Chamness, “Objective analysis of loudspeaker polar response”, EAW, One Main Street, Whitinsville, MA 01588, November, (1994).

[12] P. M. Morse and K. U. Ingard, *Theoretical acoustics*, Princeton University Press, (1986).

[13] A.D. Pierce, *Acoustics: an introduction to its physical principles and applications*, Acoutical Society of America, New York, (1994).

[14] K. Gerdes, “A review of infinite element methods for exterior Helmholtz problems”,*Journal of Computational Acoustics*, **8**(1), 43-61, (2000).

[15] F. Fahy, *Foundations of Engineering Acoustics*, Academic Press, (2001).

[16] L. G. Copley, “Fundamental results concerning integral representations in acoustic radiation”, *Journal of the Acoustical Society of America*, **44**, 28-32, (1968).

[17] H. A. Schenek, “Improved integral formulation for acoustic radiation problems”, *The Journal of the Acoustical Society of America*, **44**, 41-58, (1968).

[18] A. J. Burton and G. F. Miller, “The application of integral equation methods to the numerical solutions of some exterior boundary-value problems, *Proceedings of the Royal Society of London A*, **323**, 201-210, (1971).

[19] R. Martinez, “The thin-shape breakdown (TSB) of the Helmholtz integral equation”, *Journal of the Acoustical Society of America*, **90**(5), 2728-2738, (1991).

[20] S. Marburg, “Six boundary elements per wavelength: is that enough?”, *Journal of Computational Acoustics*, **10**(1), 25-51, (2002).