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A SOURCE TERM FORMULATION FOR THE NON LINEAR EULER EQUATIONS IN A CONSERVATIVE AND PERTURBATION FORM

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Abstract

The numerical simulation of aerodynamic noise is a complex task that can hardly be conducted through a single method / solver. Thus, one generally uses hybrid techniques that associate CFD and CAA computations, the latter being devoted respectively to the generation and the propagation of acoustics events. Such a methods / solvers association requires a coupling that can be achieved by two distinct approaches; the first one is the "surfacic coupling" that was initially developed at ONERA, and successfully applied within the framework of airframe noise. However, such a coupling presents some drawbacks that may render it inapplicable to realistic (and, thus, complex) configurations. Then, recently, another approach has been investigated, which is the "volumic coupling" technique. If such an technique is potentially more promising than the previous one, it however asks for more formulation / implementation work. In particular, it requires the establishment of a source term that can suit the CAA formulation and solver to be used with. In the present case, it then was necessary to derive a proper source term for the specific formulation (*Conservative, Perturbed, Non linear Euler's equations*) and numerical tool (*sAbrinA*) that are used at ONERA for all the CAA purposes. This (innovative) source term constitutes the matter of the present paper; after a necessary reminder about the CFD / CAA hybrid approach and coupling techniques, a brief state-of-the-art of the 'source term' question will be made. After what the specific source term this paper is about will be detailed, the "volumic coupling" it allows being then illustrated / early validated with two academic test cases applications.

1. INTRODUCTION

The aerodynamic noise is a complex phenomenon associating mechanisms of very different space and temporal scales; in particular, the acoustic generation is driven by turbulent structures presenting both high amplitudes and small space-time correlations, while the acoustic propagation is characterized by waves of low amplitude and large space-time correlation. If the aerodynamic noise generation can (partially) be simulated with an unsteady CFD (Computational Fluid Dynamics) method, its propagation can only be computed through a CAA (Computational AeroAcoustics) technique. Hopefully, a chaining of these two methods may be done through a hybrid approach [3] (see Figure 1) such as the one generally used at ONERA for the airframe noise prediction [3, 4]. This CFD/CAA chaining can be seen as a "weak" (or "one way") CFD→CAA coupling, where a preliminary CFD calculation provides all the generated noise events to a CAA computation which then makes them propagate. Therefore, it implies that a correct transmission of the unsteady events is conducted from the CFD domain towards the CAA one. Such a task can be done through both the "surfacic" and the "volumic" coupling techniques that are presented hereafter.

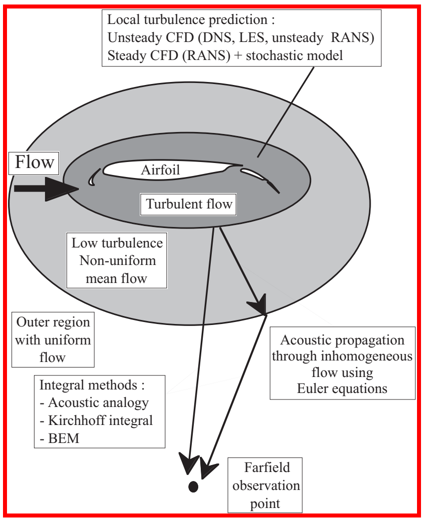

Figure 1: Hybrid approach, for the numerical simulation of airframe noise.

2. CFD/CAA COUPLING TECHNIQUES

2.1 CFD/CAA "Surfacic" Coupling

The surfacic coupling consists in injecting within a CAA calculation the acoustic perturbations that could have been previously generated and (partially) propagated by an unsteady CFD calculation - such injection being made through a boundary condition applied along the interface on which the unsteady events could have been stored. Initially developed at ONERA, this surfacic coupling approach gave very satisfactory results when applied to the numerical prediction of the airframe noise characterizing an in-flight NACA0012 with a blunted trailing edge [3, 4]. However, this technique presents a certain number of disadvantages which make it not easy to be handled.

First of all, the positioning of the coupling interface is delicate; if the latter is located too far away from the profile, it may cover an area where part of the acoustic events to be stored/injected will have be filtered/dissipated by the CFD calculation (especially in the medium and high frequency ranges). On another hand, if this interface is located too close from the profile, it may intercept zones where the acoustic events to be stored/injected will be contaminated by some "under-resolved" hydrodynamic events that could not be injected properly within the CAA field [3]. The solution consisting in a compromise to be found between these two things, the result will probably be always partially conclusive.

Secondly, as said before, such a surfacic coupling is of "boundary condition" nature, consisting in a peripheral / temporal "forcing" of the perturbed field to be solved within the CAA domain. Therefore, it inhibits the application of any other boundary condition that could be needed at the concerned frontier; in particular, no free-field B.C. can be longer prescribed so that is allowed the exit of possible waves which could have been retro-propagated within the computational domain - a thing likely to arrive as soon as the latter will include potential sources of reflection / refraction effects such as solid bodies / mean flow heterogeneities. This strongly restricts the generality of the method, as well as its applicability to the realistic configurations that are aimed at (high-lift wing, etc.).

Finally, the acoustic events to be stored/injected being by nature of large spatial correlations, a surfacic coupling implies that the interface is wide enough to properly recover their features; one is thus generally obliged to apply a strategy of “global” coupling, where one single large interface surrounds all the acoustical generation areas. From a methodological point of view, this does not allow to consider independently each one of the latter, as it could be useful to do for studying separately every region (and possible associated mechanism) of noise production. From a more technical point of view, the CFD and CAA calculations still having to be conducted successively, the necessary data storage to be performed between the two becomes then consequent both in terms of CPU memory (storage) and time (restitution).

These are the reasons that led to consider another approach of CFD/CAA chaining, which is the “volumic coupling” technique detailed hereafter.

2.2 CFD/CAA “Volumic” Coupling

Instead of being injected within the CAA domain through an explicit forcing of its peripheral border, the beforehand CFD computed (and stored) events are now built into a volumic “source term” which, then, will constitute the right-hand side member of the equations’ set to be CAA-solved. The acoustic perturbed field remaining free of evolving accordingly to such the left-hand side propagation kernel, this volumic coupling can be seen as “indirect” forcing that simply “encourages” the CAA solution to be locally driven (or not...) by the CFD one.

First of all, this coupling technique being no longer of “boundary condition” nature, it does not contradict anymore the possibly required “non reflective” character of the domain frontier(s). Therefore, it becomes applicable to all types of configurations whatever they include or not diffracting solid bodies and/or refracting flow regions. Secondly, this coupling process being now applicable anywhere within the CAA domain, independently of the injected events nature (acoustic, hydrodynamic, etc…), it does not require no more precision/stability compromise that could have been inherited of the CFD-solving constraint. Finally, this volumic coupling being intrinsically a “local” process, it can now be specifically applied to well-localized/identified acoustic generation zones/mechanisms; coming in addition to a possibility of finer investigations, this allows to possibly reduce considerably the storage/coupling operations (in term of CPU volume/time).

3. A VOLUMIC SOURCE TERM FOMULATION

If the volumic coupling seems to be more attractive than its surfacic counterpart, its implementation requires more work; in particular, it implies to derive a ‘suitable’ source term that *has* to be coherent / consistent with the “propagation kernel” used by the CAA solver. This explain why literature abounds in various source term formulations, the possible propagation kernels being numerous - depending on what are the chosen equations’ set (Helmholtz or Euler) and formulation (conservative or not, linear or not, perturbed or not, etc.). Nevertheless, one can notice that no source term had been proposed yet for the Conservative Formulation of Non Linear Perturbed Euler’s equations (as the one solved by the ONERA’s *sAbrinA* solver [1, 2, 3, 4]). The present paper and the (original) formulation it proposes aim at answering this need.

3.1 The Source Term Wriggle

From a mathematical point of view, the so-called “source term” corresponds to the right-hand side member of the *non-homogenous* equation to solve. However, from a physical point of view, in absence of remote forces, the equation to be solved is *homogeneous* (the right-hand side term is *null*). Thus, authors classically use the artifice that consists in constituting a right-hand side member with the terms they could wish to evacuate from the left-hand side one, so that the latter reduces itself to the desired propagation kernel. It suffices then to feed this artificial source term with any physical quantities (that could have been previously obtained by analytical or numerical ways), to see them driving (indirectly and locally) the field to be solved by the propagation kernel.

3.2 A Few Source Terms

The initial idea comes from works of Sir James Lighthill [5] who derived his famous analogy by re-writing the Navier-Stokes equations, so that the left-hand side member reduces itself to the (possibly convected) *Helmotz Equation* - which is the uniform media's propagation kernel. The remaining terms were thus evacuated from the left to the right-hand side and, after having been drastically simplified (into the well-known Lighthill's tensor "$\rho u_i u_j$"), they were built up into a source term. Obviously, because it did not comprise any of the quantities related to mean low gradients, such a propagation kernel could not account for refraction effects. Moreover, the source term having been reduced to its simplest expression, it did not allow to deal with a certain class of problems, such as the highly entropic ones (hot jets).

Bailly et al. (from the Ecole Centrale de Lyon) were among the firsts to compute the acoustic propagation over heterogeneous flows; they thus naturally sought to establish a source term that could suit their propagation kernel – which was given by the *Non Conservative*, *Perturbed* and *Linear Euler Equations*. Following the same process, they moved to the right-hand side all the (non linear) terms left unused by the propagation kernel (that could now handle any heterogeneous mean flow), and degraded them into a proper source term (corresponding to the Lighthill's one [6]). More recently, Billson et al. (Chalmers University) extended this work by re-establishing it under a *Conservative* form, and by re-enriching the corresponding source term with all the non-linear quantities (the viscous ones remaining solely unconsidered) [7]. Despite both its propagation kernel and related source term present a quite general character, such formulation remains restricted to the sole class of linear (or slightly non-linear) configurations – all the non linear features having been frozen within the right-hand side member.

This is one of the reasons that led to the establishment of the hereafter proposed source term, which is specifically devoted to the *Conservative*, *Perturbed* and *Non linear Euler*'s equations that constitute the propagation kernel of the ONERA' *sAbrinA* solver.

3.3 A Source Term For The Conservative, Perturbed and Non linear Euler's Equations

Classically, the Navier-Stokes equations were re-written so that their left-hand side reduced itself to the (*Conservative*) *Non Linear Euler*'s equations – this being done by transferring to the right-hand side all the viscous terms. A classical mean flow / *perturbed* splitting was then applied to both the left and the right sided quantities, which was made accordingly to the "small perturbations" hypothesis usually adopted in CAA [1, 2]. All that led to the following system, whose left member constitutes the propagation kernel, while its right counterpart constitute the source term:

$$\partial_t \mathbf{u}_\mathrm{p} + \nabla \cdot \mathbf{F}_\mathrm{p} = \nabla \cdot \mathbf{F}_\mathrm{p}^{\nu} \tag{1}$$

The propagation kernel **(**which complete expression can be found in [1, 2]) is given by the *Conservative*, *Perturbed* and *Non linear Euler*'s equations. Its corresponding source term is given by the *divergence* of the *perturbed viscous flux*. The latter can be expressed so that its linear and non linear parts are distinguished:

$$\mathbf{F}_{\mathrm{p}}^{\nu} = \mathbf{F}_{\mathrm{l}}^{\nu} + \mathbf{F}_{\mathrm{nl}}^{\nu} \tag{2}$$

with, for the linear component,

$$\mathbf{F}_{\mathrm{l}}^{\nu} = \frac{1}{\mathrm{Re}} \begin{bmatrix} {}^{t}\mathbf{0} \\ \mu_{\mathrm{o}}\boldsymbol{\sigma}_{\mathrm{p}} + \mu_{\mathrm{p}}\boldsymbol{\sigma}_{\mathrm{o}} \\ \mu_{\mathrm{o}}\,{}^{t}\!\left(\boldsymbol{\sigma}_{\mathrm{o}}\cdot\mathbf{v}_{\mathrm{p}} + \boldsymbol{\sigma}_{\mathrm{p}}\cdot\mathbf{v}_{\mathrm{o}} + \frac{C_p}{\mathrm{Pr}}\nabla T_{\mathrm{p}}\right) + \mu_{\mathrm{p}}\,{}^{t}\!\left(\boldsymbol{\sigma}_{\mathrm{o}}\cdot\mathbf{v}_{\mathrm{o}} + \frac{C_p}{\mathrm{Pr}}\nabla T_{\mathrm{o}}\right) \end{bmatrix} \tag{3}$$

and, for the non linear one,

$$\mathbf{F}_{\mathrm{nl}}^{\nu} = \frac{1}{\mathrm{Re}} \begin{bmatrix} {}^{t}\mathbf{0} \\ \mu_{\mathrm{p}}\boldsymbol{\sigma}_{\mathrm{p}} \\ \mu_{\mathrm{o}}\,{}^{t}\!\left(\boldsymbol{\sigma}_{\mathrm{p}}\cdot\mathbf{v}_{\mathrm{p}}\right) + \mu_{\mathrm{p}}\,{}^{t}\!\left(\boldsymbol{\sigma}_{\mathrm{o}}\cdot\mathbf{v}_{\mathrm{p}} + \boldsymbol{\sigma}_{\mathrm{p}}\cdot\mathbf{v}_{\mathrm{o}} + \boldsymbol{\sigma}_{\mathrm{p}}\cdot\mathbf{v}_{\mathrm{p}} + \frac{C_p}{\mathrm{Pr}}\nabla T_{\mathrm{p}}\right) \end{bmatrix} \tag{4}$$

In the previous expressions, the subscripts “o” and “p” respectively flag the mean and the perturbed quantities. In addition to a Reynolds number (directly linked to the dimensionalization values - Re = $\rho_{ref}\, x_{ref}\, \mathbf{v}_{ref}/\mu_{ref}$) and a Prandlt number (Pr = 0.72, for air), these expressions make appear some well-known quantities such as the mean ($\boldsymbol{\sigma}_{\mathrm{o}}$) and perturbed ($\boldsymbol{\sigma}_{\mathrm{p}}$) stress tensors:

$$\boldsymbol{\sigma}_{*} = \left(\nabla\mathbf{v}_{*} + {}^{t}\nabla\mathbf{v}_{*}\right) - \frac{2}{3}\left(\nabla\cdot\mathbf{v}_{*}\right)\mathbf{I} \qquad \textit{with} \quad * = \mathrm{o,p} \tag{5}$$

where **v** stands for the velocity. Also appear the mean (T_o) and perturbed (T_p) temperatures which, after some manipulations, can be put under the following (and exact) forms:

$$T_{\mathrm{o}} = \frac{1}{C_p}\frac{\gamma}{\gamma-1}\left(\frac{p_{\mathrm{o}}}{\rho_{\mathrm{o}}}\right) \quad \text{and} \quad T_{\mathrm{p}} = \frac{1}{C_p}\frac{\gamma}{\gamma-1}\frac{\left(\rho_{\mathrm{o}}p_{\mathrm{p}} - \rho_{\mathrm{p}}p_{\mathrm{o}}\right)}{\rho_{\mathrm{o}}\left(\rho_{\mathrm{o}} + \rho_{\mathrm{p}}\right)} \tag{6}$$

In the previous expressions, ρ and p respectively denote the density and the pressure, while γ stands for the specific heat ratio (which value is $\gamma = 1{,}4$ for the air considered in normal conditions of pressure and temperature). The latter and the characteristic constant of perfect gases (R = 287.06 J.Kg^{-1}.K^{-1}) define the specific heat ration coefficient (C_p) given by

$$C_p = R\frac{\gamma}{\gamma-1} \tag{7}$$

Finally, the perturbed viscous flux expression makes appear the mean (μ_o) and perturbed (μ_p) dynamic viscosities which (approximate) expressions can be obtained by submitting the Sutherland's law to first order's limited developments (that are legitimated by the "small perturbations" hypothesis):

$$\mu_{o} = \mu_{Suth}\left(\frac{T_{o}}{T_{Suth}}\right)^{\frac{3}{2}} \frac{\left(T_{Suth}+Cte\right)}{\left(T_{o}+Cte\right)} \quad \text{and} \quad \mu_{p} = \mu_{o}\left[\frac{T_{o}+3Cte}{2T_{o}\left(T_{o}+Cte\right)}\right]T_{p} \tag{8}$$

with, for air in normal conditions of pressure and temperature, μ_{Suth} = 1,711 10^{-5} kg.m^{-1}.s^{-1}, T_{Suth} = 273,16 K and Cte = 110,4.

Once theoretically established, this source term formulation was used to implement a CFD/CAA "volumic coupling" functionality within the ONERA' *sAbrinA* CFD/CAA environment. Once again it should be noted that, thanks to both the non linear nature of the propagation kernel and the exhaustive expression of its corresponding source term, such functionality should theoretically be applicable to a wide range of problems (including strongly nonlinear and / or highly entropic configurations, etc.). Nevertheless, for a first validation attempt, this feature was preliminary tested on a few academic tests cases (given below).

ILLUSTRATIONS AND EARLY VALIDATION

A first illustration / early validation of the present work was attempted with the 2D injection / propagation of an elementary acoustic event within a medium at rest. For this purpose, an acoustic dipole was constituted, by associating two cylindrical waves of same amplitude and frequency (f = 30 *kHz*), but of opposite phase. Once generated (by analytical means), these (purely acoustic) fluctuations were built-up into a proper source term using the formulation detailed above. A CAA simulation was then performed with the *sAbrinA*'s CAA module (namely *sAbrinA_v0*), over a Cartesian monodomain grid of 130 × 130 cells. For such a calculation, the volumic coupling functionality was activated over a closer region (80 × 80 cells) and fed, at each time step, with the source term temporal quantities.

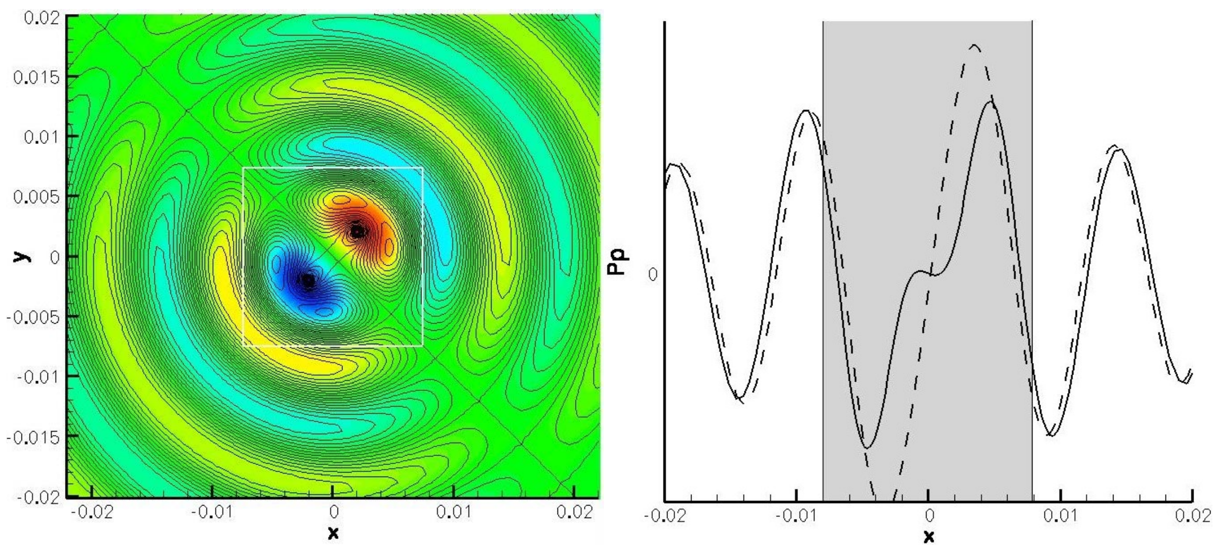

Figure 2: Injection / propagation of an acoustic dipolar emission, over a medium at rest; perturbed pressure field of both the expected (analytical, in flood colours / dashed lines) and the CAA-computed (in black isocontours / lines) solutions, plotted over the computational domain (left side) and along the *x*-axis (right). The injection zone is delimited by a white square (left) / grey area (right)

Figure 2 provides a comparison between the expected (analytical) and the computed (injected / propagated) fields of perturbed pressure. Except over the source region (for which some differences are expected to occur), the two solutions are in very good qualitative agreement. A quantitative adjustment was necessary to match the CAA solution to the exact analytical solution. This adjustment is not fully justified at the moment, but current investigations are under progress, aiming at checking if the 'numerically injected' acoustic energy corresponds to the 'analytically generated' one, as it should be.

Such a validation effort was then pursued with the 2D injection / propagation of a more complex aero-acoustic source mechanism. For this purpose, a co-rotating vortices pairing was simulated by DNS (Direct Numerical Simulation) over a Cartesian monodomain of 280 × 280 cells, this being made with the *sAbrinA*'s CFD module. The corresponding (aerodynamic and acoustic) fluctuations were then built-up into a source term, still accordingly to the present formulation. Another *sAbrinA_v0* calculation was then ran over the same computational 280 × 280 cells monodomain, with a volumic coupling functionality activated over a compact region (of 60 × 60 cells) and temporally fed with (a time-interpolated solution of) the source term quantities. Figure 3 provides a comparison between the expected (CFD simulated) and the computed (CAA injected & propagated) fields of perturbed pressure.

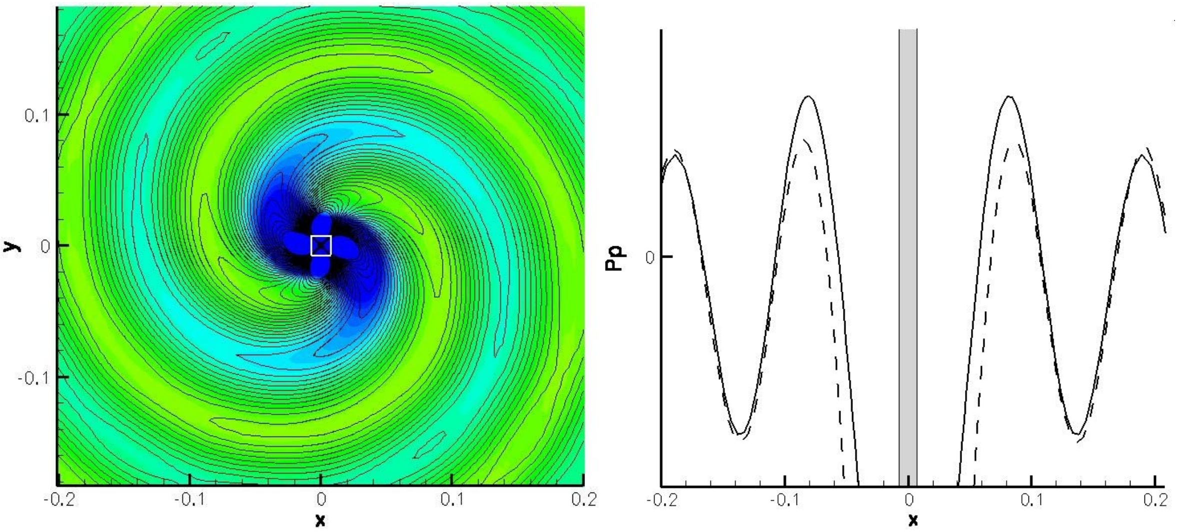

Figure 3: Injection / propagation of a vortex pairing aero-acoustic emission, within a medium at rest; perturbed pressure field of both the CFD-simulated (in flood colours / dashed lines) and the CAA-computed (in black isocontours / lines) solutions, plotted over the computational domain (left side) and along the *x*-axis (right). The injection zone is delimited by a white square (left) / grey area (right)

Here too, the two solutions are in very good agreement - at least from a qualitative point of view; in particular, both the frequency (35 *kHz*) and the vortical pattern of the acoustic emission pre-computed by DNS are fully recovered by the CAA calculation. However, a level adjustment was required again to match the two solutions. Moreover, a phase shift (equal to one fourth of a period) was also observed, and corrected. At this point, such a phase shift is not fully understood, but it may be simply due to some inevitable time delay / transitory period of the CFD/CAA coupling process launching. The above-mentioned investigation on energy conservation might provide additional clues regarding this issue.

5. CONCLUSIONS

Hybrid CFD-CAA methods used to simulate aerodynamic noise problems raise the question of how to couple (or to chain) unsteady CFD and CAA computations / solvers - which can be done either via a 'surfacic' or a 'volumic' coupling technique. In the last years, ONERA has gained experience on the 'surfacic' coupling, in particular within the framework of airframe noise prediction. Nevertheless, for several reasons detailed in the present paper, attention has been recently focused on the 'volumic' coupling. This led to the development of an innovative source term formulation specifically devoted to the Conservative, Perturbed and Non Linear Euler's equation. Such a formulation was then integrated within the ONERA's *sAbrinA* CFD/CAA platform, providing to the latter a suitable 'volumic' coupling functionality. An illustration / early validation of this work was done through the hybrid solving of two aero-acoustic academic test cases. Despite some quantitative uncertainness, the qualitative validation is very satisfying, and thus encouraging. This validation work is still under progress, and should hopefully be achieved soon, allowing an application of this volumic coupling technique to realistic configurations; in particular, is planned the CAA exploitation of two (already computed) LES calculations of the unsteady flow characterizing (i) the trailing edge region of a NACA0012 airfoil and (ii) the slat region of a high-lift wing section. Another perspective is to apply the volumic coupling to unsteady flow simulations based on stochastic turbulence modeling (performed upon RANS results). ONERA is currently working on this very promising approach, with short-term applications focusing on jet noise, but with mid-term views into the airframe noise domain.

ACKNOWLEDGEMENTS

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