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# OPTIMIZATION IN ACTIVE NON-STATIONARY RANDOM VIBRATION CONTROL FOR SMART TRUSS STRUCTURES WITH UNCERTAINTY

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**Abstract**

This paper presents the reliability-based optimization of the location and feedback gains of a piezoelectric active bar in a closed loop control system for smart truss structures with random parameters under non-stationary random excitation. The mathematical model with reliability constraints on the mean square value of the structural random dynamic displacement and stress response is developed based on maximization of dissipation energy due to the control action. The randomness of the structural physical materials and geometry are included in the analysis, and the applied forces are considered as non-stationary random excitation. Numerical examples of smart truss structures are presented to demonstrate the active control model.

## 1. INTRODUCTION

A smart truss structure is a self-adaptive structure that is utilized in some important fields [1]. In this kind of truss structure, a piezoelectric (PZT) bar is the active structural active member used to suppress mechanical vibrations, and is not only a sensor but also an actuator. Optimal placement of the PZT active bar is an important factor in the process of the structural design phase, and its shape as is vibration control. The location of active bars in the smart truss structure directly affects the performance of active vibration control. The field of smart or intelligent structures has raised much interest over the past decade [1]. Recently, there has been much work published on the optimization of smart structures [2-4].

To date, the majority of modelling on optimization of active vibration control using piezoelectric smart structures has used deterministic models to model the dynamic response of smart structures, and optimal placement of the PZT actuators and sensors. In these cases, the structural parameters, applied loads and control forces are regarded as known parameters. However, deterministic models of the dynamic response associated with smart structures cannot reflect the influence of the randomness of the structural parameters. The dynamic response of an engineering structure can be sensitive to randomness in its parameters arising from variability in its geometric or material parameters, or randomness resulting from the assembly process and manufacturing tolerances. In addition, applied loads can be random process forces, such as wind, earthquakes and blast shock. The problem of stochastic smart structures subject to random applied excitation is of great significance in realistic engineering applications.

In this paper, optimization of the location of the active bar and feedback gain in piezoelectric smart truss structures with random parameters are investigated. The randomness of the structural materials, geometry and damping are simultaneously considered. The applied force is taken as a non-stationary random excitation. The performance function due to the control action is based on maximization of the dissipation energy. To formulate the optimal control problem, the algorithm for a linear quadratic regulator with output feedback has been employed in this paper. An optimal mathematical model with reliability constraints on the mean square value of structural dynamic displacement and stress response is developed. Numerical examples of stochastic smart truss structures are given and some useful conclusions are obtained.

## 2. OPTIMAL MATHEMATICAL MODEL

Following the finite element formulation, the equation of motion for a smart structure is given by

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=v(t)\{F(t)\}+[B]\{F_C(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are displacement, velocity and acceleration vectors respectively. $\{F(t)\}$ is a stationary random load force vector, $v(t)$ denotes the non-stationary characteristics of the random force $\{F(t)\}$. $\{F_C(t)\}$ is the control force vector. The matrix $[B]$ defines the location of the active bar on the smart structure under consideration. In the following analysis, Wilson's damping hypothesis is adopted. Using the modal expansion $\{u(t)\}=[\phi]\{z(t)\}$, the equation of motion takes the form

$$[I]\{\ddot{z}(t)\}+[D]\{\dot{z}(t)\}+[\Omega]\{z(t)\}=[\phi]^T\{F(t)\}+[\phi]^T[B]\{F_C(t)\} \tag{2}$$

where $[D]=diag[2\zeta_j\omega_j]$, $[\Omega]=diag[\omega_j^2]$ for $j=1\ldots n$. $[\phi]=[\phi_1\cdots\phi_n]$ is the normal modal matrix of the structure, and $\omega_j$, $\zeta_j$ are the $j^{\text{th}}$ order natural frequency and damping ratio respectively.

For active control of the truss bar, a velocity feedback control law is considered. Since each active bar can be considered as a collocated actuator/sensor pair, the output matrix is the transpose of the input matrix. The output vector $Y(t)$ and control force vector $\{F_C(t)\}$ can be respectively expressed as

$$Y(t)=[B]^T[\phi]\{\dot{z}(t)\} \tag{3}$$

$$\{F_C(t)\}=-[G]Y(t)=-[G][B]^T[\phi]\{\dot{z}(t)\} \tag{4}$$

where $[G] = diag\{g_j\}$ is the gain matrix [5]. Substituting equation (4) into equation (2) yields the equation of the closed-loop system

$$[I]\{\ddot{z}(t)\} + ([D] + [\phi]^T [B][G][B]^T [\phi])\{\dot{z}(t)\} + [\Omega]\{z(t)\} = [\phi]^T \{F(t)\} \tag{5}$$

In the state-space representation, the equation of motion becomes

$$\{\dot{u}(t)\} = [A]\{u(t)\},\ \{u\} = \{z(t)\ \ \dot{z}(t)\}^T,\ [A] = \begin{bmatrix} 0 & [I] \\ -[\Omega] & -([D] + [\phi]^T [B][G][B]^T [\phi]) \end{bmatrix} \tag{6}$$

Both the optimal location of the active bar and the optimal gain of the closed-loop control system are determined such that the total energy dissipated in the system is maximized. The total energy dissipated in the system is taken as the performance criterion and it can be expressed as

$$J = \int_0^\infty \{\dot{z}(t)\}^T [\phi]^T ([D] + [B][G][B]^T)[\phi]\{\dot{z}(t)\} dt \tag{7}$$

Equation (7) can also be expressed as

$$J = \{u(0)\}^T \int_0^\infty e^{[A]^T t} [Q] e^{[A]t} dt \{u(0)\} \tag{8}$$

where $[Q] = \begin{bmatrix} [\Omega] & 0 \\ 0 & [I] \end{bmatrix}$. Making use of the method described in ref. [5], the performance function can be expressed as

$$J = tr[W] \tag{9}$$

where the matrix $[W]$ can be obtained by solving the following equation

$$[A]^T [W] + [W][A] = [Q] \tag{10}$$

For the smart truss structure with random parameters, and where the load is a non-stationary random excitation, an optimization program is written with reliability constraints that implements the following steps. For a fixed gain ($g = g_j$), the optimal location of the active bar (that is, the optimal $[B]$ matrix) is obtained such that the total energy dissipated, $J$ is maximized. After the optimal placement of the active bar is determined, the feedback gain is then optimized. This is achieved by calculating the mean square displacement for each $k^{th}$ degree of freedom and mean square dynamic stress for each $e^{th}$ element. Reliability constraints are placed on the mean square displacement and stress respectively as follows:

$$R^*_{\psi^2_{\sigma e}} - P_r\left\{\psi^{2\ *}_{\sigma e} - \psi^2_{\sigma e} \geq 0\right\} \leq 0, \qquad e = 1, 2, ..., m \tag{11}$$

$$R^*_{\psi^2_{uk}} - P_r\left\{\psi^{2\ *}_{uk} - \psi^2_{uk} \geq 0\right\} \leq 0\ , \qquad k = 1, 2, ..., n \tag{12}$$

$$\left[B\right] \subset \left[B^*\right], \quad \left[G\right] < \left[G^*\right] \tag{13}$$

where $\left[B\right]$ and $\left[G\right]$ are the design variables. $R^*_{\psi^2_{\sigma e}}$ and $R^*_{\psi^2_{uk}}$ are given values of reliability of the mean square stress and displacement responses, respectively. $P_r\{\cdot\}$ is the reliability obtained from the actual calculation. $\psi^{2\ *}_{\sigma e}$ and $\psi^{2\ *}_{uk}$ are given limit values of the mean square stress and displacement responses, respectively. In this model, $\left[B\right]$, $\left[G\right]$, $R^*_{\psi^2_{\sigma e}}$, $R^*_{\psi^2_{uk}}$, $P_r\{\cdot\}$, $\psi^{2\ *}_{\sigma e}$ and $\psi^{2\ *}_{uk}$ can be random variables or deterministic values. $\psi^2_{\sigma e}$ and $\psi^2_{uk}$ are the mean square dynamic stress of the $e^{\text{th}}$ element, and displacement of the $k^{\text{th}}$ degree of freedom, respectively. $\left[B^*\right]$ and $\left[G^*\right]$ are the upper bounds of $\left[B\right]$ and $\left[G\right]$ respectively.

## 3. SMART STRUCTURAL NON-STATIONARY RANDOM RESPONSE

Suppose that there are $m$ elements in the smart truss structure under consideration. In the structure, any element can be taken as either a passive or active bar, where a PZT bar is used as the active bar. The stiffness matrix $\left[K\right]$ and the mass matrix $\left[M\right]$ of the smart truss structures in global coordinates can be expressed as

$$\left[K\right] = \sum_{e=1}^{m}\left[K_e\right] = \sum_{e=1}^{m}\{\left[T_e\right]^T[\theta\frac{E^P_e A_e}{l_e} + (1-\theta)\frac{c_{33e} + (e_{33e})^2/\varepsilon_{33e}}{l^C_e}A^C_e]\left[G\right]\left[T_e\right]\} \tag{14}$$

$$\left[M\right] = \sum_{e=1}^{m}\left[M_e\right] = \sum_{e=1}^{m}\{\frac{1}{2}(\theta\rho_e A_e l_e + (1-\theta)\rho^C_e A^C_e l^C_e)\left[I\right]\} \tag{15}$$

where $\theta$ is a Boolean algebra value defined by the following: when $\theta$=0, the mixed element is a PZT active element bar; when $\theta$=1, the mixed element is a passive element bar. $\left[K_e\right]$ and $\left[M_e\right]$ are the stiffness matrix and mass matrix of the $e^{\text{th}}$ element, respectively. $\rho_e$, $A_e$ and $l_e$ are the density, cross-sectional area and length respectively of the $e^{\text{th}}$ passive bar element. $\rho^C_e$, $A^C_e$ and $l^C_e$ are the density, cross-sectional area and length respectively of the $e^{\text{th}}$ active bar element. $E^P_e$ is the Young's modulus of the $e^{\text{th}}$ passive bar element. $c_{33e}$, $e_{33e}$ and $\varepsilon_{33e}$ are the Young's modulus, piezoelectric force/electrical constant and dielectric constant respectively of the $e^{\text{th}}$ active bar element. $\left[I\right]$ is a $6^{\text{th}}$ order identity matrix. $\left[G\right]$ is a $6\times 6$ matrix, where $g_{11} = g_{44} = 1$, $g_{14} = g_{41} = -1$ and other elements are zero. $\left[T_e\right]$ is the transformation matrix that translates the local coordinates of the $e^{\text{th}}$ element to global coordinates, and $\left[T_e\right]^T$ is its transpose.

In the closed loop control system, since the control force $\left\{F_C(t)\right\}$ is determined by the applied force $v(t)\left\{F(t)\right\}$, these two variables have full positive correlation. Let

$$g(t)\left\{P(t)\right\} = v(t)\left\{F(t)\right\} + \left[B\right]\left\{F_C(t)\right\} \tag{16}$$

Equation (1) can be re-written as

$$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=g(t)\{P(t)\} \tag{17}$$

From equation (17) and by using the structural random vibration analytic theory [6], the mean square displacement of the $k^{\text{th}}$ degree of freedom can be expressed as

$$\psi_{uk}^2=\vec{\phi}_k\cdot\int_0^\infty\ [H(\omega)][\phi]^T g(t_1)[S_P(\omega)]g(t_2)\ [\phi][H^*(\omega)]d\omega\cdot\vec{\phi}_k^{\ T}\quad k=1,2,...,n \tag{18}$$

where $\vec{\phi}_k$ is the *kth* line vector of the matrix $[\phi]$. $[H(\omega)]$ is the frequency response function matrix of the structure and can be expressed as

$$[H(\omega)]=diag[H_j(\omega)],H_j(\omega)=\frac{1}{\omega_j^2-\omega^2+i\cdot 2\zeta_j\omega_j\omega}\qquad (i=\sqrt{-1})\quad j=1,2,\cdots,n \tag{19}$$

The mean square value matrix of the $e^{\text{th}}$ element stress response $[\psi_{\sigma e}^2]$ can be expressed as

$$[\psi_{\sigma e}^2]=E_e[B_1][\psi_{ue}^2][B_1]^T E_e \qquad e=1,2,\cdots,m \tag{20}$$

where $[B_1]$ is the element strain matrix, $[\psi_{ue}^2]$ is the mean square value matrix of the displacement of the nodal point of the $e^{\text{th}}$ element, and $E_e$ is the Young's modulus of the $e^{\text{th}}$ element.

## 4. NUMERICAL CHARACTERISTICS OF NON-STATIONARY RANDOM RESPONSE OF STOCHASTIC SMART TRUSS STRUCTURES

The following parameters corresponding to $\zeta_j$, $\rho_e$, $A_e$, $l_e$, $E_e^P$, $\rho_e^C$, $A_e^C$, $l_e^C$ and $c_{33e}$ are simultaneously considered as random variables. The randomness of physical parameters and geometrical dimensions will result in randomness of the matrices $[K]$ and $[M]$, and consequently the natural frequencies $\omega_j$ and natural modal matrix $[\phi]$. The mean value and standard deviation can be obtained by using the random factor method [7]. The randomness of the structural damping, natural frequencies, mode shapes and excitations will result in randomness in the structural dynamic responses of the closed loop control system, corresponding to the displacement and dynamic stress. From equation (18) and (20), by means of the functional moment method of random variables and algebra synthesis method [8], the mean value and standard deviation of the mean square value of the structural dynamic displacement and stress response can be determined.

## 5. EXAMPLE

A 20-bar planar smart truss structure shown in Figure 1 is used to illustrate the method. A ground level acceleration acts on the structure [8]. The material properties of the active and passive bars are given in Table 1.

Table 1. Intelligent truss structure physical parameters

| | Active bar (PZT-4) | Passive bar (steel) |
|---|---|---|
| Mean value of mass density $\rho$ (kg/m$^3$) | 7600 | 7800 |
| Mean value of elastic modulus $c_{33}$ (N/m$^2$) | $8.807\times10^{10}$ | $2.1\times10^{11}$ |
| Piezoelectric force/electric constant $e_{33}$ (C/m$^2$) | 18.62 | — |
| Dielectric constant $\varepsilon_{33}$ (C/Vm) | $5.92\times10^{-9}$ | — |
| Cross section area $A$ (m$^2$) | $3.0\times10^{-4}$ | $3.0\times10^{-4}$ |

In order to solve the optimal problem, two steps are adopted [5]. In the first step, the reliability constraints of dynamic stress and displacement are neglected, and the feedback gains are kept constant. Then, each element bar is taken as an active bar in turn and the corresponding performance function value is calculated. Based on the computational results for the dissipated energy, the optimal location of the active bar can be determined. In the second step, after the optimal placement of the active bar is obtained, the reliability constraints are imposed, and the optimization of feedback gain, that is, minimization of feedback gain will be developed.

For the first step, and letting the closed loop control system feedback gains be $g = g_j = 50$, each element bar is taken as active bar in turn; the corresponding performance function value is given in Table 2.

Table 2. The computational results of performance function ($g$=50).

| Element | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $J$ | 123.1 | 117.8 | 117.8 | 123.1 | 96.5 | 85.8 | 78.5 | 78.5 | 85.8 | 68.0 |
| Element | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| $J$ | 71.2 | 60.0 | 60.0 | 71.2 | 58.3 | 44.3 | 36.8 | 36.8 | 44.3 | 31.2 |

From Table 2, it can be seen that if the first or fourth element is used as the active bar, the active control performance of the smart truss structure is the best because the performance metric of energy dissipated is maximized. The effect of active vibration control of the smart truss structure is the worst if the 20th element is used as active bar. These results are not surprising since the control performance is the greatest when the active bar is closest to the primary ground excitation of the truss structure.

In order to assess the control performance with the reliability constraints imposed and optimization of the feedback gain, the control results using the 20th and 1st elements as the active bar respectively are compared. The structural parameters (material properties, geometric dimensions, structural damping) and the limit values of the mean square stress and displacement, $\psi_{\sigma e}^{2\,*}$ and $\psi_{uk}^{2\,*}$, are all taken to be random variables, where $\mu_{\psi_{\sigma e}^{2\,*}} = 2000\,\text{MPa}^2$, $\mu_{\psi_{uk}^{2\,*}} = 3.000\,\text{mm}^2$ and $R^*_{\psi^2_{\sigma e}} = R^*_{\psi^2_{uk}}$ =0.95. Values from both deterministic and random models are obtained. In the deterministic model (DM), the mean values of the random variables are unity, and their standard deviations are zero. The optimal results for the feedback gains, and the mean displacement and stress responses are given in Table 3, where $R_{\psi^2_{\sigma e}} = P_r\left\{\psi^{2\,*}_{\sigma e} - \psi^2_{\sigma e} \geq 0\right\}$ and $R_{\psi^2_{uk}} = P_r\left\{\psi^{2\,*}_{uk} - \psi^2_{uk} \geq 0\right\}$. In the random model (RM), the variation coefficients of all random variables are equal to 0.02. In addition, in order to verify our method, stationary random responses obtained using the Monte-Carlo simulation method (MCSM) are also presented in Table 3, in which 10000 simulations are used.

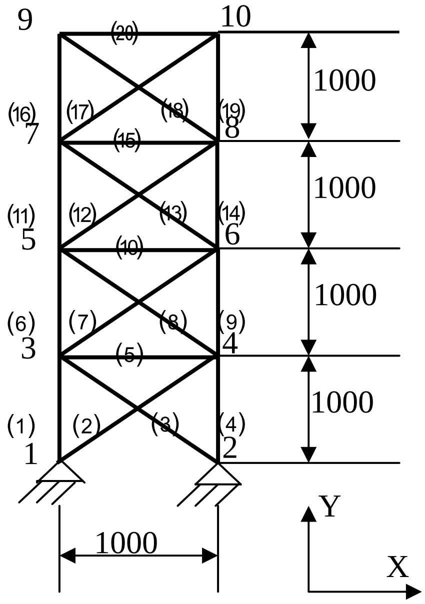


Figure 1. 20-bar planar smart truss structure (units: mm)

Table 3. Computational results of the feedback gains (*dynamic analysis by the MCSM method)

| | 1st element as an active bar | | | 20th element as an active bar | | |
|---|---|---|---|---|---|---|
| Design variables | Original value | DM | RM | Original value | DM | RM |
| G | 50 | 49.27 | 69.51 | 50 | 62.23 | 81.79 |
| *G | | | *69.57 | | | *81.88 |
| $\mu_{\psi^2_{\sigma e}}$ | 1737.6 | 1999.7 | 1582.8 | 2179.3 | 1999.1 | 1582.5 |
| $^*\mu_{\psi^2_{\sigma e}}$ | | | *1583.1 | | | *1582.9 |
| $\mu_{\psi^2_{uk}}$ | 2.7493 | 2.8454 | 2.3741 | 3.3079 | 2.8468 | 2.3739 |
| $^*\mu_{\psi^2_{uk}}$ | | | *2.3743 | | | *2.3744 |
| $R_{\psi^2_{\sigma e}}$ | | 0.47 | 0.98 | | 0.47 | 0.98 |
| $R_{\psi^2_{uk}}$ | | 0.51 | 0.95 | | 0.51 | 0.95 |

## 6. CONCLUSIONS

The results from the method presented in this paper are in good agreement with results obtained from the Monte-Carlo simulation method. The optimal results obtained with deterministic model and the random model are different. The optimal results of the deterministic model fulfil the normal constraints, but the results can not fulfil the reliability constraints.

In order to attain the same effect of active vibration control for intelligent truss structure, different elements are utilized as active bar, the corresponding optimal results of feedback gain are remarkably different.

The results of the example show that the areas of the system where the most energy is stored are the optimal location of an active bar in order to maximize its damping effect.

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