ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

THEORETICAL AND EXPERIMENTAL RESULTS OF THE TRANSMISSION LOSS OF A PLATE WITH DISCRETE MASSES ATTACHED

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Abstract

A mathematical model is developed to calculate the transmission loss of a simply-supported plate with an array of masses attached. Experimental tests were conducted and compared with the theoretical predictions and showed that there was good agreement. The results showed that the transmission loss of the plate can be improved by the addition of the masses, greater than that which would be predicted from the 'mass-law' model.

1. INTRODUCTION

This paper describes a mathematical framework for the calculation of the transmission loss of a plate with discrete masses attached. It is shown through both experimental results and theoretical predictions that the transmission loss of the plate can be improved by adding numerous discrete masses to the plate. The transmission loss that can be achieved by the addition of the discrete masses exceeds that would be obtained by merely 'smearing' the added mass across the plate, which is similar to increasing the thickness of the plate.

This publication provides an opportunity to correct some errors in a conference paper on a similar topic. A mathematical model was presented by Howard [1] for the transmission loss through a *clamped* plate with an array of discrete masses attached. The work presented in the paper was a "work in progress" and presented some promising experimental results. The mathematical model was not sufficiently accurate at the time, but did indicate similar trends as compared with the experimental results. The work presented in this paper describes the transmission loss of a *simply supported* plate with an array of discrete masses attached, including the corrections to the previous erroneous mathematical model.

The author was unable to find papers describing the transmission loss of a rectangular plate with an array of discrete masses attached. There are numerous papers describing the sound radiation from plates with simply-supported conditions that include closed-form solutions, such as Wallace [2], Lomas et.al. [3], Roussos [4]. Sound radiation from clamped edge conditions are less common and the author was unable to find in the research literature a closed-form solution for the sound radiation from a clamped-plate. Most researchers use numerical integration techniques to solve transmission loss problems to calculate the acoustic power radiated from the plate.

2. MATHEMATICAL MODEL

This section describes a mathematical model to enable the calculation of the transmission loss of a simply supported plate with an array of rigid masses attached at arbitrary locations on the plate. The theoretical derivation presented in this paper is similar to the work by Roussos [4], with the addition of the effects of the discrete masses, and extends the work by including the derivation of the diffuse-field transmission-loss.

The mathematical derivation starts with the structural behaviour of a simply-supported plate, and then includes the effect of translational and rotational inertia of discrete masses attached to the plate. Next, the vibro-acoustic coupling of the plate is considered by considering the pressure loading that occurs from an incident plane-wave striking the plate, and finally the acoustic power that is radiated from the vibrating plate.

2.1. Equation of Motion of the Plate

The vibration response of a general structure can be written in terms of its modal responses as [5]

$$\ddot{w}_s + 2\eta_s \dot{w}_s + \omega^2 w_s = \Gamma_s \tag{1}$$

For the simply-supported plate under consideration here, the displacement w of the plate can be written in terms of an infinite sum of its vibration modes multiplied by the modal participation factor for each mode $w_{m,n}$ as

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{m,n} \sin\left(m\pi x/L_x\right) \sin\left(n\pi y/L_y\right) = \mathbf{w}\boldsymbol{\Psi} \tag{2}$$

where L_x, L_y are the lengths of the plate along the x and y axes, $w_{m,n}$ are the modal participation factors, **w** is the corresponding vector of all the modal participation factors, $\boldsymbol{\Psi}$ is the corresponding matrix of mode shapes, and a particular combination of m, n indices is called the sth index where the resonance frequency of the m, n mode has been sorted into increasing resonance frequencies. The resonance frequencies of a simply-supported plate $\omega_{m,n}$ is given by

$$\omega_{m,n}^2 = \omega_s^2 = \frac{D\pi^4}{\rho h}\left[\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2\right]^2 \tag{3}$$

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{4}$$

where D is the bending stiffness of the plate, E is the Young's modulus, h is the thickness, ρ is the density of the plate, ν is the Poisson's ratio. The modal force Γ_s is given by

$$\Gamma_s = \frac{1}{\rho h N_s} \int_0^{L_x} \int_0^{L_y} \left[q_z U_{zp} + T_x U_{zp} + T_y U_{zp} \right] dydx \tag{5}$$

where q_z represent the vertical point force and T_i point moments applied along axes $i = x, y$ and are defined using the Dirac delta functions δ, as follows:

$$\begin{aligned} q_{zJ} &= F_{zJ} \delta(x - x_J) \delta(y - y_J) e^{j\omega t} \\ T_{iJ} &= (M_{iJ}/R^2) \delta(x - x_J) \delta(y - y_J) e^{j\omega t} \end{aligned} \tag{6}$$

where F_z are the vertical forces and M_i moments applied on the plate in the directions, U_z is the modal response in the vertical direction where $U_{zs} = \mathbf{w}\mathbf{\Psi}$. The term $\rho h N_s$ in Eq. (5) is the modal mass of the structure Λ_s, where

$$N_s = \int_0^{L_x} \int_0^{L_y} U_{zs}^2 dydx = L_x L_y / 4 \tag{7}$$

and hence the modal mass of a simply-supported plate is one-quarter of the mass of the plate. By making use of the relationship

$$\int_\epsilon F(\epsilon) \frac{\partial}{\partial \epsilon} \left[\delta(\epsilon - \epsilon^*) \right] d\epsilon = \frac{\partial F(\epsilon^*)}{\partial \epsilon} \tag{8}$$

the expression for the modal force in Eq. (5) can be written as

$$\Gamma_s = \frac{1}{\Lambda_s} \left[\mathbf{\Psi} F_J - \frac{\partial \mathbf{\Psi}}{\partial y} M_{Jx} + \frac{\partial \mathbf{\Psi}}{\partial x} M_{Jy} \right] \tag{9}$$

It will be shown now that the partial differentials of the mode shape matrix in Eq. (9) are equivalent to the mode shapes for the rotation of the plate. The rotations of the plate are given by [6]

$$\theta_s = \frac{v}{R} - \frac{1}{R} \frac{\partial w}{\partial \theta} \tag{10}$$

$$\theta_\theta = -\frac{1}{R} \frac{\partial w}{\partial s} \tag{11}$$

which describe the partial differentials of the mode shapes $\mathbf{\Psi}$ along each of the translational axes. Hence Eq. (9) can be written as

$$\Gamma_s = \frac{1}{\Lambda_s} \left(\mathbf{\Psi}_J F_J - \mathbf{\Psi}_{J\theta_x} M_{Jx} + \mathbf{\Psi}_{J\theta_y} M_{Jy} \right) \tag{12}$$

where $\boldsymbol{\Psi}_{J\theta_x}$ and $\boldsymbol{\Psi}_{J\theta_y}$ are the rotational mode shapes about the θ_x and θ_y axes respectively and are given by

$$\boldsymbol{\Psi}_{J\theta_x} = \frac{\partial \boldsymbol{\Psi}_J}{\partial y} = \sin\left(m\pi x/L_x\right)\,\left(n\pi/L_y\right)\cos\left(n\pi y/L_y\right) \tag{13}$$

$$\boldsymbol{\Psi}_{J\theta_y} = \frac{\partial \boldsymbol{\Psi}_J}{\partial x} = \left(m\pi/L_x\right)\cos\left(m\pi x/L_x\right)\,\sin\left(n\pi y/L_y\right) \tag{14}$$

The impedance of the Jth mass attached to the plate is included as translational and rotational inertias as

$$F_J = \omega^2 m_J \tag{15}$$

$$M_{J\theta_x} = \omega^2 J_{J\theta_x} \tag{16}$$

$$M_{J\theta_y} = \omega^2 J_{J\theta_y} \tag{17}$$

where m_J is the mass of the block, $J_{J\theta_x}$, $J_{J\theta_y}$ are the rotational inertias of the blocks along the θ_x, θ_y axes, respectively.

2.2. Vibro-acoustic coupling of the plate

Roussos [4] describes a modal summation method to calculate the transmission loss of a simply-supported plate, which provides a good framework to incorporate the effects of an array of discrete masses attached to the plate. The method involves (1) calculating the modal force that is applied to a plate due to an incident plane-wave striking the plate, (2) applying this modal force to the plate and calculating the vibration response, as per the previous section, (3) calculating the pressure, intensity, and radiated power from the plate, and (4) calculating the transmission loss of the plate by using the ratio of the incident acoustic power striking the plate and the radiated power from the plate.

Consider a plane wave incident on a simply supported plate as shown in Figure 1. The

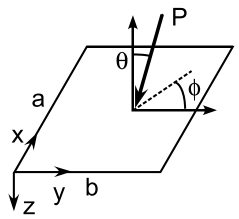

Figure 1. Plane wave striking a simply supported plate.

incoming pressure wave has an amplitude P_i and strikes the plate at angles θ_i normal to the plate and ϕ_i in the plane of the plate. The pressure that is incident on the plate $P(x, y)$ is given by

$$P(x, y) = P_i \exp[j(\omega t - kx\sin\theta_i\cos\phi_i - ky\sin\theta_i\sin\phi_i)] \tag{18}$$

and can be written in terms of a modal pressure that acts on the plate, which is calculated by multiplying the pressure distribution by the mode shape matrix (and dividing by the modal mass matrix $\boldsymbol{\Lambda}$ to be consistent with Eq. (1)) for the structure as $\boldsymbol{\Psi}P(x, y)/\boldsymbol{\Lambda}$, which can be written as $p_{m,n} = L_x L_y \overline{Y}_m \overline{Y}_n$ where [2]

$$\overline{Y}_m = (m\pi)\frac{1-(-1)^m e^{-j\alpha}}{(m\pi)^2-\alpha^2} \tag{19}$$

$$\overline{Y}_n = (n\pi)\frac{1-(-1)^n e^{-j\beta}}{(n\pi)^2-\beta^2} \tag{20}$$

where $\alpha = kL_x \sin\theta\cos\phi$, $\beta = kL_y \sin\theta\sin\phi$, $k = \omega/c$ is the wavenumber, ω is the frequency, c is the speed of sound in air.

The transmitted pressure at a point remote from the plate due to the vibration of the plate is calculated using the Rayleigh integral and can be written as [2, 7]

$$p^t_{m,n} = -j(j\omega w_p)k\rho c\frac{e^{jkr}}{2\pi r}L_x L_y Y_m Y_n \tag{21}$$

where [2]

$$Y_m = (m\pi)\frac{1-(-1)^m e^{-j\alpha}}{(m\pi)^2-\alpha^2} \tag{22}$$

$$Y_n = (n\pi)\frac{1-(-1)^n e^{-j\beta}}{(n\pi)^2-\beta^2} \tag{23}$$

and the transmitted intensity is calculated as $I^t = |\sum_m \sum_n p^t_{m,n}|^2/(2\rho c)$. The total power Π^t that is radiated by the plate is calculated as the integral of the intensity over an imaginary far-field hemisphere as

$$\Pi^t = \int_{\phi_t=0}^{2\pi}\int_{\theta_t=0}^{\pi/2} I^t r^2 \sin\theta_t d\theta d\phi \tag{24}$$

The power that is incident on the plate is given by

$$\Pi^i = (|P_i|^2 L_x L_y \cos\theta_i)/(2\rho c) \tag{25}$$

Finally, the transmission loss TL for a plane wave striking the plate is given by

$$\mathrm{TL} = 10\log_{10}(\tau(\theta_i,\phi_i)) = 10\log_{10}(\Pi^i/\Pi^t) \tag{26}$$

A diffuse field is characterised by an infinite number of uncorrelated plane-waves. The sound field inside the reverberation chamber used in the experimental part of the work conducted here is assumed to be a diffuse field. The transmission loss for a diffuse field is calculated as [8]

$$\mathrm{TL}_{\mathrm{diffuse}} = \frac{\int_0^{2\pi}\int_0^{\pi/2}\tau(\theta_i,\phi_i)\sin\theta_i\cos\theta_i\, d\theta_i d\phi_i}{\int_0^{2\pi}\int_0^{\pi/2}\sin\theta_i\cos\theta_i\, d\theta_i d\phi_i} = \frac{\int_0^{2\pi}\int_0^{\pi/2}\tau(\theta_i,\phi_i)\sin 2\theta_i\, d\theta_i d\phi_i}{2\pi} \tag{27}$$

3. EXPERIMENT

The transmission loss of a clamped aluminium plate was experimentally measured. The plate had the properties described in Table 1.

Figure 2(a) and (b) show a picture of an aluminium plate installed in the transmission loss test facility at the University of Adelaide, and a close-up picture of the rigid-blocks attached to the plate with 'super-glue', respectively. Forty-nine rigid-blocks were cut from bar-stock such that they had masses as shown in Figure 3. The blocks were arranged on the plate as shown in Figure 2(c), such that the lightest mass (block 1) was located in the top left corner and the heaviest mass (block 49) located in the bottom right corner. The purpose of this experiment was not to determine an optimum arrangement for the blocks, but to compare the theoretical predictions with experimental results.

Table 1. Geometry of the plate

Width L_x	1.0	m
Height L_y	1.5	m
Thickness h	0.0015	m
Density ρ	2700	kg/m^3
Young's Modulus E	70	GPa
Poisson's ratio ν	0.33	No units
Loss factor η	0.01	No units

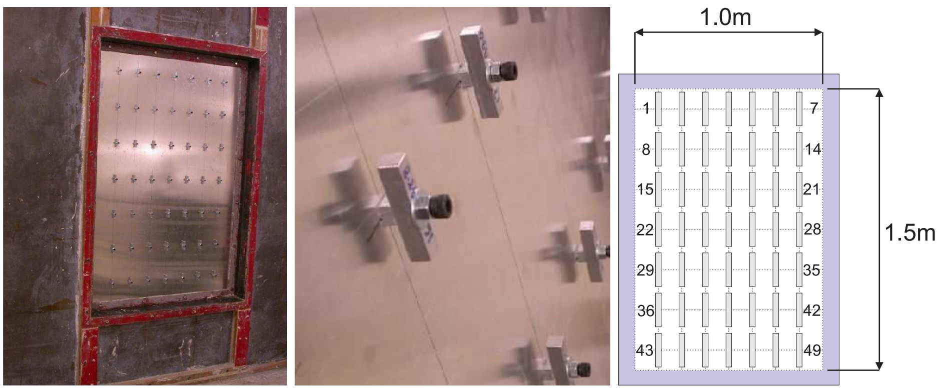

Figure 2. (a) picture of the plate installed in the transmission loss test facility, (b) close-up picture of the rigid blocks attached to the plate, (c) arrangement of the masses on the plate.

Figure 4(a) shows the transmission loss of the plate for the experimental results and theoretical predictions based on Sewell's finite plate theory [7, 9] and the model presented here, when no masses were attached to the plate. The results show that there good agreement between the Sewell's theory for the finite plate and experiment. It can be seen that the modal method has poor agreement above approximately 1000Hz, which is caused by including only 2000 structural modes in the analysis, up to a frequency of about 5000Hz. Generally, modal summation methods require inclusion of modes two octaves higher than the frequency range of interest.

Figure 4(b) shows the transmission loss of the plate with and without the masses for the modal method described here and the experimental results. The figure shows that the experimen-

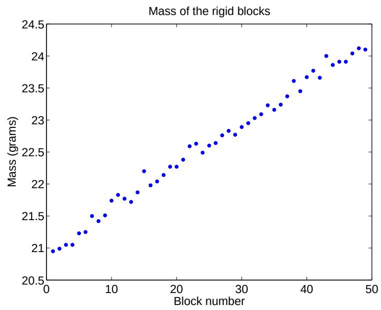

Figure 3. Masses of the 49 blocks.

tal results of the plate with the masses attached has greater transmission loss than the bare plate over the frequency range from 125-400Hz. The theoretical results also indicate improvement in the transmission loss over the frequency range from 100-300Hz.

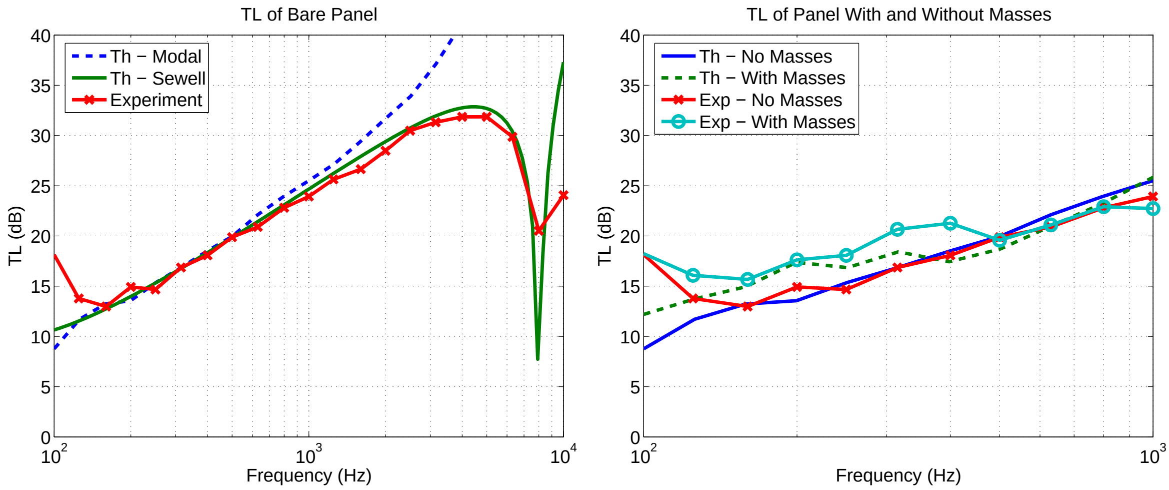

Figure 4. (a) Transmission loss of the plate for the modal method, Sewell, and experimental results; (b) Modal method and experimental results with and without masses.

Figure 5(a) shows the transmission loss of the plate predicted using Sewell's theory for the thickness of the plate used in the experiment, a plate of equivalent thickness (1.79mm) had the mass of the rigid blocks been smeared across the plate, and the experimental results of the transmission loss of the plate with and without the masses attached. The results show that the experimentally measured transmission loss of the plate with the masses attached is greater than the theoretical predictions for a thicker plate.

Figure 5(b) shows the predicted and measured improvement in the transmission loss due to the addition of the rigid masses, which is calculated as the transmission loss with the masses attached minus the transmission loss without the masses attached. The figure shows that there is good agreement between theoretical and experimental results.

4. CONCLUSIONS

A theoretical model was developed to enable the prediction of the transmission loss of a simply-supported plate with rigid masses attached. Theoretical predictions were made and compared

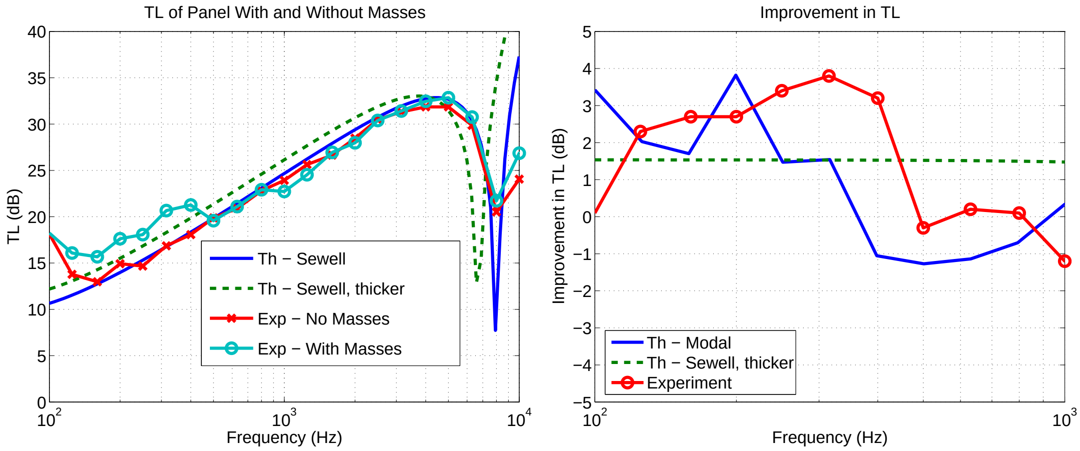

Figure 5. (a) Transmission loss of the plate; (b) Improvement in transmission loss.

with experimental results. It was found that the effect of the masses increased the TL of the plate, greater than that which would have been achieved if the added mass from the rigid blocks had been smeared across the plate, which would have the effect of increasing the thickness of the plate. Similar work to material presented in this paper was presented at an earlier conference [1], however there were a number of mistakes with the theoretical model that have been corrected in this paper.

Future work will be done to develop a theoretical model to predict the transmission loss of a plate with an array of vibration absorbers attached, which will be compared with experimental measurements. Theoretical models have been developed for fuzzy structures that comprise a *large* number of vibration absorbers attached to a master structure [10].

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