

THE DYNAMIC ANALYSIS OF A BUILDING STRUCTURE – ACOUSTIC VOLUME INTERACTION SYSTEM EXCITED BY HUMAN FOOTFALL IMPACTS

Jing Tang Xing, Ye Ping Xiong and Mingyi Tan

School of Engineering Sciences, University of Southampton, Southampton SO17 1BJ, England. jtxing@soton.ac.uk

Abstract

A numerical model is developed using a mixed finite element method to simulate building structure – acoustic volume interaction systems excited by human walking impacts. The pressure in the air volume and the displacement in the structure are chosen as fundamental variables to describe structure – air interaction dynamics. The governing equations and corresponding variational formulation are presented. Based on available experimental results on the footfall load history, an approximate load function is proposed to simulate the measured dynamic footfall load as a moving load with a walking speed and dynamically applied to each foot contact point. A simple example is presented to illustrate the method which reveals the mechanism of low-frequency vibration produced by human walking impacts using an air-structure interaction method. The benefits of the proposed method are summarized to provide a guideline using the method to practical house designs.

1. INTRODUCTION

Assembly structures such as stadiums, theatres, and dance studios are primarily subjected loads produced by crowds of people. These loads increase the possibility of excessive and annoying vibrations when high-strength and light-weight construction materials are used [1-5]. As described in [6], when occupants were asked to describe the "character" of the footstep noise that they found, most objectionable terms, such as "thuds", "thumps" and "booming" were the most frequently used. Many people also complained about rattling light fixtures, closet doors and wall-hung furniture, which they identified with feel-able structural vibration created by footfall impacts. Thus from these interviews it became obvious that the principal factor responsible for complaints involved the low-frequency impulse response of the structural floor / ceiling system when excited by foot-steps.

To avoid these annoying vibrations, many researches have been reported, which covers all aspects involving experimental, analytical and numerical investigations as well as practical structural designs. For example, Ungar & White [7] and Ungar et al [8] have developed a basis for estimating levels of footfall-induced floor vibration that has achieved wide

recognition as a useful tool in the structural design of floor systems to minimize lowfrequency disturbances, whereas the document 11 of the steel design guide series publications by AISC and CISC [9] presented the detailed design methods considering floor vibrations due to human activity. Blazier & DuPree [6] contributed a very detailed investigation of lowfrequency footfall noise in wood-frame, multifamily building construction. To name but too many, the interested reader may wish to consult the 254 references reviewed in a comprehensive critical review paper published by two parts [10-11]. The first part of the paper surveyed 132 general references pertinent to the three key aspects involving the vibration source, path and receiver of the problem. The second part of the paper reviewed 122 references on the approaches which integrated the three key aspects of the problem and proposed the ways for its mathematical modelling.

The knowledge learnt from our daily living experiences suggests that vibrations of a building structure are strongly influenced by the vibrations of the air volumes included in the building. This implies that an integrated building vibration system consists of its solid structure and air volume included, and therefore this is an air-acoustic volume-structure interaction problem. However, while reading the available references on building vibrations induced by human activities, we have not found any references investigated the problem considering air-structure interactions. This paper intends to address this problem to understand the mechanism on the annoying noises, such as "thuds", "thumps" and "booming" claimed by many people using a numerical method on fluid-structure interaction dynamics developed in [12-13].

2. MATHEMATICAL MODELLING



2.1 Governing Equations of Building Structure – Acoustic Volume Interaction System

Figure 1. Schematic illustration of a building structure – air acoustic volume interaction system.

Fig.1 shows a generalized building structure – acoustic volume interaction system consisting of a flexible structure of mass density ρ_s within a domain Ω_s of boundary $S_T \cup S_w \cup \Sigma$

with its unit normal vector v_i and the air in a domain Ω_f of boundary $\Gamma_p \cup \Sigma$ with a unit normal vector η_i . The system is excited by external dynamical forces $\hat{T}_i, \hat{F}_i, \hat{f}_i, \hat{p}$ and ground acceleration \hat{w}_i . The Cartesian tensor notations [14] with subscripts *i*, *j*, *k* and *l* (=1, 2, 3) obeying the summation convention are used in the paper. For example, u_i , v_i , w_i , e_{ij} , σ_{ij} and E_{ijkl} represent displacement, velocity, acceleration vector, strain, stress and elastic tensor in the solid, respectively, *p* denotes the pressure in the air volume and \hat{p} is defined as the given dynamic atmospheric pressure, say $\hat{p}=0$. The governing equations describing the dynamics of the coupled air-structure interaction problems are as follows.

2.1.1 Building structure

Dynamic equation

$$\sigma_{ij,j} + \hat{F}_i = \rho_s w_i, \qquad (x_i, t) \in \Omega_s \times (t_1, t_2).$$
(1)

Strain-displacement

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad (x_i, t) \in \Omega_s \times (t_1, t_2).$$
⁽²⁾

Constitutive equation

$$\sigma_{ij} = E_{ijkl} e_{kl}, \qquad (x_i, t) \in \Omega_s \times (t_1, t_2). \tag{3}$$

and, assuming linearity, we have

$$v_i = u_{i,t}, \ w_i = v_{i,t}, \ d_{i,j} = e_{ij,t} = \frac{1}{2}(v_{i,j} + v_{j,i}).$$
 (4)

Boundary conditions

acceleration:
$$w_i = \hat{w}_i, \quad (x_i, t) \in S_w \times [t_1, t_2],$$
 (5-1)

traction:
$$\sigma_{ij}\nu_j = T_i$$
, $(x_i, t) \in S_T \times [t_1, t_2]$, (5-2)

2.1.2 Air acoustic volume

Dynamic equation

$$p_{,tt} = c^2 p_{,ii}, \quad (x_i, t) \in \Omega_f \times (t_1, t_2).$$
 (6)

Boundary condition
pressure:
$$p = \hat{p}$$
, $(x_i, t) \in \Gamma_n \times [t_1, t_2]$, (7)

2.1.3 Air – structure interaction interface

The following consistent and equilibrium conditions should be satisfied.

$$w_i v_i = p_i \eta_i / \rho_f, \quad (x_i, t) \in \Sigma \times [t_1, t_2],$$
(8-1)

$$\sigma_{ii} v_i = p \ \eta_i , \quad (x_i, t) \in \Sigma \times [t_1, t_2].$$
(8-2)

2.2 Variational Formulations and FEA Equations

The variational principle [13] describing this building structure – acoustic volume interaction system is as follows:

$$H_{sf}[p,w_{i}] = \int_{t_{1}}^{t_{2}} \{\int_{\Omega_{s}} (\frac{1}{2}\rho_{s}w_{i}w_{i} - \frac{1}{2}E_{ijkl}d_{ij}d_{kl} - \hat{F}_{i}w_{i})d\Omega_{s} - \int_{S_{T}}\hat{T}_{i}w_{i}dS - \int_{\Sigma} p w_{i}\eta_{i}d\Gamma + \int_{\Omega_{f}} \frac{1}{2}[\frac{p_{,t}p_{,t}}{\rho_{f}c^{2}} - \frac{p_{,i}p_{,i}}{\rho_{f}}]d\Omega_{f}\}dt$$

$$(9)$$

Using FEA interpolations [15-16] as well as the method given by [12-13], from equation (9) we obtain a finite equation describing the air – structure interaction system

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{m} \end{bmatrix} \begin{bmatrix} \mathbf{\ddot{U}} \\ \mathbf{\ddot{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}^T \mathbf{M}^{-1} \mathbf{K} & -\mathbf{K}^T \mathbf{M}^{-1} \mathbf{R}^T \\ -\mathbf{R} \mathbf{M}^{-1} \mathbf{K} & \mathbf{k} + \mathbf{R}^T \mathbf{M}^{-1} \mathbf{R}^T \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{f} \end{bmatrix},$$
(10)

where U, M and K represents the displacement, mass and stiffness matrices of the structure, respectively; \mathbf{p} , \mathbf{m} and \mathbf{k} denote the pressure, mass and stiffness matrices of the fluid domain; \mathbf{R} is the fluid-structure interaction matrix. For natural vibrations of the system, the force vector on the right of equation (10) vanishes. The developed computer code [17] is used to solve this equation.

3. FOOTFALL LOAD SIMULATIONS

There are many publications involving human walking loads measurement and simulations. Fro example, references [18-27] investigated and developed various experimental measurement approaches to obtain the time histories of the dynamic loads induced by human activities. The experiment results were used to construct approximate load functions to simulate footfall loads used in calculations or designs. ASTM E 492-77 [28], ASTM E 989-84 [29] and ASTM 1991 [30] presented some standards used in these experiments.

The time impulse function used in the simulation is shown in figure 2 which is an approximation of the experimental result given in [24]. This load will be dynamically added at each different floor point at which the walking person foot contacts. To establish a relationship between the walking speed and the starting time of the walking load at each point, we assume that a person is walking along x direction, shown in figure 3, from the origin x_0 in a step length L = 80cm and a walking speed v = 143cm/s. This gives a period $\tau = L/v = 0.56$ s of walking time impulse function represented by

Here *t* denotes a relative time from the starting time $t_N = 3(N-1)\tau/4$ at which the *N-th* step starts and the walking impulse function is added at point $x_N = x_0 + (N-1)L$. Therefore, there is about a short time period about $\tau/4$ in which the walking loads are applied at two feet contacting points x_{N-1} and x_N to meet the practical cases as measured in the experiments reported in [24].

Using this numerical approach developed herein, there is no need to find a continuous walking load function derived from the experiment results but the measurement loads data can be directly used in the simulation. It is also very convenient to simulate more complex cases, such as several people walking in the different directions.





Figure 2. The time history of human walking impact load function.

Figure 3. The mesh structure of a wood chamber – air interaction system.

$$f(t) = \begin{cases} 4t/\tau, & 0 \le t \le \tau/4, \\ 1-2(t-\tau/4)/\tau, & \tau/4 \le t \le \tau/2, \\ 1/2+2(t-\tau/2)/\tau, & \tau/2 \le t \le 3\tau/4, \\ 1-4(\tau-3\tau/4)/\tau, & 3\tau/4 \le t \le \tau, \\ 0, & t \ge \tau. \end{cases}$$
(11)

4. NUMERICAL EXAMPLE

Figure 3 shows a system of a wood chamber – air interaction system. The origin of the coordinate system O - XYZ is located at the centre of the bottom. The geometrical size in X, Y and Z directions of this chamber is 480 X 480 X 240cm. The top and walls of this chamber are made of wood plates of thickness 6cm and gravity density $\gamma_s = 5.0 \times 10^{-4} \text{ Kg/cm}^3$. The Young's modulus and Poisson ratio of the wood are assumed as $E = 1.2 \times 10^5 \text{ Kg/cm}^2$ and $\mu = 0.315$, respectively. The material properties of the air inside the chamber are used as the gravity density $\gamma_a = 1.225 \times 10^{-6} \text{ Kg/cm}^3$ and the speed of sound $C = 3.4 \times 10^4 \text{ cm/s}$. A person is walking along the central symmetrical line, parallel to X axis, on the top plate of the chamber from node488 to node500 in a step length L = 80 cm and speed $\nu = 143 \text{ cm/s}$. The footfall load is a time impulse function with a period $\tau = 0.56\text{ s}$ shown in figure 2 and expressed by equation (11). To simulate human walking, this load is added at node488 at time t = 0 and at node490 at time $t = 3\tau/4$ which moves along X-direction in the person walking speed. There is a time period of $\tau/4$ during which the two loads are simultaneously added at the two connecting nodes at which the human feet contact.

In the simulation, FSIAP computer code [17] is used to complete the air-structure interaction analysis. The sold structure is modelled by 432 four node plate elements and the air volume is modelled by 864 three dimensional pressure elements. The corresponding interface elements on the air-structure interfaces between the structure and the air are used to generate the coupling matrix involving the air acoustic volume and structure. The computer

code provides a convenient way allowing users to choose any interested points in the structure or in the air volume to obtain the displacement, stress on the structure and the dynamic pressure in the air.



Figure 4. The sound pressures at nodes 420 (-160, 0,160), 422 (-80, 0, 160), 424 (0, 0,160), 426 (80, 0, 160) and 428 (160, 0,160) in the chamber (cm).



Figure 5. Dynamic displacement responses at Point 488, 494 and 500 on the top floor of the chamber.

Figure 4 shows the dynamic pressure of sound at four nodes in the air acoustic volume inside the chamber. It is assumed that four people of same height of 160cm stand at Node 420, 422, 424 and 426, respectively. These four people can hear the noise pressure produced by the human walking on the top plate, which are shown by the four curves in figure 4. From this result, it is found that although the used fundamental walking load frequency is about 1/0.56=1.786Hz, the dynamic pressure of sound excited by the walking loads includes some higher frequency components, for example, there is a pressure component of about 17Hz appearing in the curves. It has been realised that this frequency is one of natural frequency of the air – structure interaction system. This simulation incorporating air acoustic volume –

structure interaction provides a deep understanding on the reason of low-frequency walking noise often happening in wooden buildings.

Figure 5 shows the dynamic displacements at three points on the top floor of the chamber. For this system, the displacements in Y-direction at three points are very small. The displacement in X-direction at the point 494 is also very small. However, the displacements in Z-direction at all points are larger than other directions.

Although the presented example is very simple but the fundamental mechanism of air acoustic volume – building structure interactions is considered. The drum-like mechanism in wooden building structures caused by human walking step loads is revealed using the proposed fluid-structure interaction method. It has realized that the real structures of wooden walls and floors are more complex than the case of the simple example considered herein. However, as indicated in finite element books [15-16], it is not difficult to create complex structure meshes and suitable elements to address the requirements from practical house designs. The computer code for dynamic analysis of general fluid-structure interaction systems [17] provides an essential program to address some practical problems for better building designs to reduce low-frequency vibration and noise caused by human walking impacts.

5. CONCLUSIONS AND DISCUSSIONS

The numerical method developed to analysis the dynamic response of buildings subject human working impacts has the following benefits.

- 1) An integrated acoustic volume structure interaction system allows air structure interactions to be considered, and therefore provides an approach to reveal the mechanism of low frequency vibration noises like "thuds", "thumps" and "booming" induced by human footfall loads.
- 2) The time history of human walking loads is simulated using a measured time history of a single step load of which the moving speed and path can be defined by any practical cases. Therefore, it is no need to generate a time function representing the walking loads.
- 3) The numerical method provides a convenient way to simulate the loads introduced by several people walking in different directions with various speed and body weights.
- 4) It is convenient to adjust any designs of structures, such as considering windows and doors, different wall materials, stiffeners, damping materials etc. It is able to simulate multi-storied buildings and their couplings as well as vibration transmission between them.
- 5) The model allows to find effective vibration isolation or control methods or to further check current design from a view point of vibration / noise levels and comfortable living environment.

The example studied herein to illustrate the developed method is quite simple compared with practical house structures and excitations. However, it is not difficult to use complex meshes and loads distributions to address practical requirements. The resultant dynamic response histories can be transferred to noise level required in noise analysis. The computer code developed provides various elements to simulate complex house structure – air acoustic volume interactions.

ACKNOLEDGEMENT

Authors acknowledge Dr G. Pernica, IRC, National Research Council Canada and Dr L. Hu, FCC, Canada for providing information and discussions on the research topic.

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