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PHYSICS-BASED NUMERICAL METHOD FOR ANALYZING MID-FREQUENCY ACOUSTIC RADIATION

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Abstract

As industrial noise sources have increased year by year, tolerance for noise levels by industry, government, and the general population has decreased. Ever more stringent noise regulations are driving up the cost of design and manufacturing. To reduce this cost, designers look to the NVH community for sophisticated numerical tools to predict noise over a broad frequency range. This work focuses on mid-frequency acoustic analysis tools.

Boundary, finite, and infinite element methods have proven useful for solving the 3D acoustic wave equation at low frequencies. For high frequency, localized formulations such as plane-wave and ray-tracing methods have been applied. These methods are impractical, inaccurate, or both for treating mid-frequency problems. Low-frequency methods make very large demands on computer processing while high-frequency methods do not account for non-local effects.

Several investigators [1-4] have proposed applying a hybrid low/high frequency approach to solve the mid-frequency problem. This approach essentially applies mathematical functions that match the fluid impedance at the low and high ends of the frequency spectrum and provide a mathematically smooth bridge between them. The drawback to this approach is that it does not address the physics unique to mid-frequency, and therefore it cannot provide very accurate solutions.

The goal of this work is to develop a method for efficiently addressing a class of mid-frequency vibration. This class deals with structural surfaces characterized by a moderate number of major sections in which the spatial wavenumber vibration content is bandlimited and known a priori. A physics-based strategy will be followed to capture the fluid-structure interaction character that is peculiar to the mid-frequency range.

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1. INTRODUCTION

The differential equations governing acoustic radiation were derived in the late 19th century [5],6]. Complete, closed-form analytical solutions were tractable mainly for the simplest of geometries, for example the sphere, as other geometries produce integrands unsuitable for analytical integration. Investigators interested in finding solutions for more complex geometries focused on low- and high-frequency ranges for which asymptotic approximations greatly simplified the integrands. With the advent of computers, numerical techniques expanded the geometric complexity for which solutions could be found, but still remained within the frequency restriction of low or high frequencies. While ever increasing speed and storage capacity of computers continues to raise the upper bound for low-frequency methods, for many practical problems there is a large gap in the frequency range for which asymptotic methods will not suffice for the foreseeable future.

Several investigators [1-4] have developed so-called hybrid methods, by which low- and high-frequency solutions are bridged mathematically. Frequency-domain polynomials are created that match the acoustic impedances asymptotically at low and high frequencies, while remaining smooth in the mid-frequency range. This method has found some practical application in time-domain shock-wave problems, in which the spectral content is dominated by the low and high frequency. However, for problems with predominantly mid-frequency excitation, these techniques have not proved accurate.

Asymptotic methods (and their hybrid offspring) do not work well in mid-frequency because mid-frequency physics is significantly different from low- and high-frequency physics. In mid-frequency pressure at any surface point is highly affected by motion in a finite neighbourhood surrounding that point. This is quite different from low-frequency, where the motion of every surface point can significantly contribute to the surface pressure, and from high-frequency, where only the motion of the surface point itself contributes to its pressure. This difference in spatial influence cannot be captured by following the hybrid approach of applying a weighted average of the low- and high-frequency solutions.

The difficulty in developing a general numerical method for mid-frequency acoustic problems has been that a wide range of surface velocity wavenumbers poses insatiable demands for processing and storage. However, there is a large class of acoustic radiation problems for which the surface velocity wavenumber content is fairly narrowband. Vibrations of large shells for example. By limiting the scope to problems with bandlimited surface velocity wavenumbers, some practical progress can be made. This paper presents the formulation of a numerical method for analysing acoustic radiation for spatially-bandlimited surface vibrations.

2. FORMULATION

The surface pressure P generated by a surface S vibrating in an infinite acoustic space is expressed by the Helmholtz Integral Equation (1),

$$P(\vec{x},k) = \int_S [\rho G(r,k) V(\vec{y},k) - H(r,k) P(\vec{y},k)] dS(\vec{y}) \tag{1a}$$

$$r \equiv \|\vec{x} - \vec{y}\| \tag{1b}$$

$$G(r,k) \equiv \frac{ikc}{2\pi r} e^{-ikr} \tag{1c}$$

$$H(\vec{x},\vec{y},k) \equiv \frac{1}{2\pi}\left[\vec{\nabla}\left(\frac{1}{r}e^{-ikr}\right)\cdot\vec{n}\right] \tag{1d}$$

where $\vec{x}$ and $\vec{y}$ are the position vectors of the field and source points, respectively, V is the outward normal velocity, and $\vec{n}$ is the unit normal vector directed into the fluid. Numerical analysis requires the discretization of the integral equation to form a complete set of algebraic equations.

The numerical approach developed in this paper for mid-frequency analysis is an extension of the boundary element analysis (BEA) method that has been developed for low-frequency problems. Hence it is instructive to summarize BEA to set the context for mid-frequency analysis.

2.1 Low Frequency Boundary Element Analysis

For low-frequency analysis, we partition the surface into a mesh of surface elements and suppose the mesh spacing small enough that the normal velocity and pressure are approximately constant over each element. Then the Helmholtz Integral Equation transforms into a set of discrete algebraic equations.

$$P_n(f) = \sum_{m=1}^{N}\left[G_{nm}(k)|S_m|V_m(k) - H_{nm}(k)|S_m|P_m(k)\right], \quad n = 1,\ldots,N \tag{2a}$$

$$G_{nm}(k) \equiv \frac{1}{|S_n||S_m|}\int_{S_n}\int_{S_m} G(r,k)\,dS(\vec{y})\,dS(\vec{x}), \quad m,n = 1,\ldots,N \tag{2b}$$

$$H_{nm}(k) \equiv \frac{1}{|S_n||S_m|}\int_{S_n}\int_{S_m} H(r,k)\,dS(\vec{y})\,dS(\vec{x}), \quad m,n = 1,\ldots,N \tag{2c}$$

in which $|S_n|$ is the nth element surface area. Assembling the surface element pressures, velocities, areas, and associated coefficients into matrices produces a matrix equation that can be solved for the pressures for known distributions of velocities:

$$\{P\} = [G][S]\{V\} - [H][S]\{P\} = [[I] + [H][S]]^{-1}[G][S]\{V\} \tag{2d}$$

The numerical integrations involved to compute the matrix coefficients (2b) and (2c) require special algorithms removing singularities for the diagonal components (coincident field and source elements). This singularity-removal step is one of two major numerical tasks in setting up the boundary element problem. The second major task is inverting the matrix $[[I]+[H][S]]$ in (2d). Special care must be taken in limiting the element size so this matrix is non-singular.

2.2 Modifying The Discrete Helmholtz Integral Equation For Mid-Frequency

If we take a boundary element mesh suitable for low-frequency analysis and raise the frequency, the assumption of constant normal velocity and pressure over an element begins to fail as the acoustic wavelength decreases to the element span.

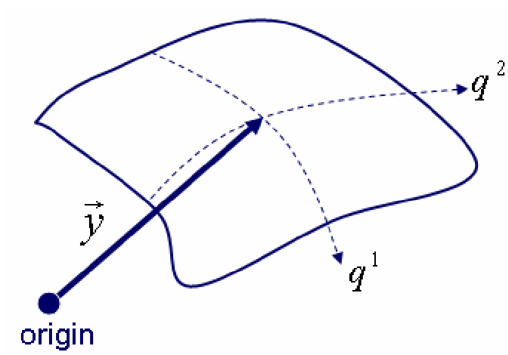

Figure 1: Surface element geometry

Consider the geometry of a general surface element depicted in Figure 1. Let the 3D space position vector $\vec{y}$ of a point on the surface element be specified by two surface coordinates[1] $q^\alpha \ (\alpha = 1,2)$. The major condition imposed by this method is to restrict the normal velocity to be sinusoidally-distributed over the element,

$$V(\vec{y},k) \equiv V_m(k)\exp\left[i\vec{k}_m \cdot (\vec{y} - \vec{y}_{0m})\right], \quad \vec{y} \in S_m, \quad m = 1,\ldots,N \tag{3a}$$

where $\vec{y}_{0m}$ is the 3D space vector of the position of a reference point, and $\vec{k}_m$ is the velocity wavenumber 3D space vector, on the m[th] surface, respectively. The velocity amplitude can be found by integrating the velocity with the same sinusoidal weighting magnitude but with opposite phase,

$$V_m(k) = \frac{1}{|S_m|}\int_{S_m} V(\vec{y},k)\exp\left[-i\vec{k}_m \cdot (\vec{y} - \vec{y}_{0m})\right] dS(\vec{y}), \quad m = 1,\ldots,N \tag{3b}$$

The major assumption is that the pressure over the element has the same sinusoidal distribution,

$$P(\vec{y},k) \approx P_m(k)\exp\left[i\vec{k}_m \cdot (\vec{y} - \vec{y}_{0m})\right], \quad \vec{y} \in S_m,\ m = 1,\ldots,N \tag{4}$$

Inserting equations (3) and (4) into equations (2b) and (2c) modifies the coefficients of the discrete HIE matrices.

$$\hat{G}_{nm}(k;\vec{k}_n,\vec{k}_m) \equiv \frac{1}{|S_n||S_m|}\int_{S_n}\int_{S_m} G(r,k)\exp\left[i\vec{k}_m \cdot (\vec{y} - \vec{y}_{0m})\right] dS(\vec{y})\exp\left[-i\vec{k}_n \cdot (\vec{x} - \vec{x}_{0n})\right] dS(\vec{x}) \tag{5a}$$

$$\hat{H}_{nm}(k;\vec{k}_n,\vec{k}_m) \equiv \frac{1}{|S_n||S_m|}\int_{S_n}\int_{S_m} H(r,k)\exp\left[i\vec{k}_m \cdot (\vec{y} - \vec{y}_{0m})\right] dS(\vec{y})\exp\left[-i\vec{k}_n \cdot (\vec{x} - \vec{x}_{0n})\right] dS(\vec{x}) \tag{5b}$$

in which hatted quantities refer to mid-frequency coefficients. Comparison of Equation (5) with Equation (2) shows the mid-frequency coefficients approach the low-frequency coefficients as the velocity wavenumbers decrease to zero.

2.4 Evaluating Surface Integrals

The number of elements must be small enough for the matrix inversion in equation (2d) to be performed accurately without excessive demands on computer memory. This practical limitation requires element sizes of the same order as for low-frequency BEA. A further step for keeping the method practical is to use a mesh suitable for both low- and mid-frequency analyses.

[1] To distinguish between 3D space and 2D surface coordinates, indices for space coordinate values use Roman letters and indices for surface coordinate values use Greek indices.

As designed, the integrands in equation (5) will be highly varying over the surface element, thus presenting a challenge for efficient, accurate integration. Performing purely numerical integration would offer little computational savings over merely performing low-frequency BEA over a much finer mesh. Instead we pursue a strategy of increasing the amount of integration that can be performed analytically and make use of pre-tabulated functions.

Examine the double surface integral (over source and field element surfaces) in Equation (5). The integrands $G(r,k)\exp\left[i\vec{k}_m\cdot(\vec{y}-\vec{y}_{0m})\right]$ and $H(r,k)\exp\left[i\vec{k}_m\cdot(\vec{y}-\vec{y}_{0m})\right]$, and the differential surface area $dS(\vec{y})$ can be expressed explicitly in terms of q^α. These analytical expressions contain products of complex exponentials of polynomials in q^α of the general form

$$\exp i\left[M_0+M_1\left(q^1\right)+M_2\left(q^2\right)+M_3\left(q^1\right)^2+M_4\left(q^1\right)\left(q^2\right)+M_5\left(q^2\right)^2+\cdots\right] \qquad (6)$$

in which $M_0, M_1, M_2,\ldots$ are constants. Closed-form analytical integration is not possible for these terms, so these integrals are pre-computed numerically for a number of parameters and stored in lookup tables. The second integration, over the field surface, also involves these types of expressions and the lookup tables would be used here as well.

2.5 Status

The author has developed algorithms for defining surface elements and extracting the surface geometric information required for expressing the integrands in terms of surface coordinates. To facilitate application to general problems, the equations are formulated in general curvilinear coordinates. Rules of tensor calculus [7] are applied to express and compute vector quantities and their spatial derivatives. The author is using the programming language MATLAB with special emphasis on using the elements of MATLAB most akin to the C++ programming language such as class data structures.

The project reached its first milestone of being capable of performing standard low-frequency boundary element analysis. The second milestone, scheduled for October 2007, is to perform mid-frequency analysis on a submerged sphere. Subsequent milestones in 2008 and 2009 are to perform and validate mid-frequency analyses on a variety of surfaces.

4. CONCLUSIONS

A method for solving the mid-frequency acoustic radiation problem for a spatially-bandlimited surface velocity has been presented. Code development is underway, with expected completion of a mid-frequency analysis capability for spherical surfaces by October 2007, cylindrical and planar surfaces in 2008, and general surfaces in 2009.

REFERENCES

[1] Geers, IL, 1978, “Doubly asymptotic approximations for transient motions of submerged structures”, J. Acoust. Soc. Am. 64, 1500-1508.

[2] X. Zhao, N. Vlahopoulos, “Mid-Frequency Vibration Analysis of Systems Containing one Type of Energy based on a Hybrid Finite Element Formulation,” SAE Paper No. 2001-01-1620, *SAE Journal of Passenger Car – Mechanical Systems*, 2001, Vol. 110, pp. 2119-2134.

[3] X. Zhao, N. Vlahopoulos, “A Hybrid Finite Element Formulation for Mid-Frequency Analysis of Systems with Excitation Applied on Short Members,” *Journal of Sound and Vibration*, Vol. 237(2), October 19, 2000, pp. 181-202.

[4] P. J. Shorter, R. Langley, “Modeling methods for mid-frequency vibro-acoustics”, S/C and L/V Dynamic Environments Workshop, El Segundo, CA, 22-24 June 2004.

[5] J. W. S. Rayleigh, The Theory of Sound, 1st edition, 1877.

[6] Lamb, Horace, Treatise on the Mathematical Theory of the Motion of Fluids, Cambridge University Press, 1879.

[7] McConnell, A.J., Applications of Tensor Analysis, Dover Publications, Inc., New York, 1957.