ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

IDENTIFYING CHAOTIC DYNAMICS IN A FLYING VIBRATORY SYSTEM

Jin-Wei Liang [1], Shy-Leh Chen [2], and Ching-Ming Yen [3]

[1]Associate Professor, Dept. of Mech. Eng., Mingchi University of Technology, Taiwan
[2]Professor, Dept. of Mech. Eng., National Chung-Cheng University, Taiwan
[3]Researcher, System Development Center, Chung-Shan Inst. of Science and Tech., Taiwan,
liangj@mail.mit.edu.tw (email address of lead author or presenter)

Abstract
This paper determines whether chaotic dynamics exists in a flying vibratory system. Acceleration signals were measured at nine different locations or orientations of the flying object during a test flight. Steady-state acceleration data were extracted to reconstruct pseudo phase-space trajectories from which two dynamical indices including the correlation dimension and the maximum Lyapunov exponent are calculated. Although generally the correlation dimension depends on the embedding dimension, it is found that in three out of the nine-channel acceleration signals, the correlation dimension saturates when the embedding dimension reaches a critical value. The phenomenon indicates a possible existence of chaotic motion. The maximum Lyapunov exponents calculated for the same three-channel data are all positive which again implies the possible existence of chaos. To determine whether the experimental time series that demonstrate chaotic characteristics are in fact deterministic (rather than random noise), a sequence of two statistical tests is applied. Based on these tests, the possibility of those three-channel acceleration data being random noise is excluded.

1. INTRODUCTION

Chaotic vibration of a flying system is an important issue since it may jeopardize the structure of the flying object and cause instability subsequently [1]. It can also cause uncomfortable experience for passengers in a passenger airplane and inaccurate targeting for a missile system. The vibration behaviors of a flying system can be studied via either analytical or experimental approach. For some systems, such as large-scale high order ones, the analytical approach is difficult. For some others, such as black box systems or micro/nano scale systems, modeling of the system dynamics is difficult or impossible. In these cases, the experimental approach is preferred. The experimental approach is based on a discrete set of observed quantities called a time series [2-5]. Useful information can be extracted from the time series. In general, an equally sampled time series can be written as $v(y_0), v(\varphi_\tau(y_0)), v(\varphi_{2\tau}(y_0)),......... \equiv v_0, v_1, v_2,......$ Here, $v(\varphi_t(y_0)) = v(y(t))$ denotes the value of an observable state v at time t.

Identification of chaotic dynamics from experimental time series is a nontrivial task because the data could be contaminated by random noise whose properties are very similar to chaotic data. Fortunately, a sequence of two statistical tests was proposed in [6] that can be adopted to diagnose dynamical structures inherent in time series. The first of these tests is known as the BDS test. The test can determine whether a time series, pre-whitened from linear structure, is independently and identically distributed noise (iid, or loosely denoted as random noise). If the series is not iid, the β test (also proposed in [6]) is then applied to compare estimates of correlation dimension values of a series with estimates obtained from the series' own random shuffles. The test is designed to detect whether changes in the intrinsic properties of the series take place as a result of shuffling the data. The β test is found to be able to judge whether a time series behaves like strictly deterministic process with low attractor dimension.

2. DESCRIPTIONS OF THE SYSTEM AND DATA

The missile system under study is shown in Fig. 1. It is composed of 3 main sections, where seeker is located at the front section, guidance panel is at the middle, and the actuator is at the rear section. Nine channels of acceleration signals, denoted as V_1 to V_9, were measured simultaneously at various locations and orientations of the missile during a test flight. The locations of the signals are also indicated in Fig. 1. Approximately, the signals V_1 to V_4 were measured from the front section, while V_5 and V_6 were from the middle, and V_7 to V_9 were from the rear section.

The data were sampled at 10,000 Hz for more than 5 minutes. To avoid transient effects, only those data that have reached a steady-state condition were taken for the analysis. Fig. 2 shows features of the steady-state nine-channel data. Among the nine-channel signals, V_1 to V_9, it was observed that V_2, V_4, V_5 and V_7 were dominated by a considerable amount of spikes. These spikes were very likely caused by external disturbance instead of the actual vibration signals. We also speculate that these signals were taken near nodal surfaces of some flexural vibrations, although more thorough investigations are required to turn these speculations conclusive. Therefore, in what follows, only V_1, V_3, V_6, V_8 and V_9 will be analyzed.

3. PROCEDURES FOR NONLINEAR TIME-SERIES ANALYSIS

To explore whether chaos exists in the obtained experimental data, the phase trajectory needs to be reconstructed. To this aim, the method of delay is applied. The method can be described as follows [7,8]. First, a single physical quantity $x(t)$ is measured. Then, an m dimensional pseudo state vector is formed by taking m measurements with consecutive time delays by τ, i.e., $\xi(t) = \left(x(t), x(t+\tau), ..., x(t+(m-1)\tau)\right)$. Here, m is called the embedding dimension. In practical experiments, $x(t)$ can only be measured at discrete time with a sampling period Δt. Thus, the delay time τ can only be chosen as a multiple of Δt, i.e., $\tau = l\Delta t$. where l is a positive integer. Now, let $x(n) = x(n\Delta t)$ to simplify the notation. A pseudo state vector at $t = n\Delta t$ can be re-expressed as $\xi(n) = (x(n), x(n+l), ..., x(n+(m-1)l))$. The pseudo state vector $\xi(n)$ will form the required pseudo state trajectory. To apply the method of delay, two parameters need to be determined, namely, the delay index l and the embedding dimension m. The delay index l can be determined by the average mutual information, which is a quantity measuring the mutual information in $x(n)$ and $x(n+l)$. As l increases, the mutual information will decrease. The delay index is chosen as the first minimum of the average mutual information. It is conceivable that this l will make $x(n)$ and $x(n+l)$ relatively independent.

The embedding dimension m is determined by the method of false nearest neighbors. Conceptually, when the embedding dimension is insufficient, the state trajectory is compressed into a lower dimensional space. Then, some points which are close to each other in the lower-dimension space may turn out to be far away if a larger embedding dimension is adopted. These points are called false nearest neighbors. Thus, one can gradually increase the embedding dimension while examining the number of FNN. When the number of FNN saturates at a minimum value, the very dimension will be the embedding dimension required to fully unfold the trajectory.

Two dynamical analysis indicators including the correlation dimension and the maximum Lyapunov exponent are employed in this study to detect the possible existence of the chaotic behaviors in the flying vibratory system. The correlation dimension is a measure of the complexity of the reconstructed phase trajectories. The idea is that a point located in a densely populated neighborhood will contribute more to the correlation dimension than those located in a sparsely populated neighborhood. To calculate the correlation dimension, one calculates the distances between pairs of points in the pseudo space, i.e., $s_{ij} = \left\| x_i - x_j \right\|$, using the conventional Euclidean distance. For a data set, $\{x_1, x_2, ..., x_N\}$, with the total number of points equal to *N*, the correlation function is defined as

$$C(r) = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} H\left(r - \left\| x_i - x_j \right\|\right), \tag{1}$$

where *r* is a chosen ball size and H denotes the Heaviside function. For many attractors, the correlation function has been found to exhibit a power law dependence on r as $r \to 0$, namely, $\lim_{r \to 0} C(r) = ar^d$. Thus, the correlation dimension is defined by

$$d = \lim_{r \to 0} \frac{\log C(r)}{\log r} \tag{2}$$

In general, the correlation dimension calculated from the reconstructed phase trajectory depends on the embedding dimension. However, for a chaotic system, the correlation dimension tends to saturate when the embedding dimension is larger than a critical value [8]. The saturation phenomenon can serve as an indicator for detecting the existence of chaotic dynamics.

The Lyapunov exponent is a measure of the sensitive dependence on initial conditions. Two points initially separated by an infinitesimal distance ε along a direction will, on the average, have their separation growth (or shrinkage) as $\varepsilon e^{\lambda t}$. The rate λ is the Lyapunov exponent for the direction. If the embedding dimension is *m*, then there will be *m* independent evolving directions in the phase space, leading to *m* Lyapunov exponents. The Lyapunov exponent can be positive or negative depending on whether the trajectory expands or contracts along one specific direction. The system is chaotic if the maximal Lyapunov exponent is positive while the rest of the exponents are negative.

4. IDENTIFICATION RESULTS

The procedures outlined in the previous section are applied to the five signals V_1, V_3, V_6, V_8, and V_9. Although not shown here, all five signals have more than one natural frequency, and the higher harmonics are not exact integer multiples of the fundamental ones. Therefore, to avoid possible influences of the higher harmonics, the dynamical analysis indices will not be calculated according to the Poincare maps extracted using a simple frequency. Instead, both the dynamical analysis indices are determined using the reconstructed pseudo phase trajectories. The correlation dimensions with different embedding dimensions for the 5 signals are shown in Fig. 3. In Fig. 3, the results of two benchmark signals — random signal (uniform distributed) and Lorenz signal — are also included for comparison purpose. The results reveal that V_1, V_3, and V_6 are likely to be chaotic (because their correlation dimensions saturate when *m* is larger than 8) , whereas V_8 and V_9 are random-like.

Next, the maximal Lyapunov exponent is computed using the algorithms proposed in [9] and from the reconstructed pseudo phase trajectories of the five measured acceleration data. Table 1 shows the averaged correlation dimension (averaged from the correlation dimension corresponding to *m*=8 to 13) and the maximal Lyapunov exponent of the 5 signals. It can be found that the results of correlation dimension are consistent with those of the maximum Lyapunov exponent. They both point out that V_1, V_3 and V_6 are chaotic, whereas V_8 and V_9 are random-like.

5. VERIFICATIONS USING STATISTICAL ANALYSIS

To verify the results obtained using the dynamical analysis tools, a sequence of two statistical tests are adopted. Both statistical tests are based on the correlation function. The two tests can be implemented sequentially. Firstly, the BDS test begins estimating a coefficient *W* which can be defined as follows

$$W(m,r,T) = \frac{B(m,r,T)}{S_m}$$

where $B(m,r,T) = \sqrt{T}\left[C(m,r,T) - C(1,r,T)^m\right]$ and S_m represents the standard deviation of $B(m,r,T)$. The BDS test also associates with the null hypothesis "H_o:Y_t is iid". Based on the null hypothesis, Y_t is iid if H_o is accepted. If Y_t is not iid, the absolute value of $W(m,r,T)$ will be larger than the critical Z value of the standard normal distribution assuming say 5% significance level. If Y_t is iid, no further testing is required. Otherwise, the β test will be applied to determine whether Y_t is generated from deterministic chaos or nonlinear stochastic process.

Table 2 shows the computed *W* values of the BDS test with respect to the random, the Lorenz data, and the five-channel acceleration signals. Every BDS result presented in Table 2 was conducted on 15000 points in the pseudo space with the ball size $r = 1.5\sigma$ where σ is the standard deviation of the tested data. The embedding dimension ranges from 2 to 13 while the critical Z is equal to1.96. It can be observed from Table 2 that all the *W* values of the Lorenz data are much larger than the critical *Z* value. In contrast, all the *W* values associated with the random process are well below 1.96. The results obtained from the five-channel acceleration data are more moderate. However, one can still conclude that all the acceleration signals are not iid or random data, because their *W* values are larger than 1.96. Among the five-channel data, V_1 and V_3 are the most deterministic ones while V_8 is the least deterministic one. V_6 also rejects BDS H_0 although its *W* values are not as large as those of V_1 and V_3.

To determine whether the three-channel acceleration data are deterministic chaos or nonlinear stochastic, the β test is applied next. The β test examines if the correlation dimension of the tested series changes dramatically when its data points are randomly shuffled. Intuitively, the more complex the tested series is, the lesser the effect of shuffling on the estimates of the correlation dimension will be. Here, shuffling means that the new index of the shuffled series is a random selection of the *N* index of the original time series. The β test adopts all estimates of the correlation dimension corresponding to *m*=3 to 11 and for the tested series and its shuffled counterpart. To this end, the following ratio is defined

$$\eta = \frac{\sum_{m=3}^{11} v_s(m)}{\sum_{m=3}^{11} v_o(m)} \tag{3}$$

where v_s and v_o denote the estimates of the correlation dimension of the shuffled (s) and original (o) series, respectively. Theoretically, if the tested data is iid, $\eta = 1$. In contrast, $\eta > 1$ when the process is less complex. For a large population of iid series η is normally distributed with the mean equal to one together with a constant variance. Now, let

$$\beta = \frac{\sum_{i=1}^{n} \eta_i}{n} \tag{4}$$

where $i = 1,2,..........,n$ is the number of shuffles. n is supposed to be larger than (or at least equal to) 30 if the sample mean to be approximately normally distributed based on the central limit theorem. Thus, the null hypothesis of the β test turns out to be H_o : $\beta = 1$. The null hypothesis is judged based on the standard normal distribution V defined as

$$V = \frac{|\beta_i - 1|}{\sigma_\beta / \sqrt{n}} \tag{5}$$

where σ_β (approximately equates to 0.438) is the standard deviation of the iid population of η. H_o will be rejected if $V > Z_\alpha$ (accepted, otherwise) in which Z_α =1.645. Note that in order to be checked by the β test, the tested series has presumably rejected the null hypothesis of the BDS test. To this end, the tested data must be generated from nonlinear stochastic process if it accepts the βH_o.

On the other hand, if the βH_o is rejected the data could only be generated from nonlinear deterministic or coupled nonlinear deterministic process. In such a case, one proceeds to estimate the signal to noise ratio using the following linear-regression model [6]

$$\omega = 1.70 + 0.967V \tag{6}$$

where V can be determined using Eq. (5). Eq.(6) was drawn from experiences obtained in a large amount of experiments conducted on data generated from systems with known ω values. Eq. (6) yields a ω value larger than 20 when the series is generated from a strictly deterministic process. On the other hand, the βH_o will be accepted for series with $\omega \leq 3$. Finally, a series can either be categorized as the result of a strictly deterministic process or a nonlinear stochastic process if βH_o is rejected while $3 \leq \omega < 20$.

To conduct the β test, Eq. (3) was adopted for the calculation of η where the correlation dimension ranges from 3 to 11. For each signal, 3000 data points were selected as the original series and 30 shuffles were performed to compute a single β value. Table 3 collects the results of all the β tests conducted on the benchmark and the acceleration data. Note again that every single β value listed in Table 3 is the average of η based on 30 shuffles.

Also, the βH_o is evaluated by the inequality $V > Z_\alpha$ where $Z_\alpha = 1.645$. In this sense, Table 3 shows that the βH_o will be accepted and rejected by the random and Lorenz data, respectively. Moreover, among the five-channel acceleration data, V_1, V_3, and V_6 simultaneously reject the βH_o because all the V values associated with these data are larger than 1.645. The results imply that these three-channel data could only be generated from the strictly deterministic or coupled nonlinear deterministic process. Although a further check on the signal to noise ratio reveals that all the three-channel data have ω value less than 10, the possibility of these three data being generated from the deterministic chaotic process still prevails. This is because the experimental data generated from high-order deterministic chaotic process has been known for its low signal to noise ratio [9]. Besides, the results obtained from the BDS H_o and βH_o investigations are consistent with those of the dynamical analysis tools.

6. CONCLUSIONS

This work identifies possible existence of chaotic behaviors in a missile system. Nine-channel acceleration signals were measured simultaneously from a missile during a test flight. Steady-state acceleration signals were extracted to reconstruct the pseudo phase-space trajectories from which the correlation dimension and the maximum Lyapunov exponent were computed. Based on the features observed from the computed correlation dimension and the maximum Lyapunov exponent, it is found that three out of nine channels of acceleration data possess chaotic characteristics. The chaotic characteristics include fractal correlation dimension and positive maximal Lyapunov exponents. These results are crosschecked by a sequence of statistical tests, the BDS and the β test. Through the statistical analysis, the three-channel acceleration data demonstrating deterministic chaotic features reject the possibility of being random noise, which further confirms the possibility of these data being generated from deterministic chaotic processes possessing low signal to noise ratio.

REFERENCES

[1]. Etkin, B., *Dynamics of Atmospheric Flight*, John Wiley & Sons, 1972.

[2]. Boltezar, M. and Hammond, J. K. 1999, "Experimental Study Of The Vibrational Behaviour Of A Coupled Non-linear Mechanical System," *Mechanical System and Signal Processing,* 13(3), pp.375-394

[3]. Boltezar, M. , Jaksic, N. , Simonovski, I. and Kuhelj, A. 1999, "Dynamical Behaviour Of The Planar Non-linear Mechanical System -- Part II: Experiment," Journal of Sound and Vibration, 226(5), pp.941-953

[4]. Coca, D. and Billings, S. A. 1999, "A Direct Approach To Identification Of Nonlinear Differential Models From Discrete Data," *Mechanical System and Signal Processing*, 13(5), pp.739-755

[5]. Feeny, B. F., Yuan, C. –M. and Cusumano, J. P. 2001, "Parametric Identification Of An Experimental Magneto-Elastic Oscillator," *Journal of Sound and Vibration*, 247(5), pp.785-806.

[6]. Kaboudan, M. A., 1996, "Diagnosing Time Series Dynamical Structures," *Chaos, Solitons & Fractals*, Vol.7,No.7, pp.977-990.

[7]. Takens, F. 1979-1980, "Detecting Strange Attractors in Turbulence," in *Proceedings of the Symposium on Dynamical Systems and Turbulence*, University of Warwick, edited by D. A. Rand and L. S. Young.

[8]. Abarbanel, Henry D.I. 1996 *Analysis of observed chaotic data.* New York: Springer-Verlag.

[9]. Kantz, H. and Schreiber, T., *Nonlinear Time Series Analysis*, Cambridge Nonlinear Science Series 7, Cambridge University Press, 1997.

Table 1 Correlation dimension and Lyapunov exponent for the 5-channel signals

Signal	Average correlation dimension	Maximal Lyapunov exponent
V_1	3.40	6.58
V_3	3.58	13.6
V_6	5.05	13.7
V_8	N.A.	0
V_9	N.A.	0

Table 2 The computed *W* values of the BDS test.

data / m	random	Lorenz	V_1	V_3	V_6	V_8	V_9
2	1.55	499	543	201	35	24	19
3	1.52	534	602	214	35	17	17
4	1.00	578	698	235	40	2.2	32
5	0.44	641	824	264	45	5.5	39
6	0.12	727	995	303	47	9.1	42
7	0.16	840	1227	354	48	12	46
8	0.30	985	1540	422	49	13	49
9	0.53	1172	1963	508	49	15	52
10	0.53	1411	2533	619	49	16	54
11	0.52	1716	3305	763	49	17	57
12	0.49	2107	4353	948	49	17	60
13	0.58	2610	5779	1188	48	18	61

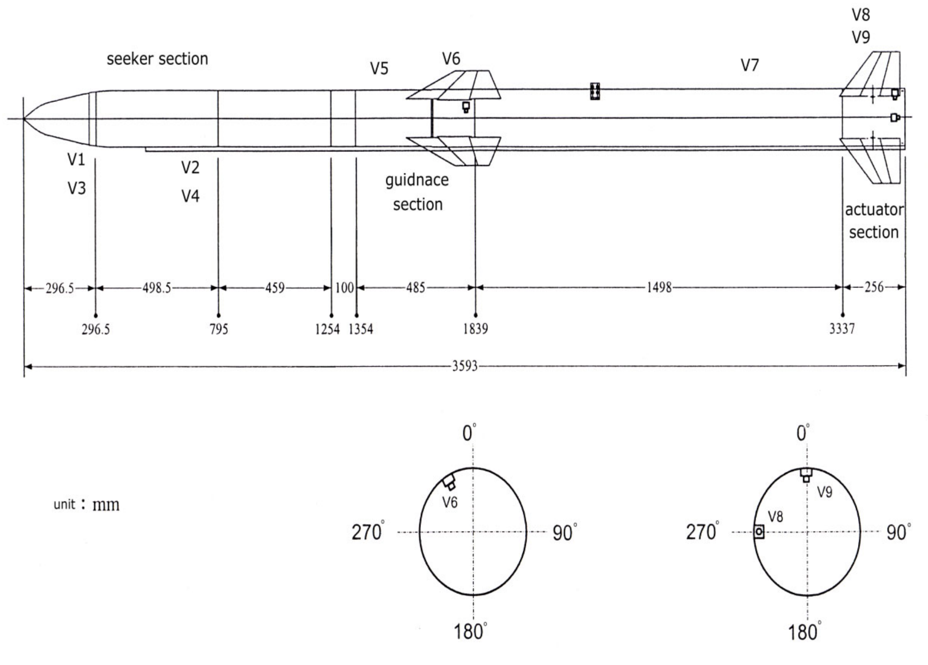

Figure 1 A schematic diagram showing the missile system.

Table 3 The results of the β test based on 3000 data points and 30 shuffles.

	random	Lorenz	V_1	V_3	V_6	V_8	V_9
V	0.12	24.26	1.96	3.75	1.75	0.25	1.50
β	0.990	2.94	1.16	1.30	1.14	1.02	1.12
ω	1.82	25.16	3.60	5.33	3.45	1.94	3.21

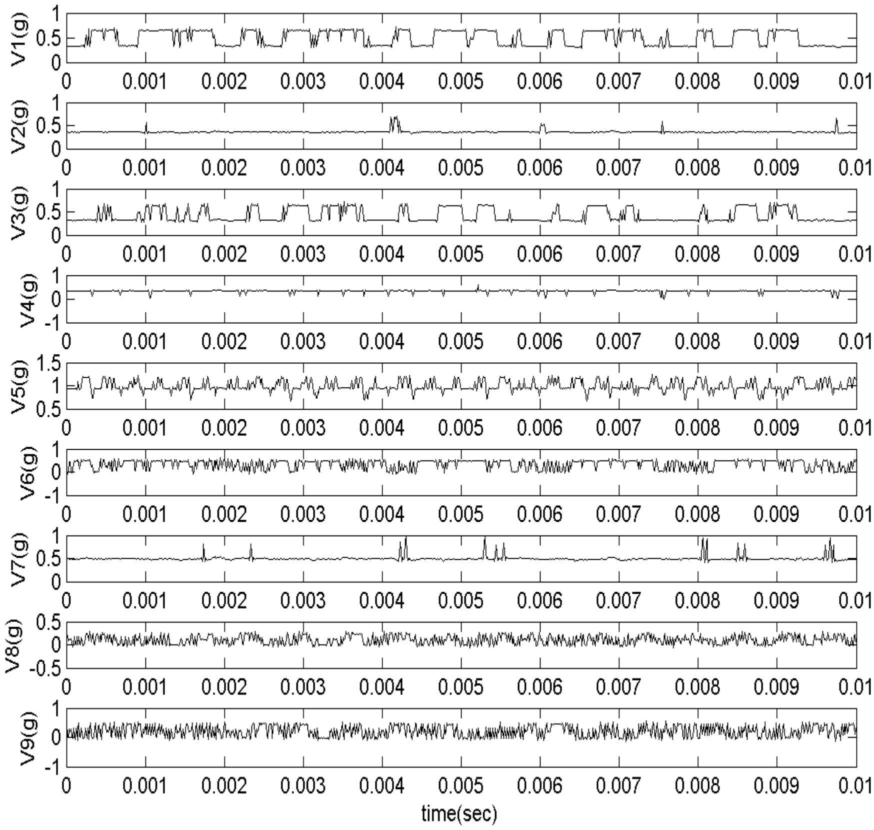

Figure 2 Features of the steady-state nine-channel acceleration signals.

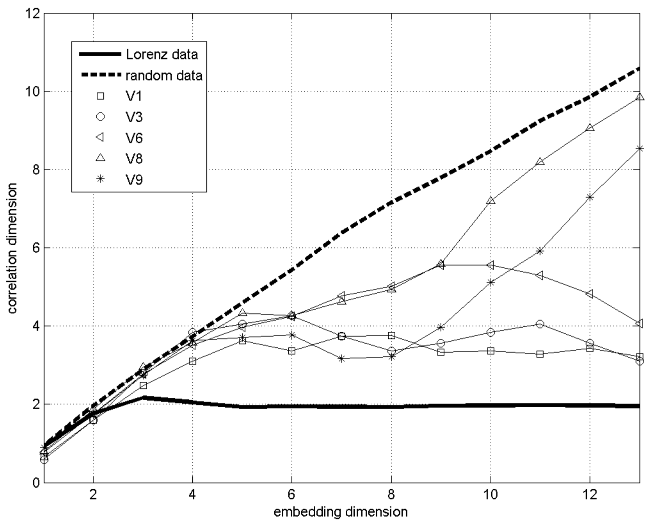

Figure 3 Correlation dimension with different embedding dimensions for various signals