ICSV14

Cairns • Australia

9-12 July, 2007

14th International Congress on Sound & Vibration

ONE-DIMENSIONAL TOPOLOGY OPTIMIZATION FOR TRANSMISSION LOSS MAXIMIZATION OF MULTI-LAYERED ACOUSTICAL FOAMS

Eun Il Kim[1], Joong Seok Lee[1], Yoon Young Kim[1], Jung Soo Kim[2] and Yeon June Kang[2]

[1]National Creative Research Initiatives Multiscale Design Center,
School of Mechanical and Aerospace Engineering, Seoul National University
Shinlim-Dong San 56-1, Kwanak-Gu, Seoul 151-742, Korea
[2]Advanced Automotive Research Center,
School of Mechanical and Aerospace Engineering, Seoul National University
Shinlim-Dong San 56-1, Kwanak-Gu, Seoul 151-742, Korea
idea0221@idealab.snu.ac.kr

Abstract

The topology optimization of one-dimensional multi-layered acoustical foams for transmission loss maximization is presented. The goal of this investigation is to find an optimal sequence and thickness of layers consisting of air gaps and certain poroelastic materials. Multi-layered acoustical foam sequences are optimized to maximize transmission loss for a target frequency or a range of frequencies. Though the one-dimensional multi-layered acoustical foam has been a popular subject in automobile and aircraft applications, the simultaneous design of sequencing and thickness has not been conducted yet. In this respect, the proposed design method can serve as a new optimal design method of multi-layered foams. For the calculation of transmission loss for a specific multi-layered foam appearing during optimization, the differential Biot equations are not solved directly, but an efficient transfer matrix approach is employed. The transfer matrix is a direct relationship between sound pressures and acoustic particle velocities on the incident and transmitted sides of a layer or a sequence of layers. The transfer matrix approach can significantly enhance computational efficiency of the topology optimization. By developing a unified model representing air gaps and poroelastic materials, the whole optimization process is performed with an efficient gradient-based optimizer. In the proposed model, the air layer is treated as a poroelastic material having limiting material properties. Several numerical case studies confirmed the effectiveness of the proposed design method for finding optimal multi-layer sequences.
Keywords: Multi-layered acoustical foams, Transmission loss maximization, Topology Optimization

1. INTRODUCTION

One-dimensional multi-layered acoustic foams have been used to attenuate sound noise in automobiles, aircrafts and others. The performance of multi-layered acoustic foams is affected by the amount of poroelastic layers and acoustic air layers. When the total thickness of the multi-layered foam is set, it is difficult to determine the optimal layer distribution maximizing transmission loss. In this work, we propose a new design method of one-dimensional multi-layered acoustic foams using topology optimization. This method can give an optimal layer sequence in multi-layered acoustic foams.

The proposed topology optimization design method for multi-layered acoustic foams, based on Biot's theory [1], introduces a new modelling technique to treat two different layers (poroelastic material layers and acoustic air layers) only with a single set of governing equations. In this approach, an acoustic air layer is treated as a limiting poroelastic layer. A transfer matrix method [2, 3] is employed for efficient analysis of one-dimensional multi-layered acoustic foams.

2. MATHEMATICAL DESCRIPTIONS

Biot's theory is used to analyze a poroelastic material that contains solid and fluid states. This theory formulates the propagation of elastic waves in a fluid-saturated porous material. The governing equations of poroelastic acoustic foams can be written as follows [1]:

$$\begin{aligned} N\nabla^2\mathbf{u} + grad\left[(A+N)e + Q\varepsilon\right] &= \frac{\partial^2}{\partial t^2}\left(\rho_{11}\mathbf{u} + \rho_{12}\mathbf{U}\right) + b\frac{\partial}{\partial t}\left(\mathbf{u}-\mathbf{U}\right) \\ grad\left[Qe + R\varepsilon\right] &= \frac{\partial^2}{\partial t^2}\left(\rho_{12}\mathbf{u} + \rho_{22}\mathbf{U}\right) - b\frac{\partial}{\partial t}\left(\mathbf{u}-\mathbf{U}\right) \end{aligned} \tag{1}$$

where $\mathbf{u}$ is the vector-field solid displacement and $\mathbf{U}$ is the vector-field fluid displacement in a poroelastic material. The symbol $e = \nabla\cdot\mathbf{u}$ denotes the volumetric strain of solid phase and $\varepsilon = \nabla\cdot\mathbf{U}$, the volumetric strain of fluid phase. The elastic shear modulus is denoted by N and the first Lamé constant, by A. The symbols Q and R express coupling coefficients between the volume change of the solid phase of a poroelastic material and that of interstitial fluid. The coefficients ρ_{11}, ρ_{12} and ρ_{22} are mass coefficients accounting for the effects of non-uniform relative fluid flow through pores in a poroelastic material. The parameter b represents the effect of viscous coupling [1].

The transfer matrix, which is expressed as a 2 × 2 matrix, represents the relationship between pressures and particle velocities of the input and output sides of a layer. Symbolically, the transfer matrix of a multi-layered acoustic foam may be written as [5]

$$\begin{Bmatrix} p \\ v_x \end{Bmatrix}_i = [T]_{total}\begin{Bmatrix} p \\ v_x \end{Bmatrix}_t \tag{2-a}$$

$$[T]_{total} = \left[T_f\right]_1\left[T_f\right]_2\cdots\left[T_f\right]_n \tag{2-b}$$

where variables p, v_x denote pressure and horizontal particle velocity. The subscripts i and t denote a wave-incident surface and a wave-transmitted surface, respectively. The total transfer matrix $[T]_{total}$ is calculated by simple multiplication of transfer matrices of single layers as in Eq. (2-b). One can obtain the transmission coefficient of a multi-layered acoustic foam from

$[T]_{total}$. More detailed descriptions about the transfer matrix method for multi-layered acoustic foams can be found in Refs [2-4].

3. TOPOLOGY OPTIMIZATION FORMULATION

Poroelastic materials are characterized by several parameters [2, 3]. If material interpolation is appropriate for topology optimization, the value of design variable χ_e can vary the transmission loss monotonically as shown in Fig.1. The state of $\chi_e = 1$ corresponds to poroelastic material while the state $\chi_e = 0$, to acoustic air. More detailed topology optimization formulation for optimal multi-layered foam design can be found in [4].

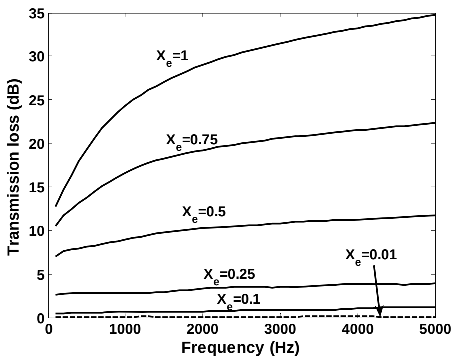


Figure 1. Transmission loss of a poroelastic material layer for various values of the design variable χ_e .

With the material interpolation scheme yielding the results shown in Fig. 1, we can formulate a topology optimization problem to find an optimal layer sequence of one-dimensional multi-layered acoustical foams to maximize sound transmission loss. The sound transmission loss (TL) is defined as $\mathrm{TL} = 10\log 1/|T|^2$, where T is the sound transmission coefficient . The topology optimization for optimal multi-layered acoustical foams for some single frequencies or a range of frequencies can be stated as

$$\min_{\chi_e} F(\chi_e; f_l, f_u) \;=\; \min_{\chi_e} \sum_{f=f_l}^{f_u} \left|T(\chi_e; f)\right| \tag{3-a}$$

subject to a mass constraint

$$\sum_{e=1}^{N_e} \chi_e \le V_0 \tag{3-b}$$

In Eq. (3-a), F denotes an objective function expressed in design variables χ_e $(1 \le e \le N_e;\ \ N_e$: number of discretized layers). The design variable χ_e that is assigned to each layer of the discreized foam can vary from 0 to 1. When a range of frequencies between the lower frequency f_l and the upper frequency f_u is considered, Transmission loss $T(\chi_e; f)$ at some selected frequencies within the frequency band is used as the objective function F . Equation (3-b) implies that the maximum volume usage of a poroelastic material is limited by V_0 .

4. NUMERICAL EXAMPLES

We applied the proposed topology optimization design method to find optimal one-dimensional multi-layer sequences. As shown in Fig. 2, a one-dimensional design domain may be expressed as $[0,\ 8\text{cm}] \times [-\infty,\ \infty]$. In this work, incident plane waves are assumed to be normal to the left surface of a design domain, i.e., the incident angle is $\theta = 0$. The design domain is discretized by 80 layers, each of which must be either a given poroelastic material or acoustic air. The total poroelastic volume is constrained not to excess 70% of the volume of the whole design domain.

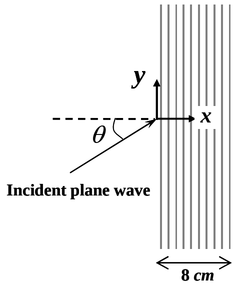


Figure 2. Design domain for topology optimization.

The optimized one-dimensional multi-layered acoustic foams at several interesting frequencies and ranges of frequency are shown in Figure 3. Figure 3 shows that the optimized foam layouts are formed by multiple layers of a poroelastic material. Also note that acoustic air gaps inserted between poroelastic layers under a limited mass usage constraint help improve the ability of the multi-layered acoustic foams; see Figure 4. As frequencies of interest increase, the number of poroelastic layers increase, but the length of each layer decreases.

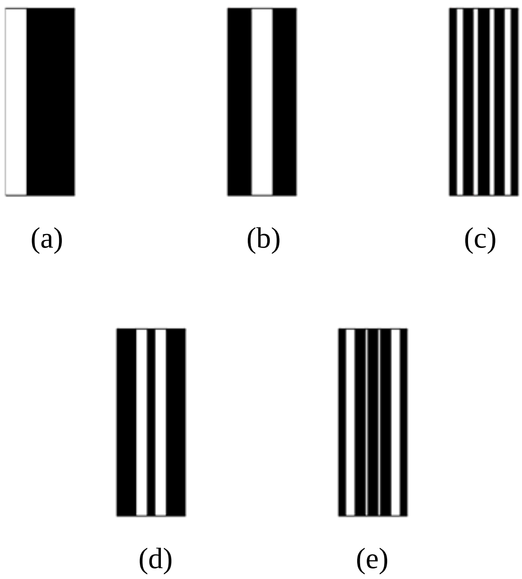
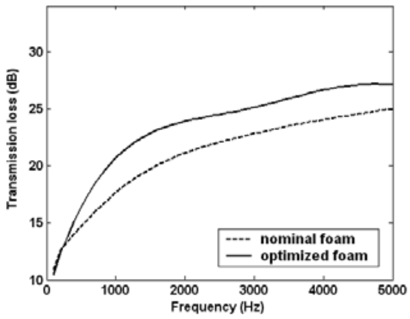
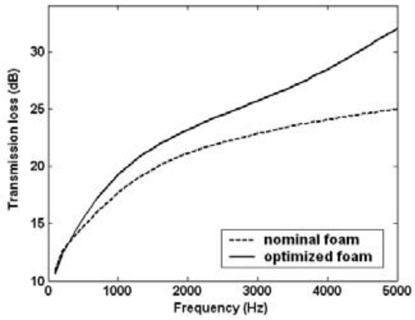


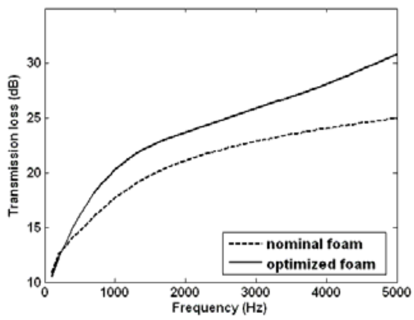
Figure 3. Comparison of the topology optimized multi-layered acoustic foams with a nominal one-layered foam. (a) A nominal layer filled with poroelastic material filling 70% of the design domain, (b) optimized multi-layer sequence at 1.0 kHz, (c) optimized multi-layer sequence at 5.0 kHz, (d) optimized multi-layer sequence for a frequency range from 1.0 kHz to 2.0 kHz, (e) optimized multi-layer sequence for a frequency range from 4.0 kHz to 5.0 kHz



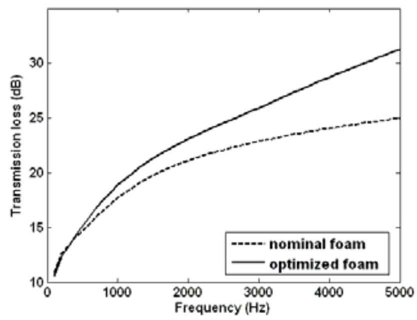
(a)



(b)



(c)



(d)

Figure 4. Transmission loss by the proposed topology optimization design method for (a) 1.0 kHz, (b) 5.0 kHz, (c) a frequency range from 1.0 kHz to 2.0 kHz and (d) a frequency range from 4.0 kHz to 5.0 kHz

5. CONCLUSIONS

We proposed a new method to design one-dimensional multi-layered acoustic foams for maximizing transmission loss. A transfer matrix method based on the Biot's theory was used for acoustic analysis and a topology optimization formulation was set up for optimal design. Biot's theory with variable material properties was used to express both air layers and poroelastic layers. The proposed topology optimization method successfully found optimal layer sequences at several single frequencies and for ranges of frequencies.

ACKNOWLEDGMENTS

This research was supported by the National Creative Research Initiatives Program (Korea Science and Technology Foundation grant No. 2006-033) contracted through the Institute of Advanced Machinery and Design at Seoul National University.

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