

## DESIGN OF COMMAND SHAPER USING GAIN-DELAY UNITS AND PARTICLE SWARM OPTIMISATION FOR VIBRATION CONTROL OF A FLEXIBLE SYSTEM

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## Abstract

Conventional command shaping method involves convolving a desired command with a sequence of impulses that may prove computationally expensive or unsuitable for a fast system. Moreover, a priori knowledge of the system parameters, such as resonance frequencies and associated damping ratios, is required to design the exact sequence of impulses to produce a command that results in zero residual vibration. A new command shaping method is proposed using gain and delay units to shape the reference input to reduce vibration of a flexible system. Assuming that, no prior information is available about the system, a new variant of particle swarm optimisation (PSO) algorithm is proposed and used to derive the gain values and the amount of delay. The effect of total number of delay and gain units is also analysed. A twin rotor system is used as the flexible system where the control strategy is applied and the results show the effectiveness of the proposed control strategy.

# **1. INTRODUCTION**

Flexible systems are lighter, faster and less expensive than rigid ones but they pose various challenges as compared to their rigid counterparts, ranging from system design, vibration control, structural optimisation etc. In order to achieve high-speed and accurate positioning, it is necessary to control the system's vibratory response in a cost effective manner. A good literature review of different control strategies for flexible systems can be found in [1]. A feedforward control scheme based on input command shaping, introduced by Singer and Seering [2], has been applied to the control of different types of flexible systems for vibration reduction or trajectory tracking [3],[4]. The conventional command shaping method involves convolving a desired command with a sequence of impulses and the amplitudes and time locations of the impulses are calculated based on the natural frequencies and damping ratios of the system [2]. In the authors' previous work [5],[6],[7], assuming no prior information was available on the system, genetic algorithms and particle swarm optimisation (PSO) algorithms [8] were employed to derive amplitude and time locations of impulses which, later on, were convolved with the reference input to form the shaped signal.

This paper presents a new command shaping method using gain and delay units where a new variant of PSO algorithm is developed and used to optimise the gain values and the amount of delay in order to reduce vibration of the system. The effect of total number of delay and gain units is also analysed. The control strategy is applied to and tested with a twin rotor multi-input multi-output system (TRMS) [9].

# 2. EXPERIMENTAL SET-UP

The TRMS is a scaled and simplified version of a practical helicopter and often used as a laboratory platform for control experiments [9]. The experimental set-up and its schematic diagram are shown in Figures 1 and 2 respectively. It consists of a beam pivoted on its base in such a way that it can rotate freely in both the horizontal and vertical planes producing two rotating movements around yaw and pitch axes, respectively. At both ends of a beam, pivoting on its base, there are two propellers driven by DC-motors. The controls of the system are the motors' supply voltages. The geometrical shapes of the propellers are not symmetric. Rotation of a propeller produces an angular momentum which is compensated by the remaining body of the TRMS beam. This results in interaction between the moment of inertia of the motors with propellers. This interaction directly influences the velocities of the beam in both planes. The measured signals are: position of the beam, that is, two position angles, and angular velocities of the rotors. The system is interfaced with a personal computer through a data acquisition board, PCL-812PG. When the rotors move, due to aerodynamic force, the rig structure undergoes deflection in the horizontal or vertical or both directions. The flexible motion due to asymmetrical mass distribution of the TRMS system causes structural vibration while in operation. As far as vibration control is concerned, the vertical channel poses more challenges compared to the horizontal channel due to higher physical diameter of the main rotor and higher aerodynamic force. Considering the importance and interesting applications, the vertical channel of the TRMS is explored in this paper. A 4<sup>th</sup> order continuous transfer function characterizing the vertical movement of the TRMS is extracted and utilized in this work. This is given as:



Figure 1: The TRMS system



Figure 2: Schematic diagram of TRMS

Pitch movement, 
$$H(s) = \frac{y(s)}{u(s)} = \frac{-0.08927s^3 + 2.249s^2 - 45.57s + 595.1}{s^4 + 3.469s^3 + 519.6s^2 + 35.95s + 2189}$$
 (1)

where u(s) represents the main rotor input (volt) and y(s) represents pitch angle (radians).

## **3. THE PROPOSED COMMAND SHAPING TECHNIQUE**

A new command shaping method is proposed, as shown in Figure 3, using gain and delay elements to shape a reference input. The unshaped reference signal is passed through multiple delay units,  $\Delta$ , and then multiplied with gain factors, K. The shaped command is formed by summing up the delayed components. For simplicity, the number of delay units and gain elements are kept the same, say n. In order to achieve the same system response level with the shaped command as with the unshaped reference, the gain values are selected in such a way to give 1 when they are added together [2], that is,

$$\sum_{i=1}^{n} K_i = 1 \tag{2}$$

In order to minimize the delay in system's response, the first delay unit is set to zero, i.e.,  $\Delta_1 = 0$  [2]. The remaining delay units,  $\Delta_2, \Delta_3, ... \Delta_n$  and gain values,  $K_1, K_2, ..., K_n$  may be derived analytically, as in the conventional command shaper. Assuming that, no prior information is available about the natural frequencies and associated damping ratios, PSO is applied to optimise the gain values and the amount of delay in order to reduce vibration of the system. The main aim of the optimisation process is to reduce vibration at vertical channel while the TRMS is in operation. Thus, the desired response of the system, d(t), is set to zero in order to achieve zero vibration while the system is in operation. So the system response, y(t), is in considered as the error signal, e(t), which in turn is used to formulate the objective function in the PSO algorithm.



Figure 3: Proposed PSO-based command shaping scheme using gain and delay elements

# 4. PARTICLE SWARM OPTIMISATION ALGORITHMS

PSO is a population-based search algorithm and is initialised with a population of random solutions, called particles and particles fly through the search space with velocities which are dynamically adjusted according to their historical behaviours. The original PSO algorithm is described as [8],[10]:

$$v_{id} = v_{id} + c_1 \times rand(\bullet) \times (p_{id} - x_{id}) + c_2 \times Rand(\bullet) \times (p_{gd} - x_{id})$$
(3)

$$x_{id} = x_{id} + v_{id} \tag{4}$$

where  $c_1$  and  $c_2$  are positive constants, and  $rand(\bullet)$  and  $Rand(\bullet)$  are two random functions in the range [0,1];  $X_i = (x_{i1}, x_{i2}, ..., x_{id})$  represents the *i*-th particle;  $P_i = (p_{i1}, p_{i2}, ..., p_{id})$ represents the best previous position (the position giving the best fitness value) of the *i*-th particle; the symbol g represents the index of the best particle among all the particles in the population;  $V_i = (v_{i1}, v_{i2}, ..., v_{id})$  represents the rate of the position change (velocity) for particle *i*. The fitness of each particle is then evaluated according to a user defined objective function. At each generation, the velocity of each particle is calculated according to equation (3) and the position for the next function evaluation is updated according to equation (4). Each time if a particle finds a better position than the previously found best position; its location is stored in the memory. The first new parameter added into the original PSO algorithm is the inertia weight,  $\omega$ , to balance between the global and local search abilities [10]. The introduction of the inertia weight also eliminates the requirement of carefully setting the maximum velocity  $V_{max}$ . Equation (3) is modified as:

$$v_{id} = \omega \times v_{id} + c_1 \times rand(\bullet) \times (p_{id} - x_{id}) + c_2 \times Rand(\bullet) \times (p_{gd} - x_{id})$$
(5)

## 4.1. Proposed variant of PSO

The global version (gbest) of PSO is relatively simpler and faster than the local best (lbest) model but the particles may lose diversity after a certain number of generations [10] and the search process may get trapped at local minima. In order to maintain diversity in the swarm (population), a fitness sharing based replacement strategy is introduced within the gbest model. The algorithm works as a conventional gbest version of PSO with time varying inertia coefficient,  $\omega$ , and constant acceleration coefficients  $c_1$  and  $c_2$ . After a certain number of generations, shared fitness of each solution is calculated. In fitness sharing technique [11], particles in the crowded region reduce fitness values of one another and thus shared fitness value reduce significantly depending on the value of niche radius,  $\sigma_s$  [11]. A certain percentage of the total population, say 25%, residing in the most crowded region are identified based on the lower shared fitness value. Then these particles are removed and the same number of new particles is introduced into the swarm. At the same time, velocities of the newly introduced particles are initialised and corresponding pbest values are reinitialised. This process is repeated after every predefined number (say, 10) of generations. It is important to note that, the global best solution, pgbest, is always preserved and passed to the next generation for further computations involving that term. The inertia factor  $\omega$  is varied from higher to lower value (0.9 to 0.4) as the algorithm progresses. The acceleration coefficients,  $c_1$  and  $c_2$  are assumed to be constants at a value of 1.5.

# **5. IMPLEMENTATION**

The control strategy was implemented in the Simulink [12] environment as shown in Figure 4. The PSO process was encoded in Matlab .m files [12]. An interfacing was made so that the gain and delay values were calculated in PSO processes and passed to the Simulink environment and after completion of simulation, system response was recorded and again

passed to the PSO process for further computation and the process was repeated based on the initial population and total generation of the optimisation process. In this example, the number of gain elements, n was selected as 3 to keep resemblance with the number of impulses with ZVD [13] type command shapers. For n = 3 the number of delay units is effectively n - 1 (=2), since the first delay unit is set to zero ( $\Delta_1 = 0$ ).

The PSO algorithm begins with a population of real numbers called swarm. The swarm has a dimension of  $(2n-1) \times N$ ; where N is the number of individuals. Each row represents a solution set called particle. A swarm of ten particles (N = 10) with five elements each, i.e.,  $10 \times 5$  was created randomly within the range of [0, +1]. The first three elements of each individual were normalized and assigned to  $K_1, K_2$  and  $K_3$ . In Matlab/Similink [12], the delay units are usually represented in terms of number of samples, which are integer numbers. So the remaining two elements of each individual are converted into integer numbers by a conversion factor of 0.01 followed by rounding operation and then assigned to  $\Delta_2$  and  $\Delta_3$ . Once all gain and delay values are calculated and passed to the model, the shaped command is formed and applied to the system (for, open loop control, see Figure 4). The error signal is calculated as: e(t) = d(t) - y(t); where d(t), is the desired response and y(t) is the system response. Taking this error signal, the objective function, f(x) is formed and the PSO process was run for a maximum generation of 200 to minimise the objective function, f(x).



Figure 4: Simulink model of command shaping using gain and delay units

### 5.1. Selection of objective function and niche radius

For any evolutionary based design procedure, the search capability of the algorithm is directly affected by the objective function. Commonly used objective functions are sum of absolute error (SAE), sum of squared error (SSE), mean squared error (MSE), root mean squared error (RMSE) and time weighted sum of absolute error (TSAE). For fitness sharing based technique, the value of niche radius,  $\sigma_s$  is crucial along with the objective function in selecting which particles would be replaced. The algorithm was run with different values of  $\sigma_s$  and with different objective functions, and time domain performance measures of the system response thus obtained with shaped commands are presented in Table 1. The output responses of the vertical channel due to shaped commands obtained with different objective functions but with a fixed  $\sigma_s$  (=0.1) are shown in Figure 5. It is observed that the same objective function with different values of  $\sigma_s$  gives different outputs. At the same time, the

algorithm with the same  $\sigma_s$  but different objective functions gives different outputs. It is noted that, the output response due to shaped command obtained with SAE as the objective function seemed to be better as far as overall performance measures are concerned.

In order to investigate the effect of number of gain and delay units on vibration reduction of the flexible system, the design procedure was repeated with different number of gain and delay units. To emulate conventional ZV-based and EI-based command shapers [13], two more command shapers were designed where the numbers of gain elements were chosen as 2 and 4 respectively. Moreover, another command shaper was designed where the number of gain units was arbitrarily chosen as 10. The PSO algorithm was applied with objective function, SAE, to find optimal solutions for these three command shapers. The responses of the vertical channel due to shaped signals obtained with the command shapers are shown in Figure 6.

Objective function	Niche radius $(\sigma_s)$	Overshoot (%)	Rise time (sec)	Settling time (sec)	Steady-state error
SAE	0.1	0.0007	0.6	0.9	0
	0.5	0.0006	1.2	1.5	0
	0.9	0.0203	1.5	1.9	0
MSE	0.1	6.0779	1.6	29.1	0
	0.5	3.45	1.5	12.9	0
	0.9	12.303	2.5	42.3	0
RMSE	0.1	3.476	1.5	13.6	0
	0.5	2.808	1.7	11.6	0
	0.9	3.654	1.6	16.1	0
Weighted sum	0.1	1.524	0.5	0.8	0
	0.5	1.5608	0.4	0.7	0
	0.9	1.5604	0.5	0.8	0

Table 1: Performance measures of output response



Figure 5: Response of vertical channel (leading edge only) with shaped commands for different objective functions ( $\sigma_s = 0.1$ )



Figure 6: Leading edges of output response with command shapers having different numbers of gain and delay units.

It is noted that the system's response due to shaped signal obtained with command shaper having minimum number of gain and delay units (gain unit =2 and delay unit =1) has recorded the fastest response at the cost of overshoot, whereas for higher number of gain and

delay units the system suffers long delay with initial oscillations. The system response due to shaped signal obtained with command shaper having moderate number of gain and delay units (gain units =3 and delay units =2), shows satisfactory results.

### 5.2. Solution and shaped command

After 200 generations, the gain values and delay units obtained with objective function SAE were:  $K_1 = 0.4015$ ;  $K_2 = 0.2785$ ;  $K_3 = 0.32$ ,  $\Delta_2 = 50$  and  $\Delta_3 = 90$  ( $\Delta_1$  is set to zero). Here the delay units are represented in terms of number of samples. Both the bang-bang input and its corresponding shaped signal due to the above gain and delay values are shown in Figure 7. The time domain responses of the vertical channel due to bang-bang input and shaped input are shown in Figure 8. It is noted that oscillation of the system was completely eliminated with shaped command and the system settled quickly to the steady state. The frequency domain representations of the bang-bang input and the shaped command are shown in Figure 9 and the corresponding system responses are shown in Figure 10. It is observed from the system's response due to bang-bang signal that the system has only one dominant mode (peak in the frequency domain representation) which appears at 0.7Hz. For bang-bang input, the total energy seems to be evenly distributed throughout the frequency band although it is higher near the dc level. On the contrary, several troughs occur in the frequency domain representation of the shaped command indicating a decrease in energy level at those frequencies. Most importantly, the first trough occurs exactly at 0.7Hz where the main resonance mode of the system lies. As a result, the shaped command, when applied to the system, reduces input energy to the system at the dominant mode to a large extent which in turn reduces system vibration significantly. At the dominant mode (0.7Hz) of the system, 31.32dB attenuation was recorded with shaped command as the input relative to that with bang-bang input.



Figure 7: Bang-bang input and shaped command (time domain)



Figure 8: Response of vertical channel due to bang-bang signal and shaped command (time domain)

# **6. CONCLUSION**

A new command shaping technique based on gain and delay units has been presented vibration reduction in flexible structures. Assuming that no prior information is available about the system, a new variant of PSO algorithm has been proposed and used to optimise the values of gain and delay units of the command shaper. A significant amount of vibration reduction has been achieved with satisfactory level of time domain performance measures

such as, overshoot, rise time, settling time and steady-state error in the system response. The results thus obtained have clearly shown the effectiveness of the proposed control strategy and the algorithm in vibration control of flexible structures. The control strategy may be extended to complex multi-input multi-output (MIMO) systems due to its simplicity in the design procedure, ease of implementation and overall performance. Since PSO is very fast, efficient and requires low computational recourses, the proposed control strategy may work in adaptive real-time systems and work is underway in this direction.



Figure 9: Bang-bang input and shaped command (frequency domain)



Figure 10: Response of vertical channel due to bang-bang signal and shaped command (frequency domain)

### REFERENCES

- [1] M. Benosman and L. Vey, "Control of flexible manipulators: A survey", Robotica, 22, 535-545 (2004).
- [2] N.C. Singer and W.P. Seering, "Preshaping command inputs to reduce system vibration", *Trans. ASME, J. Dynamic Systems, Measurement and Control*, **112**(1), 76–82 (1990).
- [3] S.M. Ahmad, A.J. Chipperfield and M.O. Tokhi, "Dynamic modelling and open-loop control of a twodegree-of-freedom twin-rotor multi-input multi-output system", *Proceedings of the IMECHE-Part I: Journal of Systems & Control Engineering*, **218**(I6), 451-463 (2004).
- [4] M.Z.M. Zain, M.O. Tokhi and Z. Mohamed, "Hybrid learning control schemes with input shaping of a flexible manipulator system", *Mechatronics*, **16**, 209-219 (2006).
- [5] M.O. Tokhi, M.S. Alam, M.Z.M. Zain and F.M. Aldebrez, "Adaptive command shaping using genetic algorithms for vibration control of a single link flexible manipulator", *Proceedings of the 12th International Congress on Sound and Vibration (ICSV12)*, 11-14 July 2005, Lisbon, Portugal.
- [6] M.S. Alam, M.O. Tokhi, M.N.H. Siddique, and M.A. Hossain, "Selection and Designing of command shaper using multi-objective genetic algorithms for vibration control of a single-link flexible manipulator", *Proceedings of the 9<sup>th</sup> International Conference on Climbing and Walking Robots and the Support* technologies for Mobile Machines, 11-14 September 2006, Brussels, Belgium.
- [7] M.S. Alam, M.H. Shaheed and M.O. Tokhi, "Modelling and vibration control of a twin rotor system: a particle swarm optimisation approach", *Proceedings of the 13th International Congress on Sound and Vibration (ICSV13)*, 2-6 July 2006, Vienna, Austria.
- [8] J. Kennedy and R. Eberhart, "Particle swarm optimization", *Proceedings of IEEE International Conference on Neural Networks (ICNN), IV*, Perth, Australia, 1995, pp. 1942-1948.
- [9] Feedback Instruments Ltd. "Twin Rotor MIMO System Manual 33-007-0", Sussex, UK, 1996.
- [10] J. Kennedy and R. Eberhart, Swarm Intelligence, Morgan Kaufmann Publishers, 2001.
- [11] K. Deb, Multi-objective optimization using evolutionary algorithms, New York; Chichester: Wiley, 2001.
- [12] MATLAB, SIMULINK Reference Guide, The Math Works, Inc., 2006.
- [13] T. Singh and W. Singhose, "Tutorial on input shaping/time delay control of maneuvering flexible structures", *Proceedings of 2002 American control conference*, Omnipress, Madison, 2002, pp. 1717– 1731.