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OPTIMUM DESIGN OF VIBRATION ABSORBER WITH VARIABLE POSITION FOR AN EULER-BERNOULLI BEAM UNDER MOVING POINT EXCITATION

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Abstract

Vibration absorbers have many applications in reducing the undesirable vibration of a system. Consider an Euler-Bernoulli beam with arbitrary supports which is under a harmonic point excitation that can be set in different positions. The effect of an absorber on reducing the vibration of this beam is studied in this paper. Also, to achieve a more accurate analysis, the effect of the spring mass on the dynamic equations of the vibrating system is considered .The optimum specifications such as spring stiffness, absorber mass and its position are determined by developing and using several algorithms under MATLAB environment. Finally, the equivalent analog circuit of the problem is simulated using the SIMULINK Toolbox of MATLAB. The advantage of this simulation is that one can find the optimum specifications such as step, ramp, etc.

1. INTRODUCTION

Dynamic vibration absorbers are used to reduce the undesirable vibrations in many applications such as electrical transmission lines, helicopters, gas turbines and engines, bridges, etc. One type of absorber is tuneable vibration absorber (TVA) which can act as a semi-active controller. Young [1] was the first one who considered the application of absorbers on beams. Snowdon [2] discussed the optimization of a discrete absorber on beams with various boundary conditions. Jacquot [3] developed a method that can find the optimal parameters of an absorber to eliminate the excessive vibration of an Euler- Bernoulli beam under sinusoidal excitation. Ozguven and Candir [4] used two dynamic absorbers for a structurally damped beam to suppress two first resonances of beams. In several similar studies, the optimal specifications of the TVA are adjusted under variable conditions [5-8]. Recently, El-Khatib et al. [9] and Brennan [10] and Clark [11] studied the application of a TVA to control the flexural waves on a beam. Also, several vibration absorbers with variable stiffness have been designed, e.g., Franchek et al. [12] changed the effective number of coils in a helical spring. Using the same vibration absorber, Buhr et al. [13] studied the noncollocated adaptive-passive vibration control. Lio et al. [14] compared two auto-tuning methods for a variable stiffness absorber and Nagaya et al. [15] developed a vibration absorber for a cantilever beam with a mass at its free end. In this paper, an Euler-Bernoulli beam with arbitrary supports under a point harmonic excitation with specific position is

considered. Using an algorithm based upon mode summation method, the optimum specifications of an absorber such as spring stiffness, absorber mass and its position can be determined such that the vibration of the beam is minimized. Also, the effect of the spring mass of the absorber is considered in this study. This effect may not be ignored specially in the problems in which the mass of the absorber spring is considerable in compare the mass of the beam. By changing the length of the spring, its mass and its stiffness can be changed. To formulate the problem, mode summation method is used which can be an effective way in the analysis of vibration of continuous systems.

2. FORMULATION OF THE PROBLEM

Structures made of several beams are common in engineering fields. Each beam constitutes of an infinite number of degrees of freedom, and the mode summation method makes their analysis possible as system of a finite number of degrees of freedom. Constraints are often found as additional supports of the structure, and they alter the normal modes of system. For forced vibration of an one-dimensional structure, a force per unit length f(x,t) or moment per unit length M(x,t) at any location of x is considered. If normal modes of the structure $\phi_i(x)$ are known, its deflection at any point x can be presented by:

$$y(x,t) = \sum_{i} q_i(t)\phi_i(x)$$
(1)

Where the generalized coordinate $q_i(t)$ must satisfy following equation:

$$\ddot{q}_{i}(t) + \omega_{i}^{2} q_{i}(t) = \frac{1}{M_{i}} \left[\int f(x,t)\phi_{i}(x)dx + \int M(x,t)\phi_{i}'(x)dx \right] \text{ in which } M_{i} = \int \rho \phi_{i}^{2}(x)dx$$
(2)

In Eq.2, ρ is mass per unit length of beam. As an example, for a beam with simple support at both ends, the eigenvector at any mode is given as $\phi_i(x) = \sqrt{2} \sin(i\pi x/l)$ and M_i will be equal to mass of beam M. Natural frequencies of the beam are defined as $\omega_n = \beta_n^2 (EI/\rho)^{1/2}$, where EI is flexural rigidity and β_n are coefficients for the first five natural frequencies of this beam are: $\beta_1 l = 3.14$, $\beta_2 l = 6.28$, $\beta_3 l = 9.42$, $\beta_4 l = 12.56$, $\beta_5 l = 15.7$ in which l is the length of beam. If instead of distributed load f(x,t) or moment M(x,t), at some point x = a, the concentrated force F(a,t) and moment M(a,t) exist, Eq.2 can be written as:

$$\ddot{q}_{i}(t) + \omega_{i}^{2} q_{i}(t) = \frac{1}{M_{i}} \left[F(a,t)\phi_{i}(a) + M(a,t)\phi_{i}'(a) \right]$$
(3)

2.1 Absorber Made of Mass and Spring

A two ends simply supported beam shown in Fig.1. The absorber and the exciting force are placed at positions x = a and x = b from the left support, respectively. Deflection of the beam at absorber location is y(a,t) and the displacement of the mass of the absorber is u, therefore Eq.3 becomes:

$$\ddot{q}_{i}(t) + \omega_{i}^{2} q_{i}(t) = \frac{1}{M_{i}} \left[-k[y(a,t) - u]\phi_{i}(a) + F_{0}\phi_{i}(b) \right]$$
(4)



Figure 1. Vibration system under harmonic point excitation.

Normal modes, displacement of the absorber and the exciting force can be written in exponential form as $q_i = \overline{q}_i e^{i\omega t}$, $u = \overline{u} e^{i\omega t}$, $F = \overline{F} e^{i\omega t}$. Using these exponential forms and substituting Eq.1 in Eq.4 yields:

$$\overline{q}_{i}(\omega_{i}^{2}-\omega^{2})+\alpha\phi_{i}(a)\sum_{j}\phi_{j}(a)\overline{q}_{j}-\alpha\phi_{i}(a)\overline{u}=\gamma\phi_{i}(b) \quad \text{in which} \quad \alpha=k/M, \ \beta=k/m, \ \gamma=F_{0}/M \tag{5}$$

Another vibration equation can be written for the absorber as:

$$m\ddot{u} + k[u - y(a,t)] = 0 \tag{6}$$

Similar procedure done to obtain Eq.5 can be applied on Eq.6 to get:

$$\overline{u}(\beta - \omega^2) - \beta \sum_j \phi_j(a) \overline{q}_j = 0$$
⁽⁷⁾

Eqs.5 and 7 are vibration equations of the whole system and can be written as [A][Q] = [B]. [A] is a full-rank matrix of order (n+1) with elements that are the coefficients of \overline{q}_i and \overline{u} , [Q] is a vertical matrix which its elements are \overline{q}_i and [B] is a vertical matrix of force elements available in the right hand of Eqs.5 and 7. If determinant of the matrix [A] is set to zero, the (n+1) frequencies of the whole system including the beam and the absorber can be achieved. To get generalized coordinate \overline{q}_i and the amplitude of the absorber mass displacement \overline{u} , Eqs.5 and 7 which are (n+1) equations must be solved simultaneously.

2.2 Absorber Made of Mass, Spring and Damper

An absorber made of a mass, a spring and a damper, as shown in Fig.2 is considered in this case. The distance between the spring of the absorber and the left support (a_1) is selected as a parameter for design. It is assumed (theoretically) that the distance between the damper and the spring is constant of d. So for this case Eqs.4, 6 are changed to:

$$\begin{cases} \ddot{q}_{i}(t) + \omega_{i}^{2}q_{i}(t) = \frac{1}{M_{i}} \left[-k[y(a_{1},t) - u]\phi_{i}(a_{1}) - c[\dot{y}(a_{2},t) - \dot{u}]\phi_{i}(a_{2}) + F_{0}\phi_{i}(b) \right] \\ m\ddot{u} + c[\dot{u} - \dot{y}(a_{2},t)] + k[u - y(a_{1},t)] = 0 \end{cases}$$
(8)



Figure 2. Vibration system with absorber contains mass- spring- damping

With similar steps done in the section 2-1, it can be shown that the vibration equations of the system shown in Fig.2 cab be expressed as:

$$\overline{q}_{i}(\omega_{i}^{2}-\omega^{2})+\alpha\phi_{i}(a_{1})\sum_{j}\phi_{j}(a_{1})\overline{q}_{j}+j\xi\omega\phi_{i}(a_{2})\sum_{j}\phi_{j}(a_{2})\overline{q}_{j}-\alpha\phi_{i}(a_{1})\overline{u}-j\xi\omega\phi_{i}(a_{2})\overline{u}=\gamma\phi_{i}(b) \quad (9)$$

$$\beta \sum_{j} \phi_{j}(a_{1})\overline{q}_{j} + j\eta \omega \sum_{j} \phi_{j}(a_{2})\overline{q}_{j} + \overline{u}(\omega^{2} - \beta - j\eta\omega) = 0$$
(10)
Where, $\alpha = k/M$, $\beta = k/m$, $\gamma = F_{0}/M$, $\xi = c/M$, $\eta = c/m$.

3. SPRING MASS EFFECT ON THE FORMULATION OF THE PROBLEM

In the analysis given in section 2, the spring mass was neglected. In many practical cases neglecting this mass has an important effect on the dynamic analysis of the system. Velocity of spring at connected point to the beam is considered as $\dot{y}(a,t)$ and at connected end to the mass is \dot{u} and it is assumed change linearly as shown in Fig.3. An element with the length of $d\lambda$ at a distance of λ from the top of spring is considered. From Fig.3 the velocity of this element can be written $V(\lambda) = \dot{y}(a,t) + (\dot{u} - \dot{y}(a,t))(\lambda/L)$ in which $\dot{y}(a,t) = \sum \phi_i(a)\dot{q}_i(t)$ and L is the free length of the spring and $m_s = \rho_s L$ denotes spring mass and ρ_s is the mass per length of the spring. So potential and kinetic energy for the system are written as:

$$T = \frac{1}{2} \sum_{i} M_{i} \dot{q}_{i}^{2} + \frac{1}{2} m \dot{u}^{2} + \frac{1}{2} \int_{0}^{l} (\rho d\lambda) \left[\sum_{i} \phi_{i}(a) \dot{q}_{i}^{2} + \frac{\dot{u} - \sum_{i} \phi_{i}(a) \dot{q}_{i}}{L} \right]^{2}$$
(11)

$$U = \frac{1}{2}k \left(\sum_{i} \phi_{i}(a)q_{i} - u\right)^{2} + \frac{1}{2}\sum_{i} \omega_{i}^{2}M_{i}q_{i}^{2}$$
(12)



Figure 3. Variation of the absorber spring velocity

Generalized force is $Q_i = F_0 \phi_i(b) \sin \omega t$. Substituting Eqs.11 and 12 into the Lagrange's equation [16], the vibration equations of the whole system are expressed by:

$$\begin{cases} \overline{q}_{i}(\omega_{i}^{2}-\omega^{2})+(\alpha-\alpha_{1}\omega^{2})\phi_{i}(a)\sum_{j}\phi_{j}(a)\overline{q}_{j}-(\alpha+\alpha_{2}\omega^{2})\phi_{i}(a)\overline{u}=\gamma\phi_{i}(b) \quad (I) \\ (\omega^{2}-\beta)\overline{u}+(\beta+\alpha_{3}\omega^{2})\sum_{j}\phi_{j}(a)\overline{q}_{j}=0 \qquad (II) \end{cases}$$
(13)

In which the expressions specified with a dashed lined are added relative to Eqs.5 and 7 and:

$$\alpha = \frac{k}{M_i}, \quad \beta = \frac{k}{m + \frac{m_s}{3}}, \quad \gamma = \frac{F_0}{M_i}, \quad \alpha_1 = \frac{m_s}{3M_i}, \quad \alpha_2 = \frac{m_s}{6M_i}, \quad \alpha_3 = \frac{\frac{m_s}{6}}{m + \frac{m_s}{3}}$$
(14)

m

 \overline{q}_i and \overline{u} are found by solving Eq.13. Using Eq.1, the deflection of beam will be achieved. It can be shown that for beams with high natural frequencies, the values of α_i , i = 1,2,3 are so small that the natural frequencies of absorbed beam are remained almost constant. Similarly, for the system with an absorber made of mass, spring and damper, Eq.13 is changed to:

$$\begin{cases} \overline{q}_{i}(\omega_{i}^{2} - \omega^{2}) + (\alpha - \alpha_{1}\omega^{2})\phi_{i}(a_{1})\sum_{j}\phi_{j}(a_{1})\overline{q}_{j} + j\xi\omega\phi_{i}(a_{2})\sum_{j}\phi_{j}(a_{2})\overline{q}_{j} - (\alpha + \alpha_{2}\omega^{2})\phi_{i}(a_{1})\overline{u} \\ - j\xi\omega\phi_{i}(a_{2})\overline{u} = \gamma\phi_{i}(b) \\ \sum_{j}\phi_{j}(a_{1})\overline{q}_{j} + j\eta\omega\sum_{j}\phi_{j}(a_{2})\overline{q}_{j} + \overline{u}(\omega^{2} - \beta - j\eta\omega) = 0 \end{cases}$$

$$(15)$$

4. RESULTS AND DISCUSSIONS

A beam with simple supports is considered. Cross section of the beam is shown in Fig.4 and specifications of the beam and absorber are listed in Table.1. It should be noted that these values are selected only for start of analysis. During the procedure of the analysis of the problem, each of these parameters can be varied such that the optimum values of them are determined. All results presented in this paper are for the case of absorber with no damper. Values of five natural frequencies of the beam with/without absorber are shown in Table.2.

Table 1. Specification of the vibration system

Beam material	steel	beam mass	M =1800 kg	Absorber mass	m = 30 kg	Excitation amplitude	$F_0 = 5000N$
Young' modulus	E = 150Gpa	Beam length	<i>L</i> = 30 <i>m</i>	Spring stiffness	k = 2000 N / m	Excitation position	<i>b</i> =10 <i>n</i>
Density	$\rho' = 7800$ kg/m ³	Number of modes	$\operatorname{mod} e = 5$	Damping	c = 50 N.s/m	Excitation frequency	$\omega = 20$ rad / s
Mass per unit length of beam	ho = 60 kg/m	Number of elements	Nu = 50	Distance between spring and damping	d = 5cm	Kind of support	simple- simple

Natural frequencies of the	Natural frequencies of the		
beam without absorber \mathcal{O}_n	beam with absorber \mathcal{O}_{nn}		
3.71	3.81		
14.84	14.95		
33.40	33.62		
59.37	59.58		
92.77	92.94		



20cm

Figure 4. Cross sectional area of the beam

4.1 Finding the Best Position of the Absorber at a Given Spring Stiffness

To find the best position of the absorber, the beam is diffracted to a finite number of elements. Then, absorber moves along the beam and placed at the end of each element and deflection of midpoint of the beam is computed. Finally between all values, the minimum value of deflection and its corresponding position of absorber are determined. Considering other parameters constant, spring stiffness of the absorber is varied theoretically between 500N/m. The best position of the absorber for different values of spring stiffness is found. This procedure is repeated in various excitation frequencies and various positions of excitation, e.g., Figs.5 and 6 show the best position of the absorber versus the spring stiffness in log-scale at an arbitrary frequency and a natural frequency.



4.2 Effect of the Absorber on the Deflection of the Beam and its Optimum Design

At the beginning of the analysis the mass and spring stiffness of the absorber are selected as m=30kg, k=20KN/m respectively (these are not optimum values). Now, the excitation frequency is varied in range $1 < \omega < 100rad / s$ which contains the first five natural frequencies of the beam. Using a program written in MATLAB, in each frequency, the best position of the absorber is found such that the deflection of midpoint of the beam is minimized. The deflection curves of the beam against the excitation frequency without/with absorber are shown in Fig.7, where resonance occurs at the first and the fifth natural frequencies of beam. Theoretical values of deflection at the midpoint at these frequencies are $y_1 = 2.1m$, $y_3 = 0.12m$ that after using absorber are reduced to $y'_1 = 0.025m$, $y'_3 = 0.11m$. But the used absorber is not the optimum one. To design an optimum absorber, the mass of absorber is assumed to be constant (m = 30kg). Considering other parameters constant, three regions are considered for spring.

Table 2. Natural frequencies of the beam



Figure 7. Deflection curves of the beam at different excitation frequencies without/with absorber (Left/Right respectively)

In each region, best values of both the spring stiffness and its position are found such that the deflection of midpoint is minimized. For the beam with specifications given in Table.1 and with the excitation frequency of $\omega = 20rad/s$, the best value of stiffness is found at k = 7600N/m. For other frequencies, the optimum absorber is determined similarly, e.g., at the first natural frequency of the beam, Table 3 is achieved. The best value of stiffness is k = 9800N/m. In two arbitrary values of frequency, a frequency except the natural frequency and the natural frequency, the optimum values of absorber parameters are presented in Table 4. Deflection curves of beam without/with absorber are shown in Fig.8, in which the excitation frequency is $\omega = 3.81rad/s$. The values of midpoint deflection without/with absorber are $y_0 = 6.445m$, y = 0.0228m, respectively. The deflection of midpoint is reduced about 280 times.

k(KN/m)	Best value of k	Best position of absorber	Deflection of midpoint of beam
0.5 < k < 1	0.5	16.8 <i>m</i>	0.0475 <i>m</i>
1 < k < 10	9.8	28.2 <i>m</i>	0.0228m
10 < <i>k</i> < 100	68	28.2 <i>m</i>	0.0345 <i>m</i>

Table 3. The best values of k and the best position of the absorber for $\omega_{n1} = 3.8 rad / s$.

	Table 4.	The	optimum	values	of the	absorber	parameters
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	Mass absorber (constant)	Optimum spring stiffness	Best position of absorber
$\omega = 20 rad / s$	m = 30 kg	k = 7.6 KN / m	$a_{best} = 24.6m$
$\omega = 3.8 rad / s$	m = 30 kg	k = 9.8 KN / m	$a_{best} = 28.2m$



Figure 8. Deflection curves of the beam without/with optimum absorber (left and right, respectively)

5. SIMULATION OF THE SYSTEM

Finally, vibration system is simulated by *SIMULINK* Toolbox of MATLAB. The important advantage of this simulation is that one can find the absorber optimum specifications by several tries and errors for a beam with arbitrary supports under any other types of excitation such as step, ramp and etc. Simulated block diagram of the problem is not shown in this paper.

6. CONCLUSIONS

In this paper the effect of a tuneable vibration absorber (TVA) on reducing vibration of an Euler-Bernoulli beam is studied. This beam is under a harmonic point excitation which can have a variable position. It is shown that the best position of the absorber to minimize midpoint deflection of beam depends on both frequency and position of exciting force. The optimum specifications such as spring stiffness, absorber mass and its position are determined simultaneously by developing and using several algorithms under MATLAB environment. The effect of absorber is observed clearly in the natural frequencies of system. Also, the effect of variation of the spring length which results the variation of both absorber mass and spring stiffness can be another interesting research in passive control of beam vibration.

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