



EXPERIMENTAL IDENTIFICATION OF FRACTIONAL-DRAIVATIVE PARAMETERS OF A DAMPING MATERIAL USING THE OPTIMIZATION TECHNIQUE

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Abstract

In this paper, an efficient identification method of the material parameters is proposed using an optimization technique and applied to real materials to identify the fractional-derivative-model parameters of the material. In the proposed method, frequency response functions are measured from a cantilever beam impact test. The frequency response functions on the same points with the measured one are calculated by using an FE model with the equivalent stiffness approach. The differences between the measured and the calculated FRFs are minimized using a gradient-based optimization algorithm in order to identify the real values of the parameters. Four FRF's of a damped beam structure are measured in an environmental chamber at different temperatures and used as reference responses. An impact hammer and a laser vibrometer are used to measure the reference responses. Both linear and nonlinear relationships between the logarithmically-scaled shift factors and temperatures are examined in identifying the material parameters. The experimental results show that the proposed method accurately identifies the fractional-derivative-model parameters even for real damping materials.

1. INTRODUCTION

Material parameters of a viscoelastic damping material are very important data both in describing the responses of a damped structure and in designing the damped structure [1]. In recent years the fractional derivative model has been used to describe the dynamic characteristics of a damping material with respect to frequency and temperature [2, 3]. However, only a few researchers have studied on a method of optimal identification of material parameters of viscoelastic materials. Lekszycki *et al.* treated the problem of optimal constitutive parameter identification of viscoelastic materials of the sandwich beam using the derivation of optimality condition and one-dimensional Voigt model [4]. Deng *et al.* proposed a system identification procedure based on the direct nonlinear optimization and a sub-optimal method to estimate the polyurethane foam modeled by a fractional derivative model [5].

In previous work [6], the authors proposed an efficient identification method of the material parameters using an optimization technique and showed its applicability through numerical examples. In this study, the proposed procedure is applied to real materials to identify the fractional-derivative-model parameters of the material.

2. IDENTIFICATION OF MATERIAL PROPERTIES

2.1 Fractional Derivative Model of Viscoelastic Materials

Dynamic characteristics of the viscoelastic materials in frequency domain can be represented using the complex modulus such as:

$$\overline{\sigma} = E^* \overline{\varepsilon} = E(1 + i\eta)\overline{\varepsilon} \tag{1}$$

where $i = \sqrt{-1}$, $\overline{\sigma}$ and $\overline{\varepsilon}$ are the Fourier transforms of stress and strain, respectively. E^*, E and η are the complex modulus, the storage modulus and the loss factor, respectively.

The complex modulus of viscoelastic materials is strongly dependent on temperature as well as frequency. However, one can predict the complex modulus at any temperatures using the shift factor $\alpha(T)$ from the temperature-frequency superposition principle of viscoelastic materials. The shift factor is coupled with temperature through the linear Arrhenius equation or nonlinear William-Landel-Ferry (WLF) one such as [11]:

$$\log(\alpha(T)) = d_1 (1/T - 1/T_0)$$
(2)

$$\log(\alpha(T)) = -d_1 \frac{(T - T_0)}{(b_1 + T - T_0)}.$$
(3)

where d_1 and b_1 are material constants, and T_0 is a reference temperature in degrees absolute.

Considering the frequency variation of damping behavior as well as temperature variation, the complex modulus of the fractional derivative model in frequency domain can be written as follows [11, 12].

$$E^{*} = E(1+i\eta) = \frac{a_{0} + a_{1}(if\alpha(T))^{\beta}}{1 + c_{1}(if\alpha(T))^{\beta}}$$
(4)

Here, the four parameters a_0, a_1, c_1 and β in equation (4) are identified by a suitable empirical way.

It is well known that the four-parameter fractional derivative model is sufficient to represent the real behavior of viscoelastic materials over a wide frequency range [3]. Therefore, identifying the six or seven parameters of a viscoelastic material, the fractional derivative model can describe the dynamic characteristics of the viscoelastic materials over frequency and temperature variations. To estimate the fractional-derivative-model parameters of a real material with conventional methods, first many tests should be repeated until sufficient number of data are acquired at different frequencies and temperatures using, for example, Oberst beam test [8, 9] as shown in Fig. 1. Second, from these data, the coefficients of the fractional

derivative model can be determined using a statistical data analysis technique that minimizes the mean square error between theoretical value and the tabulated value [11]. However, the statistical data analysis process is not so efficient because it includes



Figure 1. Oberst beam test configuration.

trial-and-error steps, i.e., the shift factor is assumed and the mean square error is minimized. The trial-and-error step is repeated in turn until the minimal global error value is obtained.

2.2 A New Identification Method

A new estimation method of the fractional-derivative-model parameters starts from an assumption that if a numerical model reproduces measured responses, then material properties used in the simulation model is the real material properties of the material. Then by minimizing the response difference between the measured and simulated FRF's, one can identify the material properties using a numerical search algorithm. Therefore, the identification index function that is zero at the true values and should be minimized for the identification can be defined as follows.

$$g(b) = \sum_{i=1}^{N} \int (x_{simulated}^{i} - x_{measured}^{i})^{2} df$$
(5)

Here, x, N and f are frequency responses, number of responses and frequency, respectively. Generally, gradient-based mathematical programming techniques are used to minimize the identification index because the gradient-based methods are the most efficient although it may give a local minimum.

The convex region of the identification index function should be as wide as possible in order that the identification procedure can give true values consistently regardless of initial values. The identification index defined in Eq. (5) sometimes may fall into a local minimum if initial values far from the true values are given. To widen the stable region of the identification process, the authors introduced a new identification index and divided the identification process into two steps. The first step is a peak-alignment step and the second one is an amplitude-adjustment step. As a

result, the identification index defined in Eq. (5) is split into two as follows:

$$g_{1}(b) = \sum_{i=1}^{N} \sum_{k=1}^{M} (\lambda_{k,simulated}^{i} - \lambda_{k,measured}^{i})^{2} \quad (6)$$
$$g_{2}(b) = \sum_{i=1}^{N} \int (x_{simulated}^{i} - x_{measured}^{i})^{2} df \quad (7)$$

where λ and M are resonance frequencies and the number of resonant peaks within a concerned frequency range, respectively. Then, minimizing the first identification index function with respect to the of parameters the factional derivative model, one can expect that the response differences will be very small. Therefore, the second step that is a minimization step of magnitude-difference between the measured and simulated FRF's, can



Figure 2. The two-step identification procedure.

be started from very close values to true values, which means that the identification process has little possibility of falling in a local minimum. Figure 2 summarizes the identification procedure.

2.3 Finite Element Analysis of the Damped Beam

For the identification process a simulation model of the damped beam is necessary. In addition, the gradient information of the identification index function with respect to the unknown parameters should be provided in order to search minimum points using a numerical search algorithm.

To simulate the forced responses of the damped beam, the modal superposition method is used. The unconstrained beam is modeled by finite beam elements with the Ross, Kerwin and Ungar (RKU)'s equivalent complex flexural rigidity. The resulting eigenvalue problem becomes frequency-dependent one due to damping materials and the iteration procedure of Ref. [7] is used to solve the frequency-dependent real eigenvalue problem. The modal strain energy method is utilized to predict the loss factor of each mode. For more detail explanation of the analysis of unconstrained damped beam, one can see Ref. [7].

To identify the parameters of the fractional derivative model using a gradient-based algorithm, the sensitivity analysis for the identification indexes are needed. The parameter sensitivity information can be obtained analytically by differentiating the identification index expressions with respect to the fractional-derivative-model parameters. The resulting sensitivity equation consists of eigenvalue and eigenvector sensitivities and derivative expression of the complex modulus represented by the fractional derivative model. The details of the parameter sensitivity analysis method can be found in Ref. [7] and will not be repeated here because of lack of space.

3. APPLICATION TO A REAL MATERIAL

To verify the proposed identification method, a viscoelastic material is selected and its fractional-derivative-model parameters are identified experimentally. To obtain the reference responses of the identification method at several temperatures, impact tests in a constant-temperature chamber were fulfilled. Figure 3 shows the schematic diagram of the test set-up and the clamped beam structure. The beam-clamping structure was composed of a steel jig fixed on a test bed and an aluminum beam. The length, width and thickness of the beam were 200.0, 20.0 and 4.0 mm, respectively. The aluminum beam was clamped by the two plates





(a) Schematic diagram



Figure 3. Experimental set-up

fastened by six steel bolts with a constant torque. A Polytec laser Doppler velocimetry (LDV) and Scadas III front-end were used to acquire response signals. Only one direction perpendicular to the beam was excited and measured in the experiment. The frequency band was 3000 Hz with 1-Hz frequency resolution.

The analytical finite element model was generated and validated using the bare beam. The bare beam was modeled by 40 linear finite beam elements. The calculated frequency response function of the bare beam was correlated with that of the measured one by slightly changing Young's



Figure 4. Calculated FRF compared with measured one for the bare beam

modulus and structural damping from the typical values of aluminum property. Figure 4 shows good agreement between the two results.

A viscoelastic material, 3M-467 adhesive, was bonded on the beam with 1.2 mm thickness. Then, the fractional-derivative-model properties of the viscoelastic material were identified using the proposed method. The reference FRF's were measured at four different circumferential temperatures such as 25, 35, 40 and 55 °C. To measure the FRF's at a circumferential temperature, the temperature of the environment chamber was kept at the temperature at least 2 hours. The measured responses were averaged 7 times for each temperature. The damped beam was also modeled by 40 finite elements with the equivalent stiffness. Thereafter, the identification index function was defined and minimized in order to identify the material parameters. The lower and upper limits of frequency band were 30 and 3000 Hz, respectively. To solve the minimization problem, the commercial program DOT Ver. 5.4 [14] was employed with the analytical sensitivity information.

In identifying the fractional-derivative-model parameter in the identification procedure, one can select the linear Arrhenius or the nonlinear WLF relationships to define a relationship between the shift factor and temperature. If one uses the linear relationship, it was shown in Ref. [6] that two reference FRF's are sufficient to identify the parameters. However, a number of FRF's over an interesting temperature range must be given for the WLF relationship. First, the

parameters were identified with four reference FRF's of different temperatures using the linear Arrhenius relationship. Comparing the calculated responses with the reference FRF's in the identified results, the regenerated FRF's at 25, 35 and 40 °C showed very good agreement with the reference FRF's. However, the reference FRF and the calculated one at 55 °C showed large difference in the location of the fourth

Table 1. Identified material parameters for 3M-467 with the WLF shift factor relationship

Parameters	Initial Value	Identified
a_0	0.02	0.34065
a_1	0.02	0.49161
<i>C</i> ₁	0.02	0.0010212
d_1	1.00	19.489
b_1	100.	376.81
β	1.00	0.52618
\overline{T}_{0}	10.0	23.527



Figure 5. Identified FRF's compared with the reference FRF's



Figure 6. The shift factors compared with the linear and nonlinear relationships.

resonant frequency, which means the linear relationship of the shift factor properly describe does not the temperature effects of the material around the temperature 55 °C. Therefore, the identification process was repeated once more with the same condition except adopting the nonlinear WLF relationship. Table 1 shows the identified parameters of the fractional-derivative-model. Figure 5 also shows the regenerated FRF's with the identified parameters compared with the reference FRF's at 25 and 55 °C . As shown in Fig. 5, the

nonlinear WLF relationship describes the temperature effect very well. One can determine the shift factor value for each temperature manually by minimizing the response difference between the reference FRF and the calculated one. These results are plotted with the identified Arrhenius and WLF relationships in Figure 6. One can see in Fig. 6 that the linear Arrhenius relationship starts to deviate from the real value above 40 $^{\circ}$ C . Actually, the fractional-derivative-model parameters identified with the WLF relationship are very close to the reference values of Ref. [11]; only 2.7% difference in storage modulus at the reduced frequency 1 Hz.

4. CONCLUSIONS

In the design stage of damped structures, the properties of damping materials such as storage modulus and loss factor are essential information. In addition, the fractional-derivative model is widely used to describe the dynamics characteristics of damping materials including temperature effects. In this paper, an efficient identification method of the fractional-derivative-model parameters is proposed using an optimization technique and applied to real materials. In the proposed method, frequency response functions are measured

from a cantilever beam impact test. The frequency response functions on the same points with the measured one are calculated by using an FE model with the equivalent stiffness approach. The differences between the measured and the calculated FRFs are minimized by using a gradient-based optimization algorithm in order to identify the real values of the parameters. For a real damping material, four FRF's of a damped beam structure are measured in an environmental chamber at different temperatures and used as reference responses. A light impact hammer and a laser vibrometer are used to measure the reference responses. Both linear and nonlinear relationships between the logarithmically-scaled shift factors and temperatures are examined in identifying the material parameters. The experimental results show that the proposed method accurately identifies the fractional-derivative-model parameters even for real damping materials.

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