ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

FEEDBACK SUPERVISING FEEDFORWARD MINIMUM-VARIANCE CONTROL OF ACOUSTIC NOISE

Marek Pawelczyk

Institute of Automatic Control, Silesian University of Technology
ul. Akademicka 16, 44-101 Gliwice, Poland
E-mail: Marek.Pawelczyk@polsl.pl

Abstract

Minimum-Variance Control is one of the dominating strategies for actively suppressing acoustic noise. Feedforward structure is generally preferred if a reference signal is available. Otherwise, feedback structure is the interim solution. The purpose of this paper is to design an optimal combined control-weighted minimum variance noise control system. In the proposed approach feedforward is the primary control strategy. Feedback control is added to the already operating feedforward. It aims at controlling the residual noise of feedforward control. However, the main purpose of closing the external loop is to benefit from properties of feedback control. Appropriate controllers can be designed in the time domain, frequency domain or transform domain. The latter approach, based on polynomial operations, including Diophantine equations has been chosen. Correctly applied control weighting for both techniques guarantees stable operation for a non-minimum phase plant. Performance of the combined system is compared to performance of the individually operating feedforward system. Different time dependences are considered. Theoretical analysis is supported by simulation experiments based on data obtained from a real-world active headrest system.

1. INTRODUCTION

For active control, if a reference signal is measure-available, a fixed-parameter optimal control is preferred over corresponding feedback control. However, such statement is generally true provided the following assumptions are satisfied.
a). Time delay introduced by the primary path, i.e. the acoustic path between the reference and the error sensor is smaller than the overall time delay introduced by the reference signal measurement path, control filter and the secondary path, i.e. the plant.
b). Plant response and properties of the disturbance do not change significantly.
c). The plant is linear.

If assumption a) is not satisfied performance of the feedforward system may be poor since the control filter should work as a predictor. It can be then shown that under certain circumstances feedback system may perform better than feedforward system [1]. It is well known from control theory that violation of assumption b) or c) has usually also negative influence on performance because the compensation action cannot be correctly undertaken. Feedback control has an in-built potential to suppress negative effects of plant/disturbance variations and plant non-linearity [2]. This is the primary premise for designing the combined system [3].

A secondary question is whether the combined system may perform better for the nominal plant and disturbance. In the presented approach it is assumed that optimum feedforward Minimum Variance (MV) control system is operating and then the optimum feedback external loop is provided. Therefore, although in fact a combined system is considered as presented in Figure 1, it can be substituted for analysis by the feedback system presented in Figure 2a, where the disturbance is the output of the feedforward system presented in Figure 2b.

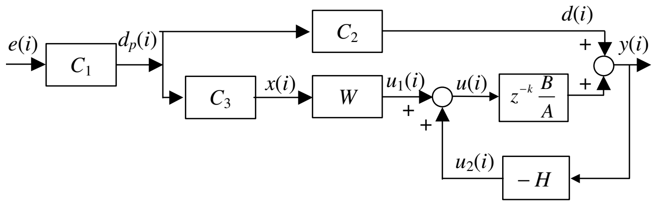

Figure 1. Combined control system.

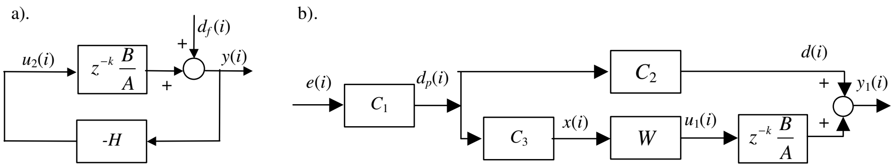

Figure 2. Separately considered control systems (a – feedback, b – feedforward).

The feedforward system is designed to minimize variance of the disturbance d at time instant i at the output of a plant modelled by an infinite impulse response (IIR) discrete time filter $z^{-k}B/A$. The plant itself is of acousto-electric nature and it is composed of a microphone, loudspeaker, appropriate amplifiers, anti-aliasing and reconstruction analogue filters, and A/D and D/A converters. All transfer functions in the figures are rational functions and the polynomials are of complex variable z. Informally, z^{-1} should also be recognized as a one-sample backward time-shift operator in difference equations. Since the measure-available reference signal $x(i)$ driving the feedforward filter W should be correlated with the output disturbance $d(i)$, both of them are assumed to originate from the same signal $d_p(i)$ being filtered by minimum phase finite impulse response (FIR) filters C_3 (of r samples discrete time delay) representing the reference path and C_2 (of s samples discrete time delay) standing for the primary path, respectively. The signal $d_p(i)$ is then assumed to be modelled as a wide-sense stationary white noise, $e(i)$, of respective variance, filtered by a minimum-phase filter C_1. The following notation is also introduced:

$$C_3 = z^{-r}\,\bar{C}_3;\; C_2 = z^{-s}\,\bar{C}_2;\; C_1 C_2 = C;\; C_1 C_3 = C_r;\; C_1\,\bar{C}_2 = \bar{C};\; C_1\,\bar{C}_3 = \bar{C}_r;\; r - s := p;\; k + p := \bar{k}\,. \quad (1)$$

It is assumed that polynomial $\bar{C}$ is monic. The assumption concerning an FIR structure of the filter modelling the output disturbance differs from designs met in the literature, where an ARX or ARMAX model of the plant is usually used [2], [4], [5]. This assumption does not limit the considerations but simplifies analysis of the control system and makes some interpretations easier.

2. FEEDFORWARD MV CONTROL SYSTEM

It is well-known that if the control objective were to simply minimise the output signal variance the direct solution would result in an unstable system for a non-minimum phase plant [2], [4]. To avoid such problem the control filter can be designed to compensate for minimum phase part of the plant, [4], or inner-outer factorization and causal-anticausal decomposition of respective transfer functions can be performed [6], [7]. In this paper another approach is applied, which requires modification of the cost function by including control signal variance

$$L_1(i) = E\left\{ y_1^2(i) + qu_1^2(i-k) \right\}, \tag{2}$$

where $q > 0$ is a weighting coefficient [1], [4]. For feedforward control, the system output is expressed as follows (Figure 2b):

$$y_1(i) = z^{-k}\frac{B}{A}C_1C_3We(i) + C_1C_2e(i). \tag{3}$$

Using the notation defined in (1) the system output can be written as:

$$y_1(i) = z^{-s}\left(z^{-\bar{k}}\frac{B}{A}\bar{C}_r W + \bar{C} \right)e(i). \tag{4}$$

Let the following Diophantine equation be defined:

$$\bar{C} = F_f + z^{-\bar{k}}G_f, \;\; \deg F_f = \bar{k}-1; \; \deg G_f = \deg \bar{C} - \bar{k}. \tag{5}$$

which simply splits polynomial $\bar{C}$ into two polynomials. Substituting (5) for $\bar{C}$ in (4), noticing that $u_1 = z^{-r}\bar{C}_r W$, and shifting by $s+\bar{k}$ samples gives:

$$y_1(i+s+\bar{k}) = \left[\frac{B}{A}\bar{C}_r \frac{1}{z^{-r}\bar{C}_r}u_1(i) + G_f e(i) \right] + \left[F_f e(i+\bar{k}) \right]. \tag{6}$$

Because $\dim F_f = \bar{k}-1$ the two terms in square brackets are independent. As a result the cost function, (2), for shifted samples takes the form:

$$L_1(i+s+\bar{k}) = E\left\{ \left(\frac{B}{A}u_1(i+r) + G_f e(i) \right)^2 + q\left[u_1(i+r) \right]^2 \right\} + E\left\{ \left[F_f e(i+\bar{k}) \right]^2 \right\}. \tag{7}$$

The second term on the RHS of this equation is unknown at time instant i and cannot be controlled by $u_1(i)$. Thus, minimisation of the cost function is equivalent to minimisation of its first term, what can be done by:

$$\frac{\partial}{\partial u_1(i+r)}\left[L_1(i+s+\bar{k}) \right] \equiv \frac{\partial}{\partial u_1(i+r)}\left\{ \left(\frac{B}{A}u_1(i+r) + G_f e(i) \right)^2 + q\left[u_1(i+r) \right]^2 \right\} = 0. \tag{8}$$

After partially differentiating, the following relation is obtained:

$$2\left(\frac{B}{A}u_1(i+r)+G_f e(i)\right)b_0+2qu_1(i+r)=0\,, \tag{9}$$

where b_0 is the first parameter of B. The optimal feedforward control law is expressed by:

$$u_{1opt}(i)=-z^{-r}\frac{AG_f}{B+q'A}e(i)\,, \tag{10}$$

where $q'=q/b_0$, what gives the optimal feedforward filter:

$$W_{opt}=-\frac{AG_f}{\bar{C}_r\left(B+q'A\right)}\,. \tag{11}$$

Combining (4) and (11) leads to an equation defining the stability condition:

$$A\,\bar{C}_r\left(B+q'A\right)=0\,. \tag{12}$$

Taking (6) and (10) into account, the following relation is obtained:

$$y_1(i+s+\bar{k})_{opt}=q'\frac{AG_f}{B+q'A}e(i)+F_f e(i+\bar{k})\,. \tag{13}$$

Because the two random variables on the RHS of this equation are independent then the variance of the system output can be expressed as follows:

$$E\left\{y_1^2(i+s+\bar{k})_{opt}\right\}=E\left\{\left[q'\frac{AG_f}{B+q'A}e(i)\right]^2\right\}+E\left\{\left[F_f e(i+\bar{k})\right]^2\right\} \tag{14}$$

3. COMBINED CONTROL SYSTEM

If a feedback is added to the feedforward system, the feedforward system output, (13), becomes the disturbance, $d_f\,(i)$, to be reduced by the feedback, i.e., $d_f(i)=y_1(i)$ (see Figure 2). Then, the combined system output is given by:

$$y(i)=z^{-k}\frac{B}{A}u_2(i)+F_f e(i)+z^{-\bar{k}}q'\frac{AG_f}{B+q'A}e(i)\,. \tag{15}$$

Let a Diophantine equation be defined to split polynomial F_f into two polynomials:

$$F_f=F_{fc}+z^{-k}G_{fc},\ \deg F_{fc}=k-1,\ \deg G_{fc}=\bar{k}-1-k=r-s-1=p-1\,. \tag{16}$$

Combining (15) and (16), and shifting by k samples ahead, give:

$$y(i+k) = \frac{B}{A}u_2(i) + G_{fc}e(i) + z^{-p}q'\frac{AG_f}{B+q'A}e(i) + F_{fc}e(i+k)\,. \tag{17}$$

Using the Diophantine equation defined for feedforward control, (5), the white noise signal can be extracted from (15) as:

$$e(i) = \frac{B+q'A}{F_f B + q'A\bar{C}}y(i) - z^{-k}\frac{B}{A}\frac{B+q'A}{F_f B+q'A\bar{C}}u_2(i)\,. \tag{18}$$

Then, substituting for $e(i)$ in (17), gives:

$$\begin{aligned} y(i+k) &= \frac{B}{A}u_2(i) + \frac{G_{fc}\left(B+q'A\right)}{F_f B+q'A\bar{C}}y(i) - z^{-k}\frac{BG_{fc}}{A}\frac{B+q'A}{F_f B+q'A\bar{C}}u_2(i) \\ &+ z^{-p}\frac{q'AG_f}{F_f B+q'A\bar{C}}y(i) - z^{-\bar{k}}q'\frac{B}{A}\frac{AG_f}{F_f B+q'A\bar{C}}u_2(i) + F_{fc}e(i+k). \end{aligned} \tag{19}$$

Separating variables and using (16) and (5) leads to

$$y(i+k) = \frac{G_{fc}\left(B+q'A\right)+z^{-p}q'AG_f}{F_f B+q'A\bar{C}}y(i) + \frac{B}{A}\frac{\left(B+q'A\right)F_{fc}}{F_f B+q'A\bar{C}}u_2(i) + F_{fc}e(i+k)\,. \tag{20}$$

Let now an additional cost function be defined as follows:

$$L_2(i+k) = E\left\{y^2(i+k) + q_{fc}u_2^2(i)\right\}. \tag{21}$$

Repeating the same steps as for the feedforward system, i.e. partially differentiating the cost function with respect to $u_2(i)$ and making the result equal zero allows for finding the following optimal controller:

$$H_{opt} = \frac{A\left[G_{fc}\left(B+q'A\right)+z^{-p}q'AG_f\right]}{B\left(B+q'A\right)F_{fc}+q'_{fc}A\left(F_f B+q'A\bar{C}\right)}, \tag{22}$$

where $q'_{fc} = q_{fc}/b_0$. The characteristic equation takes the following form:

$$A\left(B+q'_{fc}A\right)\left(BF_f + q'A\bar{C}\right) = 0\,. \tag{23}$$

Equations (12) and (23) define conditions for stable operation of the combined system and justify presence of the control signal variance in the cost functions. The output signal under optimal combined control is:

$$y(i+k)_{opt} = q'_{fc}\frac{A\left[G_{fc}\left(B+q'A\right)+z^{-p}q'AG_f\right]}{\left(B+q'A\right)\left(B+q'_{fc}A\right)}e(i) + F_{fc}e(i+k)\,. \tag{24}$$

Because the two random variables on the RHS of this equation are independent the variance of the combined system output can be expressed as follows:

$$E\left\{y^2(i+k)_{opt}\right\}=E\left\{\left[q'_{fc}\frac{A\left[G_{fc}\left(B+q'A\right)+z^{-p}q'AG_f\right]}{\left(B+q'A\right)\left(B+q'_{fc}A\right)}e(i)\right]^2\right\}+E\left\{\left[F_{fc}e(i+k)\right]^2\right\}. \quad (25)$$

4. COMPARISON OF THE CONTROL SYSTEMS

The feedback loop operating over the feedforward system has been introduced to benefit from properties of feedback control in terms of response for modelling error and non-linearity, as explained in Introduction. Nevertheless, it is interesting to compare performance of the feedforward and combined control systems. This problem can be addressed fairly if nominal and varying conditions are considered separately.

4.1 Nominal plant and disturbance

To evaluate whether the feedback loop may enhance performance of the first implemented feedforward system for time-invariant plant and stationary disturbance the optimal feedforward output variance equation, (14), should be rewritten using the Diophantine equation defined for feedback, (16):

$$E\left\{y_1^2(i+s+\bar{k})\right\}=E\left\{\left[q'\frac{AG_f}{B+q'A}e(i)\right]^2\right\}+E\left\{\left[G_{fc}e(i+p)\right]^2\right\}+E\left\{\left[F_{fc}e(i+\bar{k})\right]^2\right\}. \quad (26)$$

When compared to output variance obtained for the combined system, (25), it follows that the last term on the RHS is the same. Further analysis requires considering different time dependences.

4.1.1 The case of $k+r\le s$

According to (5) and (16) there are: $F_f = 0$, $G_f = \bar{C}$, $F_{fc} = 0$, $G_{fc} = 0$.

Superiority of any of these systems depends on the plant and disturbance. However, as simulation experiments demonstrate the combined system usually results in a smaller output variance. If the plant were minimum phase then q' and q'_{fc} could be set zero. Consequently, output variances for both systems would be equal zero resulting in perfect disturbance cancellation.

4.1.2 The case of $k+r>s$ and $r\le s$

According to (5) and (16) there are: $F_f \ne 0$, $F_{fc} = 0$, $G_{fc} = F_f$.

Superiority of any of these systems depends on the plant and disturbance. However, as simulation experiments demonstrate the combined system usually results in a smaller output variance. If the plant were minimum phase then q' and q'_{fc} could be set zero. Hence, the first terms on the RHSs of (25) and (26) would nullify and the combined system would outperform the feedforward system.

4.1.3 The case of $r > s$

Both Diophantine equations, (5) and (16), have non-trivial solutions. Superiority of any of these systems depends on the plant and disturbance. If the plant were minimum phase then *q'* and q'_{fc} could be set zero. Hence, the first terms on the RHSs of (25) and (26) would nullify and the combined system would outperform the feedforward system.

4.2. Varying plant and non-stationary disturbance

If the plant response or the disturbance spectral properties change, what is a usual case for active control, or modelling errors are significant, the forms of optimal feedforward control filter, (11), and feedback controller, (22), should be presented as follows:

$$W_{opt} = -\frac{\hat{A}\hat{G}_f}{\hat{\bar{C}}_r\left(\hat{B}+q'\hat{A}\right)}, \qquad H_{opt} = \frac{\hat{A}\left[\hat{G}_{fc}\left(\hat{B}+q'\hat{A}\right)+z^{-p}q'\hat{A}\hat{G}_f\right]}{\hat{B}\left(\hat{B}+q'\hat{A}\right)\hat{F}_{fc}+q'_{fc}\hat{A}\left(\hat{F}_f\hat{B}+q'\hat{A}\hat{\bar{C}}\right)}, \tag{27}$$

where $\hat{A}$, $\hat{B}$ are models of the denominator and numerator of the plant, $\hat{\bar{C}}_r$, $\hat{\bar{C}}$ are models of corresponding disturbance filters, and $\hat{F}_f$, $\hat{G}_f$, $\hat{F}_{fc}$, $\hat{G}_{fc}$ are solutions to Diophantine equations obtained for the models. Equations (12) - (25) and (23) - (26) are then not valid. Equations for the optimal system outputs and their variances should be found from control systems equations based on Figures 1 and 2.

5. SIMULATION ANALYSIS

To compare performance of the feedforward and combined controls systems designed in this paper simulation experiments were performed based on data obtained from a real-world active headrest system. Headrest of a chair was equipped with two secondary loudspeakers and two microphones in order to attenuate acoustic noise at the ears of a person occupying the chair [7]. Necessary electronics was also applied including A/D and D/A converters, analogue antialiasing and reconstruction filters (650 Hz cut-off frequency) as well as amplifiers. The sampling frequency was 2 kHz. For the purpose of the paper the secondary loudspeaker on the RHS only was running and the attenuation at the right error microphone was of interest, which for the nominal plant was at the distance of 15 cm to the user's ear. The plant was non-minimum phase including a 3-sample time delay. A grain mill noise was generated by a primary loudspeaker located in front of the headrest at the distance of 4 m. The reference signal was obtained from a reference microphone located in front of the headrest at the distance of 3.5 m Two types of experiments were performed. In the first experiment performance of the feedforward and combined control systems for the nominal plant was considered. In the second experiment the head was moved forward and to the left so the distance between the error microphone and the ear increased to 40 cm. For both experiments the noise was stationary. Results of the experiments are presented in Figures 3 and 4, respectively.

Comparing the experiments the plant response changed dramatically resulting in significant change of polynomials *A* and *B*, although the time delay was the same. In turn, the primary and reference paths changed marginally. Therefore, the solutions to Diophantine equations can be considered the same. It follows from Figure 3 that for the nominal plant the feedforward control system performs well yielding 15.3 dB attenuation and the combined system results in 16.4 dB attenuation. Because, due to the distance between the microphones, that was the control case discussed in section 4.1.1, the little improvement in the combined system is observed. Superiority of the combined system is clearly evident for the second experiment where there was large plant modelling error and the feedforward system could not correctly compensate for the disturbance. In that experiment attenuation levels obtained for the feedforward and combined systems are 6.1 dB and 12.7 dB, respectively.

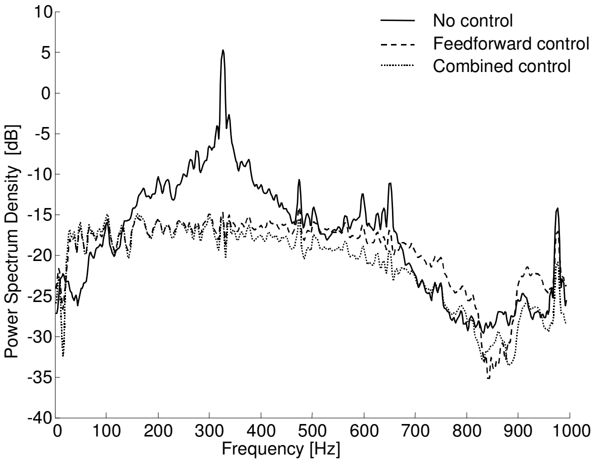

Figure 3. Control results for the nominal plant.

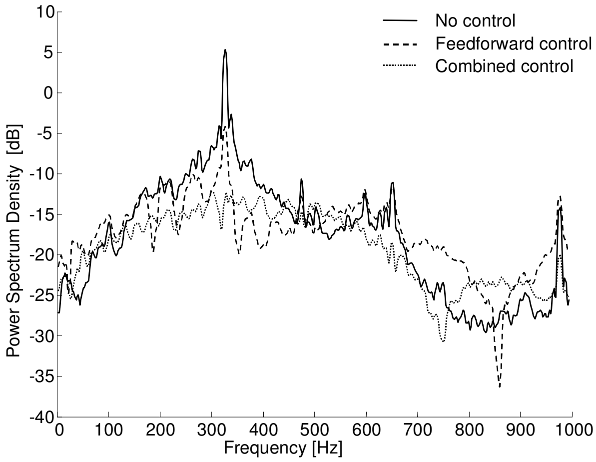

Figure 4. Control results for the changed plant.

6. CONCLUSIONS

It has been shown in this paper that a feedback loop may significantly support feedforward control system. Its contribution depends on plant response and disturbance properties and is clearly evident in case of plant modelling errors. Simulation experiments have confirmed theoretical considerations. The algorithms presented in this paper can be extend to control noise at the user’s ears by applying the idea of virtual microphones described, e.g. in [7].

ACKNOWLEDGEMENTS

Financial support from the state budget for science in Poland is gratefully acknowledged.

REFERENCES

[1] M. Pawełczyk, “Minimum variance control for acoustically-compact plants“, International Journal of Acoustics and Vibration **5**(4), 183-189 (2000).

[2] K.J Åström and B. Wittenmark, *Computer-Controlled Systems*, Prentice Hall, 1984.

[3] M. Pawelczyk, *Active Noise Control for Compact Acoustic Plants*, JSCS, 1999.

[4] Niederlinski, J. Moscinski and Z. Ogonowski, *Regulacja Adaptacyjna*, in Polish (*Adaptive Control*), WNT, 1995.

[5] M.J. Grimble and M.A. Johnson, *Optimal Control and Stochastic Estimation: Theory and Applications*, vol. 2, J. Wiley & Sons, 1988.

[6] M. Sternad and A. Ahlén, “A novel derivation methodology for polynomial-LQ controller design”, *IEEE Transactions on Automatic Control* **38**, 116-121 (1993).

[7] M. Pawelczyk, *Feedback Control of Acoustic Noise at Desired Locations*, Silesian University of Technology Press, 2005.