ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

ROTOR CRACK DETECTION FROM VIBRATION MONITORING USING A NARMAX MODELLING APPROACH*

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Abstract

An approach to detect the presence of cracks in rotors and rotor blades through the application of the Nonlinear Auto-Regressive Moving Average with eXogenous inputs (NARMAX) modelling tool, is proposed in this paper. The NARMAX methodology has previously been shown to provide excellent representation for nonlinear system dynamics in the time domain for a wide variety of processes. The initial application of this method is evaluated using rotor crack detection as the objective. In order to check whether the NARMAX approach can obtain both correct model terms and parameters for the underlying system, a developed cracked rotor model has been expanded from a differential equation to a difference equation representation. The study shows that the proposed approach provides a logical procedure for model order selection and nonlinear structure determination and high accuracy is achieved. The crack detection can then be obtained by comparing the resulting signatures with the crack free case. Also discussed in the paper is the selection of an appropriate operation status to obtain a good model fit that closely reflects the real system.

1. INTRODUCTION

For large complicated processes, it is often not possible to establish an accurate analytical system model, so it is therefore difficult to apply conventional model based damage detection approaches. As a result, data driven methodologies, such as the conventional time domain, frequency domain, time-frequency domain and polyspectra methods, play a key role in the fault detection field. The Nonlinear AutoRegressive Moving Average with eXogenous inputs (NARMAX) model, introduced in [2], provides a general parametric representation for a wide class of non-linear systems, such as: Volterra, Hammerstein, Wiener and Bilinear, for example. The NARMAX model describes a nonlinear system in terms of a group of lagged input and output terms, which admits a simple system description that is only based on measured input/output data. The work in [10] established a NARMAX model for composite materials, and proposed a damage detection algorithm based on the difference between the model

* This work is funded by Alstom Power Service and the support of Dr Mark Donne and Dr Andrew Pike are gratefully acknowledged.

parameters for the intact and damaged systems. Similarly, [7] used the parameter difference between the fault free and faulty cases to detect faults appearing in a silicon micro-accelerometer. Both of these works have illustrated the potential of using the NARMAX model representation for fault detection. As was the case in the authors' previous work [3] and [4], which studied the efficacy of a range of feature extraction methods for crack detection, the application of the NARMAX approach is initially focussed on rotor crack detection.

2. REVIEW OF THE NARMAX REPRESENTATION

The general form of the NARMAX model for an m outputs r inputs system can be expressed as a nonlinear recursive difference equation [2]:

$$y(t) = f(y(t-1),\cdots,y(t-n_y),u(t-1),\cdots,u(t-n_u),e(t-1),\cdots,e(t-n_e)) + e(t)\,, \tag{1}$$

where, $f(\bullet)$ is a vector valued nonlinear mapping for a system, which can be expressed in a polynomial, rational, wavelet or neural network form; $y(t)=[y_1(t),\cdots,y_m(t)]^T$, $u(t)=[u_1(t),\cdots,u_r(t)]^T$ and $e(t)=[e_1(t),\cdots,e_m(t)]^T$ are system output, input and prediction error respectively; n_y, n_u and n_e are associated maximum lags.

A special case of the NARMAX model is the NARX model, formed by removing the additive noise term; this can be defined as below when it is expressed in a polynomial form [5] and [9]:

$$\begin{aligned} y(t) &= f^P(y(t-1),\cdots,y(t-n_y),u(t-1),\cdots,u(t-n_u)) + e(t) \\ &= \sum_{m=0}^{l}\sum_{p=0}^{m}\sum_{n_1=1}^{n_y}\cdots\sum_{n_m=1}^{n_u} c_{p,m-p}(n_1,\cdots,n_m)\prod_{i=1}^{p} y(t-n_i)\prod_{i=p+1}^{p+q} u(t-n_i) + e(t) = \sum_{i=1}^{M}\theta_i p_i(x(t)) + e(t), \end{aligned} \tag{2}$$

where l is the maximum degree of the system nonlinearity; $M = p+q$ is the total number of the potential terms; θ_i are the coefficients; $p_i(x(t))$ are the candidate model terms of the form $\prod_{i=1}^{p} y(t-n_i)\prod_{i=p+1}^{p+q} u(t-n_i)$. To get the most representative approximation of a nonlinear system based on measured input/output data, the NARMAX identification approach generally involves three main steps: data pre-processing; term and variable selection; and model validation.

One of the main procedures, orthogonal least squares, is adopted for selecting the significant terms from the total M candidates, using Classical Gram-Schmidt, Modified Gram-Gchmidt and Householder transformation. The basic concept for the Classical Gram-Schmidt method is: At the first step, calculate ERR_i ($i=1,\cdots,M$), search through all M candidates of the model terms, find the one with the largest ERR value, denote this as the first term and remove from the candidate set. Following this, recalculate ERR_i ($i=1,\cdots,M-1$), search through the $M-1$ remaining candidates, and find the term with largest ERR. The terms are now ranked in the order of ERR and the most significant terms can be obtained by using a threshold.

The M in equation (2) could be very large for a nonlinear system, so selecting the significant model terms can be computationally expensive. A new algorithm was introduced in [8] to solve this problem whereby the linear and cross-bilinear models are used to pre-select the significant variables and the model terms are formed from these variables.

After the model terms have been selected and the associated parameters have been estimated, a model validation procedure is required to test whether the model is an adequate representation for the underlying dynamics of the system. There are four tests often adopted: One-step Ahead Prediction (OAP), Model Prediction Output (MPO), cross validation test, and residual correlation test. More theoretical details about the NARMAX approach can be found from various publications from Billings and co-workers.

3. CRACKED ROTOR MODEL

The cracked rotor model used is for a De Laval rotor with a disk of mass m supported by a mass-less elastic shaft of length L, and where it is assumed that the crack is located near the disk at the mid-span of the shaft, as shown in Figure 1. The model, equation 3, is developed based on the classical simple hinge model with some normalizations:

$$\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{pmatrix} \dfrac{x_{\tau\tau}}{x_0} \\ \dfrac{y_{\tau\tau}}{x_0} \end{pmatrix} + \begin{bmatrix} 2\zeta & \\ & 2\zeta \end{bmatrix}\begin{pmatrix} \dfrac{x_{\tau}}{x_0} \\ \dfrac{y_{\tau}}{x_0} \end{pmatrix} + \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}\begin{pmatrix} \dfrac{x}{x_0} \\ \dfrac{y}{x_0} \end{pmatrix}$$
$$-\frac{1}{2} f(\psi)\begin{bmatrix} \Delta k_{\xi/0}(1+\cos 2\Phi) + \Delta k_{\eta/0}(1-\cos 2\Phi) & \Delta k_{\xi/0}\sin 2\Phi - \Delta k_{\eta/0}\sin 2\Phi \\ \Delta k_{\xi/0}\sin 2\Phi - \Delta k_{\eta/0}\sin 2\Phi & \Delta k_{\xi/0}(1-\cos 2\Phi) + \Delta k_{\eta/0}(1+\cos 2\Phi) \end{bmatrix}\begin{pmatrix} \dfrac{x}{x_0} \\ \dfrac{y}{x_0} \end{pmatrix} \qquad (3)$$
$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \varepsilon \begin{bmatrix} a_r \sin\theta + (a_r \tau + \Omega_0)^2 \cos\theta \\ -a_r \cos\theta + (a_r \tau + \Omega_0)^2 \sin\theta \end{bmatrix},$$

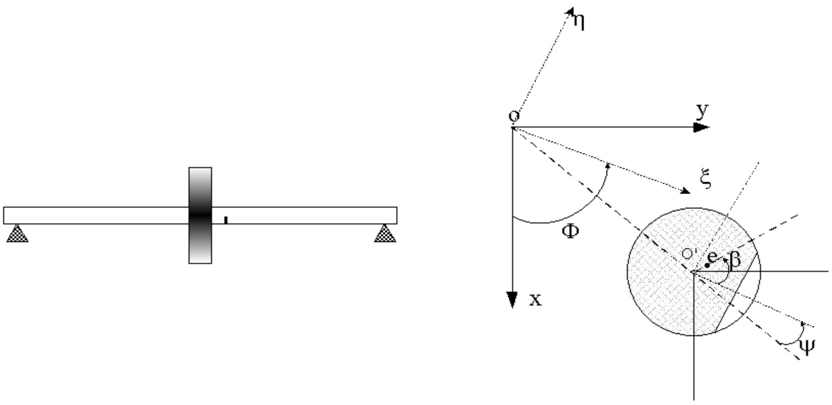

Figure 1 (left) crack position on the shaft; (right) the cross section of the cracked rotor, where $x-y$ are the stationary coordinates, $\xi-\eta$ are the rotating coordinates.

where the subscript and double subscript $\tau\tau$ denote the first and second order derivative with respect to τ, $\tau = w_n t$; $w_n = \sqrt{k/m}$ is the natural frequency; $x_0 = mg/k$ is the static deflection; $\zeta = d/2mw_n$ is the damping ratio; $\Delta k_{\xi/0} = \Delta k_\xi / k_0$ and $\Delta k_{\eta/0} = \Delta k_\eta / k_0$ is the relative stiffness variation in the ξ and η rotation directions respectively; $\varepsilon = e/x_0$ is the relative eccentricity; $\Omega_0 = w_0 / w_n$ is the relative initial angular speed; and $a_r = a/w_n^2$ is the relative angular acceleration. The function $f(\psi)$ denotes the breathing dynamics of the crack, which can be defined as:

$$f(\psi) = \begin{cases} \begin{cases} 0 & for\ \xi < 0 \\ 1 & for\ \xi > 0 \end{cases} = \dfrac{1}{2} + \dfrac{2}{\pi}\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}\cos(2n-1)\psi}{2n-1} & for\ a\ small\ crack, \\ \\ \dfrac{1+\cos\psi}{2} & for\ a\ big\ crack. \end{cases} \qquad (4)$$

$\psi = \Phi - \arctan(y/x)$ is the angle between the crack centreline and shaft displacement vector; $\Phi = \frac{1}{2}a_r\tau^2 + \Omega_0\tau + \Phi_0$ is the normalized self rotation angle of the shaft; the stiffness variation Δk_η can be expressed as:

$$\Delta k_\eta = \begin{cases} 0 & \textit{for a small crack,} \\ \Delta k_\xi / 6 & \textit{for a big crack.} \end{cases} \tag{5}$$

More detailed descriptions for the cracked rotor model can be found in references [1], [3], [4], [11] and [12].

4. FITTING A NARX MODEL WITH A CONSTANT OPERATING SPEED

For processing simplicity, the analytical expansion of the cracked rotor model into the NARX model format is obtained based on the following assumptions:

- ✧ The cracked rotor is weight dominant or $\psi = \Phi$, which makes the expansion much simpler by ignoring the $\arctan(y/x)$ component effect. This assumption is reasonable for large-scale rotating machines, such as gas/steam turbine rotor systems.
- ✧ The crack is large; this simplifies the crack breathing dynamic equation as shown in equation (4), where only one cosine function is needed rather than the series of cosine functions required for a small crack. However, the expansion procedure is very similar for a small crack.
- ✧ The cracked rotor runs at steady state, or $a_r = 0$ and $\Omega_0 \neq 0$.
- ✧ Due to the page limit, only the x-direction displacement is used as output and x-y vibration coupling is assumed to be small.

The dynamic equation for the ξ direction deflection can be extracted from equation (3) as:

$$\frac{x_{\tau\tau}}{x_0} + 2\zeta\frac{x_\tau}{x_0} + \frac{x}{x_0} - \frac{1}{2}f(\psi)\Delta k_{\xi/0}\left(\frac{7}{6} + \frac{5}{6}\cos 2\Phi\right)\frac{x}{x_0} = 1 + \varepsilon[a_r\sin\theta + (a_r\tau + \Omega_0)^2\cos\theta]\,. \tag{6}$$

Set $\sin\Phi = -\left(\frac{1}{a_r\tau + \Omega_0}\right)\frac{d\cos\Phi}{d\tau}$, then equation (6) can be modified to:

$$\ddot{y}_1 + 2\zeta\dot{y}_1 + y_1 - \frac{1}{4}\Delta k_{\xi/0}\left[\frac{1}{3}y_1 + \frac{1}{3}u_1 y_1 + \frac{5}{3}u_1^2 y_1 + \frac{5}{3}u_1^3 y_1\right] = 1 + \varepsilon\Omega_0^{\,2}\cos\beta\, u_1 + \varepsilon\Omega_0\sin\beta\,\dot{u}_1\,, \tag{7}$$

where $u_1 = \cos\Phi$; $y_1 = \frac{x}{x_0}$; $y_2 = \frac{y}{x_0}$; and $a_r = 0$. By using the Taylor series expansion, the above equation can be modified to the NARX form:

$$\begin{aligned} y_1(k) &= a_1y_1(k-1) + a_2y_1(k-2) + a_3u_1(k-2)y_1(k-1) + a_4u_1^2(k-2)y_1(k-1) \\ &\quad + a_5u_1^3(k-2)y_1(k-1) + a_6u_1(k-1) + a_7u_1(k-2) + a_8u_1(k-3) + a_9, \end{aligned} \tag{8}$$

where $a_1 = \frac{2 - \delta^2 + \frac{1}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_2 = -\frac{1-\zeta\delta}{1+\zeta\delta}$; $a_3 = \frac{\frac{1}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_4 = \frac{\frac{5}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_5 = \frac{\frac{5}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_6 = \frac{\frac{1}{2}\varepsilon\delta\Omega_0\sin\beta}{1+\zeta\delta}$; $a_7 = \frac{\varepsilon\delta^2\Omega_0^{\,2}\cos\beta}{1+\zeta\delta}$; $a_8 = -\frac{\frac{1}{2}\varepsilon\delta\Omega_0\sin\beta}{1+\zeta\delta}$; and $a_9 = \frac{\delta^2}{1+\zeta\delta}$.

The NARX model is fitted to data collected from a simulated steel rotor which comprises a 20mm diameter, 520mm length with a 1kg mass shaft and an 8kg mass disk located at mid-span. The dimensionless parameters used are: $\Omega_0 = 0.75$, $\varepsilon = 0.1$, $\beta = 0$, $\zeta = 0.05$ and $\Delta k_{\xi/0}$ = 0, 0.1, 0.3, 0.5 (representing different crack depths), respectively. The dynamic response, the displacement, is simulated by using a fourth order Runge-Kutta algorithm and with a fixed time step $\delta = 0.2$ and 60dB SNR white noise is added into each data set. A data section with 20000 samples is obtained for each crack depth and all sets are divided into two independent parts, one for identification and the other for model validation.

Normally, the maximum lags n_y, n_u and system nonlinearity degree l are assumed unknown, which is realistic for a real complex physical system. A searching strategy for the purpose of finding a reasonable model structure has also been studied by the authors, but due to the page limit this is not presented here. General guidelines for such a search strategy can be found in [8] and [9]. Here a NARX model was fitted to the simulated data sets by assuming that the system nonlinearity and lags for both input and output are known a *priori*. A model structure extracted from equation (8), ($n_{y_1} = 2$, $n_{u_1} = 3$, $l = 4$), is used to fit the data obtained when $\Delta k_{\xi/0} = 0.3$. The results in Figure 2 indicate that the model predicted outputs follow the original output well with a difference barely observable; the correlation tests are passed with the residual limited within the 95% confidence bands, and the estimated model also passes the cross validation test that has a similar form to the first plot in Figure 2. The selected model terms and the estimated parameters are listed in Table 1, which are however quite different from the original.

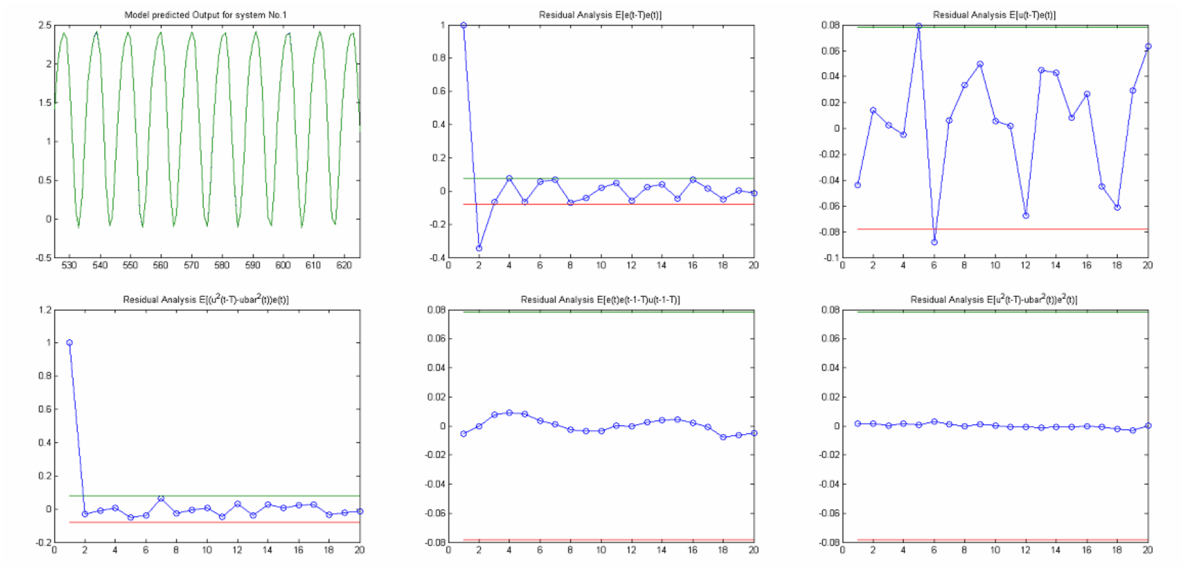

Figure 2 The results for the model predicted output and correlation tests.

Table 1 Identified model structures with degree of four for cracked case, $\Delta k_{\xi/0} = 0.3$.

Selected Terms	Parameters		E.R.R.S
	True	Estimates	
$y_1(k-1)$	1.9416	0.028117	0.99734
$y_1(k-2)$	-0.9802	0.12367	0.0025945
constant	0.0396	1.028	5.6715e-005
$y_1(k-1)y_1(k-1)u_1(k-1)u_1(k-2)$		0.098018	1.1728e-005
$y_1(k-2)u_1(k-1)u_1(k-1)$		-0.060589	6.9103e-008
$u_1(k-1)u_1(k-1)u_1(k-1)u_1(k-1)$		-0.0045132	2.826e-008
$y_1(k-2)u_1(k-2)u_1(k-2)u_1(k-2)$		0.034735	1.5126e-009

$y_1(k-2)y_1(k-2)y_1(k-2)u_1(k-1)$		0.010815	2.6561e-009
$y_1(k-1)u_1(k-2)u_1(k-2)u_1(k-2)$		-0.031127	3.4883e-010
$u_1(k-1)$	0	0.18119	1.7709e-008
$u_1(k-2)u_1(k-3)u_1(k-3)$		0.018228	1.0932e-008
$u_1(k-3)$	0	0.16498	4.3523e-009
$y_1(k-1)u_1(k-2)$		0.11403	2.4669e-010
$u_1(k-2)u_1(k-2)$		-0.14733	2.1748e-010
$y_1(k-1)y_1(k-2)u_1(k-1)u_1(k-1)$		-0.10336	4.6181e-010

Based on the above, it is clear that although the fitted NARX model contains a quite different model structure, the estimated model still captures the dynamics of the original system well for this operating point. From a system identification point of view, it is possible to represent a simple system with different estimated model structures. However, in this case, the poor parameter estimation accuracy is caused by using naturally occurring excitation which can be considered to be due to a bad "design" for the input signal [6,9]. Since the input signal, $u_1 = \cos(\frac{1}{2}a_r\tau^2 + \Omega_0\tau + \Phi_0)$, contains only a single frequency it is clearly not able to excite all of the underlying system dynamics, and therefore causes the poor identification.

5. FITTING A NARX MODEL WITH A CONSTANT ACCELERATION

It is widely accepted that it is easier to carry out fault detection during the run-up or coast down stages of operation rather than at steady state. In this section the feasibility of improving the NARX model fit using data collected from when the rotor runs up or runs down, or more generally experiences constant acceleration operation, is investigated.

All the parameters are set the same as in last section, except here a constant acceleration $a = a_1 \neq 0$ is assumed, and two new inputs: $u_2 = a_r k\delta + \Omega_0$; and $u_3 = \dfrac{1}{a_r k\delta + \Omega_0}$ are introduced.

Then equation (6) can be rewritten as:

$$\begin{aligned} y_1(k) &= a_1y_1(k-1) + a_2y_1(k-2) + a_3u_1(k-2)y_1(k-1) + a_4u_1^2(k-2)y_1(k-1) + a_5u_1^3(k-2)y_1(k-1) \\ &+ a_6u_2(k-1)u_1(k-2) + a_7u_2(k-3)u_1(k-2) + a_8u_2^2(k-2)u_1(k-2) + a_9u_2(k-2)u_1(k-1) \\ &+ a_{10}u_2(k-1)u_3(k-1)u_1(k-1) + a_{11}u_2(k-3)u_3(k-1)u_1(k-1) + a_{12}u_2(k-2)u_1(k-3) \\ &+ a_{13}u_2(k-1)u_3(k-1)u_1(k-3) + a_{14}u_2(k-3)u_3(k-1)u_1(k-3) + a_{15}, \end{aligned} \tag{9}$$

where $a_1 = \dfrac{2-\delta^2+\frac{1}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_2 = -\dfrac{1-\zeta\delta}{1+\zeta\delta}$; $a_3 = \dfrac{\frac{1}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_4 = \dfrac{\frac{5}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_5 = \dfrac{\frac{5}{12}\delta^2\Delta k_{\xi/0}}{1+\zeta\delta}$; $a_6 = \dfrac{\frac{1}{2}\varepsilon\delta\sin\beta}{1+\zeta\delta}$; $a_7 = -\dfrac{\frac{1}{2}\varepsilon\delta\sin\beta}{1+\zeta\delta}$; $a_8 = \dfrac{\varepsilon\delta^2\cos\beta}{1+\zeta\delta}$; $a_9 = \dfrac{\frac{1}{2}\varepsilon\delta\sin\beta}{1+\zeta\delta}$; $a_{10} = -\dfrac{\frac{1}{4}\varepsilon\cos\beta}{1+\zeta\delta}$; $a_{11} = \dfrac{\frac{1}{4}\varepsilon\cos\beta}{1+\zeta\delta}$; $a_{12} = -\dfrac{\frac{1}{2}\varepsilon\delta\sin\beta}{1+\zeta\delta}$; $a_{13} = \dfrac{\frac{1}{4}\varepsilon\cos\beta}{1+\zeta\delta}$; $a_{14} = -\dfrac{\frac{1}{4}\varepsilon\cos\beta}{1+\zeta\delta}$; and $a_{15} = \dfrac{\delta^2}{1+\zeta\delta}$.

1000 samples, obtained from when the rotor runs up $a = 0.001$ and passes through the 1/2 sub-harmonic range, are used for NARX model fitting. Table 2 is the identified structure obtained using ($n_{y_1} = 2$, $n_{u_1} = 3$, $n_{u_2} = 3$, $n_{u_3} = 1$ and $l = 4$). The results indicate that the NARX model fitting correctly selects the first six terms in the original equation (9), and five of them are selected with the maximum E.R.R.S. Furthermore, the estimated parameters are very close to the true values, especially the first three terms.

Table. 2 Identified model structures with 4th order degree for cracked case, $\Delta k_{\xi/0} = 0.3$

Selected Terms	Parameters		E.R.R.S
	True	Estimates	
$y_1(k-1)$	1.9416	1.9417	0.99628
$y_1(k-2)$	-0.9802	-0.9802	0.0035949
constant	0.0396	0.039473	0.00010422
$y_1(k-1)u_1(k-2)u_1(k-2)u_1(k-2)$	0.0050	0.0075494	1.5394e-005
$y_1(k-1)u_1(k-2)u_1(k-2)$	0.0050	0.0048913	1.7984e-006
$u_1(k-3)u_2(k-1)u_2(k-1)u_3(k-1)$		0.00028003	1.9514e-007
$y_1(k-1)u_1(k-1)u_2(k-1)$		0.0001418	2.3746e-008
$y_1(k-2)y_1(k-2)u_1(k-2)u_3(k-1)$		6.988e-006	6.1711e-010
$u_1(k-3)u_1(k-3)u_3(k-1)u_3(k-1)$		7.211e-006	9.7403e-011
$y_1(k-2)u_1(k-1)u_1(k-1)u_1(k-2)$		0.0023307	5.8203e-011
$u_1(k-2)u_2(k-3)u_2(k-3)$		0.0035105	2.2936e-011
$y_1(k-1)u_1(k-2)$	0.0010	0.00087349	1.1582e-010
$y_1(k-1)u_1(k-1)u_1(k-3)u_1(k-3)$		-0.0026643	2.8606e-012
$y_1(k-2)u_1(k-1)u_1(k-3)u_1(k-3)$		0.0024446	3.9363e-012
$y_1(k-2)u_1(k-1)u_1(k-2)u_1(k-2)$		-0.0047332	2.9491e-012

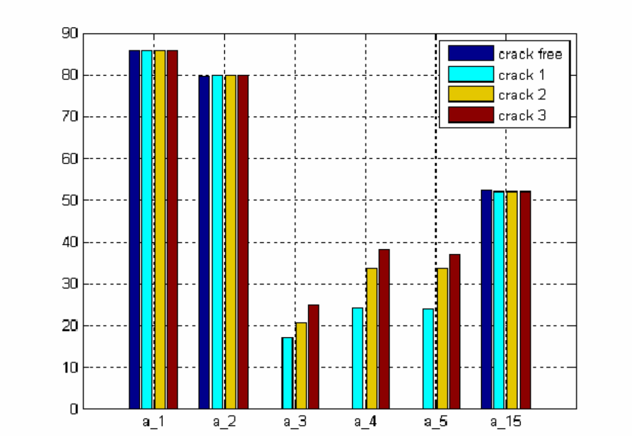

Figure 3 Comparison between the estimated parameters for different crack depths in normalised format.

The above analysis shows that it is possible to improve the model term selection and parameter estimation accuracy by changing the data set from one obtained during a steady state process to one from a constant run up process. An interesting phenomena may be observed, which is: by setting $a = a_1 \neq 0$, all the terms, which have combination with output y(t) in the original equation, have been successfully selected, but all the terms with only input u(t) are not selected. This is mainly because the designed inputs are correlated. Comparing with the coefficients and terms in equation (9), it can be seen that all the terms and parameters containing the crack dynamic $\Delta k_{\xi/0}$ are correctly selected and estimated. The estimated results can be used therefore as an indication of the crack severity. The accurately estimated parameters for different crack depths are compared in a normalised format in Figure 3, where the crack presence and subsequent increase are easily observable.

6. CONCLUSIONS

The NARMAX modelling approach has previously been proved to provide excellent representations for nonlinear system dynamics in the time domain. As a result this may lead to much better performance than traditional fault detection approaches. Here the application of the approach has been evaluated using rotor crack detection as the objective. In order to determine whether the NARX approach can obtain correct model variables and terms for the underlying system, a cracked rotor model has been expanded from a differential to a difference equation representation. The paper has covered two methods for improving the NARX model accuracy, with the intention of increasing crack detection sensitivity. Although the estimated model provides a good representation of the data, in the sense that the model predicted output closely follows the original output and both cross validation and residual correlation tests are passed, the fitted NARX model structure for the data collected from steady state operation does not match the original. As a result crack detection cannot be carried out based solely on the poorly selected terms. Therefore, a NARX model fit to data collected from a rotor undergoing a constant run-up process has been studied. The investigation shows that this does improve the accuracy of the model fit; six terms are selected correctly and the resulting parameter estimation is very accurate. Based on the accurately selected model terms and the estimated parameters, both the presence of a crack and its severity can be determined.

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