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DYNAMIC STRUCTURE MODIFICATION USING DENTS

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Abstract

In this paper, a design method to modify the vibration characteristics of a structure by creating dents, or dimples on its surface is investigated. In particular, the vibration response of a beam with several dimples is formulated using the impedance method. The dimpled beam is divided into two kinds of structural segments: one, a curved beam that is modeled as the dimple and the other, a straight beam. The frequency equation is derived by assembling the impedance of each structure segment based on conditions of force equilibrium and velocity compatibility. Then a novel method for shifting the natural frequencies of a structure to pre-assigned values by creating dimples on the structure is introduced. The dimple size and its location on the structure can be determined analytically so the time consuming process using the traditional optimal search method is thereby avoided. Several examples using this technique are demonstrated.

1. INTRODUCTION

Moving the natural frequencies of a structure away from the frequency range of an excitation is often a basic requirement in a design process. The natural frequencies of a structure can be altered through changes of the structural geometry, boundary conditions and/or the addition of auxiliary structures, such as masses, ribs, etc. [1-3]. Most studies related to shifting the natural frequency away from the forcing frequency for a structure are based on optimization methods. Here, the cost function, *e.g.*, a designated natural frequency, is minimized or maximized in an optimization algorithm. In this paper, the natural frequencies of a structure are changed to designated values by forming a series of dimples on it. The motivation for this is simply that forming dimples on a structure is a very cost-effective procedure during a manufacturing process. The dimple on a structure mainly influences its structural stiffness and hence its dynamic characteristics. The key issue that needs to be addressed is how to determine the required dimple size, number and location on the structure so the natural frequency can be shifted to the desired value. A methodology to do so based on the impedance technique is proposed in this paper.

The impedance technique [3] provides a useful method in investigating the vibration of one portion of a mechanical system independently with respect to the rest of the system. The mechanical receptance for each sub-structure is obtained first and then these receptances are integrated to determine the response of the structure after an assembly based on the conditions of force equilibrium and response compatibility. The idea of breaking up a complicated system into substructures with simple dynamic characteristics is well-known (see *e.g.*, Bishop and Johnson [4]). The traditional transfer-matrix method presented by Pestel and Leckie adopted a similar methodology [5]. These methods are ideally suited to a system with a number of sub-structures linked together in the form of a chain. The impedance method specifically aims for solving the dynamical system from the point of view of "system". The input-output relation for a dynamic system is explicitly expressed using the transfer function, called mobility. The advantage is simply that the mobility of a substructure or its inverse, the impedance can be obtained experimentally (*e.g.*, impact testing) or numerically (*e.g.*, finite element methods) when analytical models are not available due to either its irregular shape or complicated boundary conditions. In this paper, the dimpled structure is divided into two substructures: one is the dimple and the other is the straight beam. Each dimple is modeled as a curved beam and its impedance is obtained with the finite element model. The impedance of the straight beam is obtained analytically. An impedance coupling technique is used to assemble the substructures into the dimpled beam. Then a logical scheme is introduced to show how the dimple size and location can be chosen to shift the natural frequency of a beam.

2. IMPEDANCE COUPLING

Consider a beam with several dimples as shown in Fig. 1(a), where x_i, ϕ_i and R_i are the position, angle and radius for the i^{th} dimple, respectively. This dimpled beam is divided into curved and straight structural segments. Each segment is a structural subsystem and is represented by a block diagram as shown in Fig. 1(b). The link or coupling between two subsystems requires force equilibrium and velocity compatibility. For example, the vibration response of Subsystem 1 as sketched in Fig. 1(c) can be expressed as:

$$\begin{Bmatrix} \dot{\mathbf{X}}_{b1} \\ \dot{\mathbf{X}}_{b2} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\beta}_{11} & \boldsymbol{\beta}_{12} \\ \boldsymbol{\beta}_{21} & \boldsymbol{\beta}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_{b1} \\ \mathbf{F}_{b2} \end{Bmatrix}, \tag{1}$$

where $\dot{\mathbf{X}}_{b1}=\{\dot{x}_{b1}, \dot{x}_{b2}, \dot{x}_{b3}\}^{\mathrm{T}}$ represents the velocity vector that consists of the longitudinal, transverse and angular velocities, respectively; $\mathbf{F}_{b1}=\{f_{b1}, f_{b2}, f_{b3}\}^{\mathrm{T}}$ is the force vector that includes horizontal and vertical forces and a moment. Similarly, $\dot{\mathbf{X}}_{b2}=\{\dot{x}_{b4}, \dot{x}_{b5}, \dot{x}_{b6}\}^{\mathrm{T}}$ and $\mathbf{F}_{b2}=\{f_{b4}, f_{b5}, f_{b6}\}^{\mathrm{T}}$ represent the velocity and force vectors that connect with Subsystem 2. The transfer function is represented by the cross mobility, $\boldsymbol{\beta}_{12}$ and $\boldsymbol{\beta}_{21}$ and the driving point mobility, $\boldsymbol{\beta}_{11}$ and $\boldsymbol{\beta}_{22}$, respectively. For Subsystem 2 which is linked respectively with Subsystems 1 and 3, the vibration response is expressed as:

$$\begin{Bmatrix} \dot{\mathbf{X}}_{c1} \\ \dot{\mathbf{X}}_{c2} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\gamma}_{11} & \boldsymbol{\gamma}_{12} \\ \boldsymbol{\gamma}_{21} & \boldsymbol{\gamma}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_{c1} \\ \mathbf{F}_{c2} \end{Bmatrix}, \tag{2}$$

where $\dot{\mathbf{X}}_{c1}=\{\dot{x}_{c1}, \dot{x}_{c2}, \dot{x}_{c3}\}^{\mathrm{T}}$, $\dot{\mathbf{X}}_{c2}=\{\dot{x}_{c4}, \dot{x}_{c5}, \dot{x}_{c6}\}^{\mathrm{T}}$, and $\mathbf{F}_{c1}=\{f_{c1}, f_{c2}, f_{c3}\}^{\mathrm{T}}$, $\mathbf{F}_{c2}=\{f_{c4}, f_{c5}, f_{c6}\}^{\mathrm{T}}$. Based on the conditions required by the velocity compatibility and force equilibrium, Subsystem 1 with Subsystem 2 can be combined into a coupled Subsystem 1-2 as:

$$\dot{\mathbf{X}}_{d} = \boldsymbol{\alpha}_{(1-2)}\mathbf{F}_{d}, \tag{3a}$$

where $\dot{\mathbf{X}}_{d}=\{\dot{x}_{1}, \dot{x}_{2}, \dot{x}_{3}, \dot{x}_{7}, \dot{x}_{8}, \dot{x}_{9}\}^{\mathrm{T}}$, $\mathbf{F}_{d}=\{f_1, f_2, f_3, f_7, f_8, f_9\}^{\mathrm{T}}$, and $\boldsymbol{\alpha}_{(1\text{-}2)}$ denotes the mobility of Subsystem 1-2. The input-output relation of Subsystem 1-2 can be rewritten as:

$$\begin{Bmatrix} \dot{\mathbf{X}}_{d1} \\ \dot{\mathbf{X}}_{d2} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_{(1-2)_{11}} & \boldsymbol{\alpha}_{(1-2)_{12}} \\ \boldsymbol{\alpha}_{(1-2)_{21}} & \boldsymbol{\alpha}_{(1-2)_{22}} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_{d1} \\ \mathbf{F}_{d2} \end{Bmatrix}, \tag{3b}$$

where $\dot{\mathbf{X}}_{d1}=\{\dot{x}_1, \dot{x}_2, \dot{x}_3\}^T$, $\dot{\mathbf{X}}_{d2}=\{\dot{x}_7, \dot{x}_8, \dot{x}_9\}^T$, $\mathbf{F}_{d1}=\{f_1, f_2, f_3\}^T$, $\mathbf{F}_{d2}=\{f_7, f_8, f_9\}^T$, $\boldsymbol{\alpha}_{(1-2)_{11}}=\beta_{12}\beta_{22}^{-1}(\gamma_{11}^{-1}+\beta_{22}^{-1})^{-1}\beta_{22}^{-1}\beta_{21}+\beta_{11}-\beta_{12}\beta_{22}^{-1}\beta_{21}$, $\boldsymbol{\alpha}_{(1-2)_{12}}=\beta_{12}\beta_{22}^{-1}(\gamma_{11}^{-1}+\beta_{22}^{-1})^{-1}\gamma_{11}^{-1}\gamma_{12}$, $\boldsymbol{\alpha}_{(1-2)_{21}}=\gamma_{21}\gamma_{11}^{-1}(\gamma_{11}^{-1}+\beta_{22}^{-1})^{-1}\beta_{22}^{-1}\beta_{21}$, and $\boldsymbol{\alpha}_{(1-2)_{22}}=\gamma_{21}\gamma_{11}^{-1}(\gamma_{11}^{-1}+\beta_{22}^{-1})^{-1}\gamma_{11}^{-1}\gamma_{12}+\gamma_{22}-\gamma_{21}\gamma_{11}^{-1}\gamma_{12}$. By replacing β with $\alpha_{(1-2)}$ and γ with the mobility δ of Subsystem 3, the mobility $\alpha_{(1-3)}$ that combines three subsystems, i.e. Subsystem 1-3, is obtained:

$$\begin{Bmatrix} \dot{\mathbf{X}}_{e1} \\ \dot{\mathbf{X}}_{e2} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_{(1-3)_{11}} & \boldsymbol{\alpha}_{(1-3)_{12}} \\ \boldsymbol{\alpha}_{(1-3)_{21}} & \boldsymbol{\alpha}_{(1-3)_{22}} \end{bmatrix} \begin{Bmatrix} \mathbf{F}_{e1} \\ \mathbf{F}_{e2} \end{Bmatrix}, \tag{4}$$

where $\dot{\mathbf{X}}_{e1}=\{\dot{x}_1, \dot{x}_2, \dot{x}_3\}^T$, $\dot{\mathbf{X}}_{e2}=\{\dot{x}_{10}, \dot{x}_{11}, \dot{x}_{12}\}^T$, $\mathbf{F}_{e1}=\{f_1, f_2, f_3\}^T$, $\mathbf{F}_{e2}=\{f_{10}, f_{11}, f_{12}\}^T$, $\boldsymbol{\alpha}_{(1-3)_{11}}=\boldsymbol{\alpha}_{(1-2)_{12}}\boldsymbol{\alpha}_{(1-2)_{22}}^{-1}(\delta_{11}^{-1}+\boldsymbol{\alpha}_{(1-2)_{22}}^{-1})^{-1}\boldsymbol{\alpha}_{(1-2)_{22}}^{-1}\boldsymbol{\alpha}_{(1-2)_{21}}+\boldsymbol{\alpha}_{(1-2)_{11}}-\boldsymbol{\alpha}_{(1-2)_{12}}\boldsymbol{\alpha}_{(1-2)_{22}}^{-1}\boldsymbol{\alpha}_{(1-2)_{21}}$, $\boldsymbol{\alpha}_{(1-3)_{12}}=\boldsymbol{\alpha}_{(1-2)_{12}}\boldsymbol{\alpha}_{(1-2)_{22}}^{-1}(\delta_{11}^{-1}+\boldsymbol{\alpha}_{(1-2)_{22}}^{-1})^{-1}\delta_{11}^{-1}\delta_{12}$, $\boldsymbol{\alpha}_{(1-3)_{21}}=\delta_{21}\delta_{11}^{-1}(\delta_{11}^{-1}+\boldsymbol{\alpha}_{(1-2)_{22}}^{-1})^{-1}\boldsymbol{\alpha}_{(1-2)_{22}}^{-1}\boldsymbol{\alpha}_{(1-2)_{21}}$, and $\boldsymbol{\alpha}_{(1-3)_{22}}=\delta_{21}\delta_{11}^{-1}(\delta_{11}^{-1}+\boldsymbol{\alpha}_{(1-2)_{22}}^{-1})^{-1}\delta_{11}^{-1}\delta_{12}+\delta_{22}-\delta_{21}\delta_{11}^{-1}\delta_{12}$. The procedure is repeated until the first M subsystems are combined into a single system and the mobility is simply denoted as $\alpha_{(1-M)}$. Therefore for a system consisting of a number of M subsystems as shown in Fig. 1(b), the input-output relationship is simply denoted as:

$$\dot{\mathbf{X}}_M=\alpha_{(1-M)}\mathbf{F}_M, \tag{5}$$

where $\dot{\mathbf{X}}_M=\{\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{x}_{(3M+1)}, \dot{x}_{(3M+2)}, \dot{x}_{(3M+3)}\}^T$, $\mathbf{F}_M=\{f_1, f_2, f_3, f_{(3M+1)}, f_{(3M+2)}, f_{(3M+3)}\}^T$. The corresponding frequency equation is given by

$$\Delta=\det(\alpha_{(1-M)}). \tag{6}$$

It is obvious that the dimpled beam exhibits resonance if the determinant Δ approaches zero. From equations (3) and (4), the natural frequency of the dimpled beam depends on the mobility of each subsystem, which includes the dimple size, location, etc. which will be described in the following section.

3. MOBILITY OF SUBSYSTEM

The dimpled beam consists of straight and curved beam segments, and the mobility corresponding to each segment is derived in this section.

3.1 Mobility of Straight Beam

For a straight beam with free-free ends as shown in Fig. , six coordinates, a longitudinal, a transverse and an angular velocities for each end, are required to describe the link with the other structural subsystems. The mobility which represents the relation between the vibration response and the external excitation can be expressed as:

$$\dot{\mathbf{X}}_{(B1)}=\alpha_{(B1)}\mathbf{F}_{(B1)}, \tag{7}$$

where $\dot{\mathbf{X}}_{(B1)}=\{\dot{x}_{B1}, \dot{x}_{B2}, \dot{x}_{B3}, \dot{x}_{B4}, \dot{x}_{B5}, \dot{x}_{B6}\}^T$, $\mathbf{F}_{(B1)}=\{f_{B1}, f_{B2}, f_{B3}, f_{B4}, f_{B5}, f_{B6}\}^T$ and the structural mobility $\alpha_{(B1)}$ is given by [4]:

$$\boldsymbol{\alpha}_{(B1)} = j\omega \begin{bmatrix} -\frac{\cot(\mu_1 L_b)}{E_b A_b \mu_1} & 0 & 0 & -\frac{\csc(\mu_1 L_b)}{E_b A_b \mu_1} & 0 & 0 \\ 0 & -\frac{C_5}{E_b I_b \mu_2^3 C_3} & -\frac{C_1}{E_b I_b \mu_2^2 C_3} & 0 & \frac{C_8}{E_b I_b \mu_2^3 C_3} & \frac{C_{10}}{E_b I_b \mu_2^2 C_3} \\ 0 & -\frac{C_1}{E_b I_b \mu_2^2 C_3} & \frac{C_6}{E_b I_b \mu_2 C_3} & 0 & -\frac{C_{10}}{E_b I_b \mu_2^2 C_3} & \frac{C_7}{E_b I_b \mu_2 C_3} \\ -\frac{\csc(\mu_1 L_b)}{E_b A_b \mu_1} & 0 & 0 & -\frac{\cot(\mu_1 L_b)}{E_b A_b \mu_1} & 0 & 0 \\ 0 & \frac{C_8}{E_b I_b \mu_2^3 C_3} & -\frac{C_{10}}{E_b I_b \mu_2^2 C_3} & 0 & -\frac{C_5}{E_b I_b \mu_2^3 C_3} & \frac{C_1}{E_b I_b \mu_2^2 C_3} \\ 0 & \frac{C_{10}}{E_b I_b \mu_2^2 C_3} & \frac{C_7}{E_b I_b \mu_2 C_3} & 0 & \frac{C_1}{E_b I_b \mu_2^2 C_3} & \frac{C_6}{E_b I_b \mu_2 C_3} \end{bmatrix}, \tag{8}$$

where ω the force frequency, E_b the Young's modulus, I_b the moment of inertia, L_b the length of straight beam, ρ_b the density, A_b is the cross-sectional area of beam and C_1=sin($\mu_2 L_b$)sinh($\mu_2 L_b$), C_2=cos($\mu_2 L_b$)cosh($\mu_2 L_b$), C_3=cos($\mu_2 L_b$)cosh($\mu_2 L_b$)-1, C_5=cos($\mu_2 L_b$)sinh($\mu_2 L_b$)-sin($\mu_2 L_b$) cosh($\mu_2 L_b$), C_6=cos($\mu_2 L_b$)sinh($\mu_2 L_b$)+sin($\mu_2 L_b$)cosh($\mu_2 L_b$), C_7=sin($\mu_2 L_b$)+sinh($\mu_2 L_b$), C_8= sin($\mu_2 L_b$)-sinh($\mu_2 L_b$), C_{10}=cos($\mu_2 L_b$)-cosh($\mu_2 L_b$), $\mu_1^2=(\omega^2\rho_b)/E_b$, $\mu_2^4 = (\omega^2 \rho_b A_b)/(E_b I_b)$.

3.2 Mobility of Dimple

The dimple on a beam is modeled as a curved beam with a dimple angle ϕ and a radius R as illustrated in Fig. 3. The equation of motion for this curved beam is derived using the finite element method[6]. And the nodal velocity response can be expressed as:

$$\dot{\mathbf{u}}_g = \mathbf{HF}, \tag{9}$$

where $\dot{\mathbf{u}}_g$ is the nodal velocity, $\mathbf{F}$ is the harmonic nodal force and $\mathbf{H}=(\mathbf{K}-\omega^2\mathbf{M})^{-1}/(j\omega)$ is the mobility. For a curved beam with free-free ends and the external force only acting on its ends, Eq. (9) is simplified as:

$$\dot{\mathbf{X}}_{(D1)} = \begin{bmatrix} \boldsymbol{\alpha}_{(D1)11} & \boldsymbol{\alpha}_{(D1)12} \\ \boldsymbol{\alpha}_{(D1)21} & \boldsymbol{\alpha}_{(D1)22} \end{bmatrix} \mathbf{F}_{(D1)}, \tag{10}$$

where $\dot{\mathbf{X}}_{(D1)}=\{\dot{x}_{D1}, \dot{x}_{D2}, \dot{x}_{D3}, \dot{x}_{D4}, \dot{x}_{D5}, \dot{x}_{D6}\}^T$ represents the velocity of both ends , $\mathbf{F}_{(D1)}$= $\{f_{D1}, f_{D2}, f_{D3}, f_{D4}, f_{D5}, f_{D6}\}^T$ is the vector that stands for the external force acting on the both ends, and $\boldsymbol{\alpha}_{(D1)}$ is the mobility.

5. INFLUENCES OF DIMPLES ON NATURAL FREQUENCY OF BEAM

Consider a simply-supported beam with a Young's modulus E=1.89×10^{11} Pa, density ρ=7688kg/m^3, width b=0.025m, thickness h=0.001m and a length L=0.3m. A single dimple is created on it but the total mass of this beam is the same before and after the beam is dimpled, *i.e*. the dimple is thinner than the other parts of the beam. While the dimple does not change the mass of the beam, it does affect the stiffness of the beam. Thus the natural frequency of the beam can be altered by varying the dimple size and dimple location. For a beam with a single dimple whose chord is a tenth of the beam length as illustrated in Fig. 5, Figure 6 shows the change of the first natural frequency in percentage by varying the dimple angle, position and radius. In Fig. 6(a) the dimple angle varies from 0 to 180 degrees, whereas the location of this single dimple is changed from one end of the beam to the middle, i.e. from x/L=0 to x/L=0.5. As compared to a beam without a dimple, the natural frequency change in percentage is represented using gray spectrum. It is not surprising that the single dimple has its maximal influence on the first natural frequency when it is located on the middle of the beam. Furthermore, the first natural frequency is highly sensitive to the change of the dimple angle as compared to that of the dimple location if the dimple is near the anti-node of the beam. For a single dimple located on the middle of the beam, x/L=0.5, Figure 6(b) shows the influences of the dimple size, R/L and the dimple angle on the first natural frequency of the beam. It is obvious that one may use a small dimple but with a large dimple angle to have the same influence on the natural frequency of the beam as a large dimple but with a smaller dimple angle. The dimple size has little effect in changing the natural frequency when the dimple angle is small. Figures 6(a) and 6(b), show in general that the first natural frequency for a beam with a dimple is smaller than that of the beam without a dimple. The reason is simply that the dimple is thinner than the other parts of the beam; therefore, its bending stiffness is smaller than the beam.

6. NATURAL FREQUENCY TUNING OF A BEAM WITH THE ADDITION OF DIMPLES

When considering the dependency of the natural frequency of a dimpled beam on the number, angle and location of the dimples, it is evident from figures 6(a) and 6(b) that the dimple angle that has the major influence in altering the natural frequency. For a beam with a number of K dimples, the frequency equation is expressed as:

$$\Delta = f(\phi_1, \phi_2, \ldots, \phi_K, \omega_f), \tag{11}$$

where ϕ_K is the angle of the K^{th} dimple, ω_f is the excitation frequency of external loads (note that the radius of each dimple is the same for simplicity). If one needs to adjust multiple natural frequencies of a beam simultaneously by adding K dimples, equation (11) is modified as:

$$\Delta_1(\phi_1, \phi_2, \ldots, \phi_K, \omega_1)=0,\ \Delta_2(\phi_1, \phi_2, \ldots, \phi_K, \omega_2)=0,\ \ldots,\ \Delta_N(\phi_1, \phi_2, \ldots, \phi_K, \omega_N)=0, \tag{12}$$

where N is the total number of natural frequencies to be tuned. The solution of equation (12) yields the angle of each dimple required to shift N natural frequencies simultaneously to the designated values. If only one natural frequency is chosen to be changed to a designated value, *e.g.*, the m^{th} natural frequency ω_m, the dimple angle is determined using one of the equations in Eq. (12), *e.g.*, $\Delta_m(\phi_1, \phi_2, \ldots, \phi_K, \omega_m)=0$. The dimple angle is obviously different if the dimple is added at different locations on the beam. When one needs to simultaneously shift N natural frequencies simultaneously to the designated values, the dimple angle for each dimple, ϕ_1, ϕ_2, …, ϕ_K is determined uniquely for K=N. This implies that simultaneously shifting the natural frequencies to the number of N requires at least a number of N dimples; however, there would be infinite solutions if K>N.

To illustrate the applications of adding dimples on a beam in order to tune its dynamic characteristic, three examples are presented in this section. The first example demonstrates how to use a single dimple to adjust the first natural frequency of a beam to a designated value. The second example shows how to use three dimples of the same size to change the 3rd natural frequency to a designated value while the third example involves simultaneously shifting two natural frequencies to designated values using two dimples. The beam demonstrated in these examples is simply-supported at both ends and has a Young's modulus E=1.89×10^{11}Pa, density ρ=7688kg/m^3, width b=0.025m, thickness h=0.001m and length L=0.3m.

Example 1: Use a single dimple to reduce the fundamental natural frequency of a simply supported beam from ω_1 to 0.85ω_1, i.e. 15% reduction, where ω_1 is the fundamental natural frequency of the beam without the dimple.

Assume that the beam is divided into eleven segments of equal length and each segment is a candidate to be dimpled to shift the first natural frequency from ω_1 to $0.85\omega_1$. Notice that the total mass of the beam is the same before and after a single segment is dimpled as stated previously. According to the methodology of natural frequency synthesis introduced in the previous section, the angle for each dimple can be determined as:

$$\Delta(\phi_1, 0.85\omega_1)=0. \tag{13}$$

Table 1 lists the dimple angle for each segment calculated using the proposed methodology. It shows that one may use a single dimple at Segment 6 with a dimple angle of 145°, or a dimple at Segment 8 of 159° to accomplish the task. It also shows that when forming a single dimple at segments 1, 2, 10 or 11, is impossible to change the natural frequency to the designated value. Moreover, the dimple angle becomes smaller as the dimple is located near the middle of beam. It simply reveals that the dimple at the anti-node of a mode could efficiently influence the corresponding natural frequency. On the other hand, the natural frequency is not sensitive to the dimple if it is located near the node of the corresponding mode, *e.g.*, the segments 1, 2, 10 and 11 in this example.

Example 2: Use three dimples of the same size to reduce 3rd natural frequency from ω_3 to $0.9\omega_3$, where ω_3 is the 3rd natural frequency of the beam without dimples.

This example demonstrates how to change the natural frequency to a designated value by forming multiple dimples on a beam. Assume three dimples are placed at respective segments of the simply supported beam as sketched in Fig. 7 and the frequency equation required to shift the 3rd natural frequency ω_3 to $0.9\omega_3$ is

$$\Delta(\phi_1, \phi_2, \phi_3, 0.9\omega_3)=0, \text{ where } \phi_1=\phi_2=\phi_3. \tag{14}$$

The dimple angle that satisfies Eq. (14) is listed in Table 2. It shows that the dimples located respectively at segments 2, 6 and 10 have the smallest dimple angles, 86^o due to their greater sensitivities with respective to the third natural frequency. Alternatively, one may use one or two dimples to accomplish the same task. As listed in Table 3, one may use one dimple at Segment 6 with a dimple angle of 124^o; or two dimples at segments 6 and 10 with the same dimple angle of 101^o. In this specific example, the more dimples are used in tuning the natural frequency to a designated value, the smaller angle for each dimple required to satisfy Eq. (14).

Example 3: Use two dimples to shift the 1st and the 2nd natural frequencies from ω_1 to $0.9\omega_1$ and from ω_2 to $0.9\omega_2$ simultaneously.

Fig. 8 illustrates a simply supported beam with two dimples located at segments 4 and 8, respectively. In order to shift two natural frequencies at the same time, the frequency equations become:

$$\Delta(\phi_1, \phi_2, 0.9\omega_1)=0, \Delta(\phi_1, \phi_2, 0.9\omega_2)=0. \tag{15}$$

From Eq. (15), it shows that the frequency equation consists of two dimple angles ϕ_1 and ϕ_2 as the function variables. Eq. (15) are represented respectively by surfaces as illustrated in Fig. 9. The intersections between the two surfaces are the required dimple angle to simultaneously shift the 1st and the 2nd natural frequencies from ω_1 to $0.9\omega_1$ and from ω_2 to $0.9\omega_2$. In this specific example the angle for this intersection is 46° at Segment 4 and 137° at Segment 8, respectively.

7. CONCLUSIONS

The efficacy of proposed design method to modify the vibration characteristics of a structure by adding dimples was demonstrated. A beam was divided into two kinds of structural segments: one is a collection of curved beams (dimples) and the other is straight beams. The frequency equation was derived by assembling the impedance of each structure segment according to conditions of force equilibrium and displacement compatibility. The calculated natural frequency was validated by comparing it with that obtained using the traditional finite element method. Based on this impedance coupling methodology, a novel method for shifting the natural frequencies of a structure to pre-assigned values by adding dimples on the structure was introduced. The advantage of this approach is that the dimple size and its location on the structure can be determined analytically thereby circumventing the optimal search process which is highly time-consuming when the design sensitivity is unknown.

8. ACKNOWLEDGMENTS

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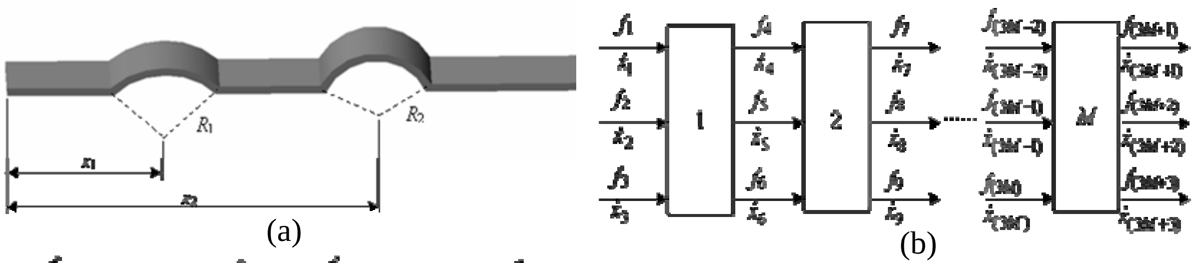
Table 1. Dimple angle for shifting the fundamental natural frequency from ω_1 to $0.85\omega_1$

Segment No.	1	2	3	4	5	6
Dimple angle(°)	NA	NA	176	158	148	145
Segment No.	7	8	9	10	11	
Dimple angle(°)	149	159	177	NA	NA	

Table 2. Dimple angle for shifting the 3rd natural frequency from ω_3 to $0.9\omega_3$

Segment No.	1, 4, 8	2, 8, 11	2, 6, 8	2, 6, 10	6	6, 10
Dimple angle(°)	123	104	97	86	124	101



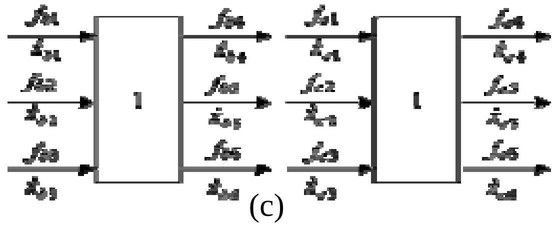


Figure 1. Schematic diagrams of dimpled beam

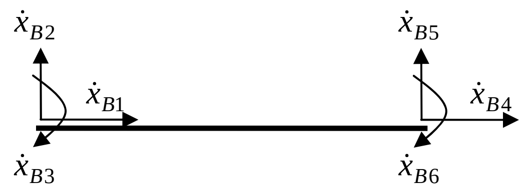


Figure 2. Impedance model of beam

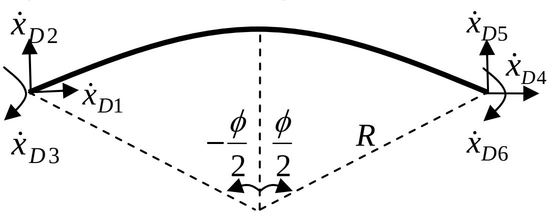


Figure 3. Impedance model of curved beam

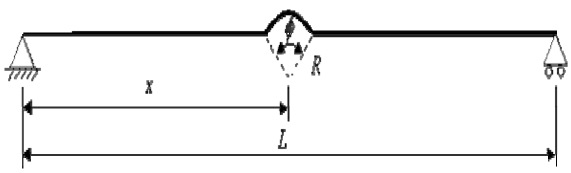
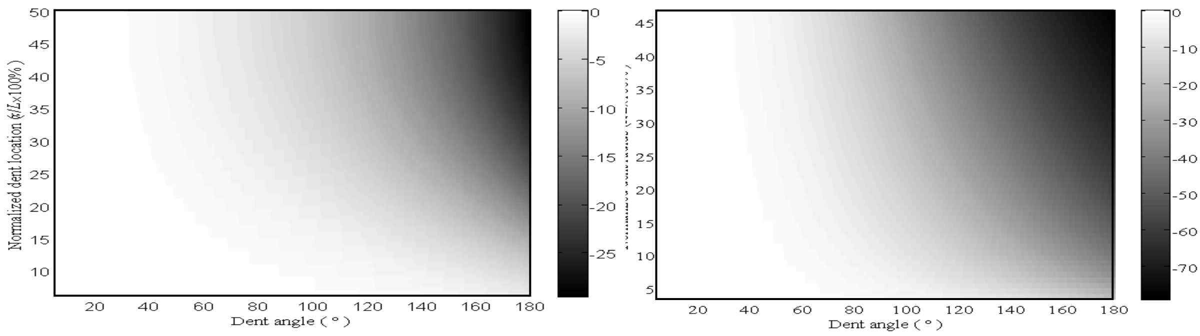


Figure 5. Schematic representation of dimpled beam



(a) (b)

Figure 6. Changes of the first natural frequency by varying the dimple location and the dimple size; (a) influence of dimple location and (b) influence of dimple size

Figure 7. Schematic representation of simply supported beam with three dimples

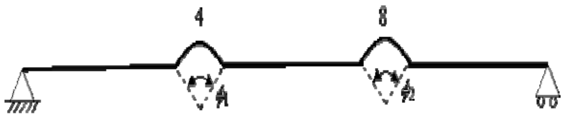


Figure 8. Schematic representation of simply supported beam with two dimples

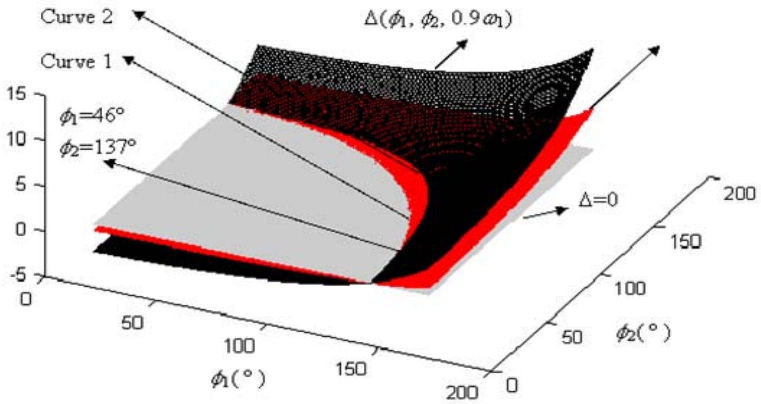


Figure 9. Surfaces of $\Delta(\phi_1, \phi_2, 0.9\omega_1)$ and $\Delta(\phi_1, \phi_2, 0.9\omega_2)$

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