



APPLICATION OF DUAL RECIPROCITY BOUNDARY ELEMENT METHOD TO PREDICT ACOUSTIC ATTENUATION CHARACTERISTICS OF SILENCERS WITH COMPLEX FLOW

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Abstract

The complex flow and sound fields inside silencers require a three-dimensional numerical method for the accurate prediction of acoustic attenuation characteristics of silencers. In the present study, the dual reciprocity boundary element method (DRBEM) is developed to predict and analyze the acoustic attenuation characteristics of the silencers with higher Mach number subsonic flow. In order to overcome the singularity in the single domain BEM for complex silencer analysis and to reduce the computational time, the substructure approach is employed. The effect of flow on acoustic attenuation performance of the silencers is investigated.

1. INTRODUCTION

In the engine exhaust silencing systems, the presence of gas flow will influence the sound propagation inside silencers, and therefore may affect the acoustic attenuation performance of the silencers [1]. In view of the complex flow and sound fields inside the silencers, a three-dimensional numerical method is needed for the accurate prediction of the acoustic attenuation characteristics of silencers. The boundary element method is an effective and powerful numerical method, and has been widely used to evaluate different types of engineering problems. The boundary element method has been developed to predict the acoustic attenuation performance of silencers without flow [2], with uniform flow [3] and low Mach number non-uniform flow [4]. However, the conventional boundary element method (CBEM) is not suitable for solving the acoustic problems of silencers with higher Mach number subsonic flow, due to the presence of domain integral.

The dual reciprocity method [5] (DRM) is a method that converts the domain integral into the boundary integral. Applying the DRM to boundary element method forms the so-called dual reciprocity boundary element method [6] (DRBEM). Lee, *et al* [7] used the DRBEM to model the acoustic radiation in a subsonic non-uniform flow field, and indicated that the Sommerfeld-radiation condition at infinite is satisfied when DRBEM is used to deal with this problem. Perrey-Debain [8] applied DRBEM to calculate the sound field in the straight ducts with uniform flow. However, the application of DRBEM to predict the acoustic attenuation performance of silencers with three-dimensional complex flow has not been reported in the literature.

For the silencers with complex internal structure, using traditional single domain BEM will generate the singular integrals, which make numerical operation more complex and lead to large computational errors. The substructure approach may be used to avoid the singular integrals and save the computational time. The substructure BEM [3, 9] divides the complex structure of a silencer into a number of substructures and then the BEM is applied to each one of these substructures leading to a system of equations. Continuity of sound pressure and normal particle velocity is then enforced at the interface between any two neighboring substructures, therefore all of unknown sound pressure and normal particle velocity on the boundary may be evaluated. The major disadvantage of DRBEM is huge computational works and long time-consuming [10]. For instance, there are 3 matrix multiplies and inverse matrix evaluation in DRBEM, the computational work needed is related to $13(N+L)^3/3$, while the computational work for CBEM is $N^3/3$, here N and L are the collocation points on the boundary and in the volume, respectively. Thus, the computational time using DRBEM is 13 times approximately of that using CBEM. The substructure approach is an effective method to reduce the computational time of DRBEM. For the acoustic analysis of silencers, we need to calculate usually the four-pole parameters and transmission loss, and it is not necessary to examine its internal sound field, so the application of substructure DRBEM to this problem may reduce the computational time and improve the numerical accuracy significantly.

The objectives of the present study are (1) to develop the substructure DRBEM to predict the acoustic attenuation performance of complex silencer with higher Mach number subsonic flow, (2) to investigate the effect of complex potential flow on the acoustic attenuation performance of silencers, and (3) to examine the efficiency of substructure approach for increasing computational speed.

2. GOVERNING EQUATIONS

The flow Mach number in engine exhaust silencing systems is usually less than 0.3, therefore the flow field inside the silencer may be considered as the incompressible potential flow which is governed by Laplace equation

$$\nabla^2 \Phi_L = 0 \tag{1}$$

where Φ_L is the velocity potential. The flow field may be obtained by solving Eq. (1).

Considering the incompressible potential flow field and homogenous medium, the harmonic wave propagation is controlled by [11]

$$\nabla^2 \Phi + k^2 \Phi - j2k(\mathbf{M} \bullet \nabla \Phi) - (\mathbf{M} \bullet \nabla)(\mathbf{M} \bullet \nabla \Phi) = 0$$
⁽²⁾

where Φ is the acoustic velocity potential, $k = \omega/c_0$ is the wavenumber, ω is the circular frequency, c_0 is the sound speed in the stationary medium, $\mathbf{M} = \mathbf{V}_0/c_0$ is flow Mach number, \mathbf{V}_0 is the flow velocity of the medium, and $\mathbf{j} = \sqrt{-1}$ is the imaginary unit.

The acoustic pressure p and particle velocity u may be expressed as

$$p = \rho_0 (j\omega \Phi + \mathbf{V}_0 \bullet \nabla \Phi), \quad u = -\nabla \Phi \tag{3,4}$$

where ρ_0 is the medium density .

3. DUAL RECIPROCITY BOUNDARY ELEMEN METHOD

Moving all terms involving the flow in Eq. (2) to the right-hand side yields

$$\nabla^2 \Phi + k^2 \Phi = b(\Phi) \tag{5}$$

where

$$b(\Phi) = j2k(\mathbf{M} \bullet \nabla \Phi) + (\mathbf{M} \bullet \nabla)(\mathbf{M} \bullet \nabla \Phi)$$
(6)

The integral formulation for Eq. (5) is expressed

$$C(P)\Phi(P) = \int_{S} \left(G\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial G}{\partial n}\right) dS - \int_{V} Gb(\Phi) dV$$
(7)

where

$$C(P) = \int_{S} \frac{\partial G_{L}}{\partial n} dS$$
(8)

 G_L is the fundamental solution of Laplace equation, and G is the fundamental solution of Helmholtz equation. For the three-dimensional problem,

$$G = \frac{1}{4\pi r} e^{-jkr}$$
, $G_L = \frac{1}{4\pi r}$ (9, 10)

Eq. (7) contains volume integral, which may be converted to the boundary integral by using DRM. $b(\Phi)$ may be approximated by the following expression

$$b(\Phi) = \sum_{i=1}^{N+L} \alpha_i f_i \tag{11}$$

where α_i are the undetermined coefficients, f_i is a simple source (approximate) function. For each point on the boundary and in the volume, N is the number of collocation points on the boundary, and L is the number of collocation points in the volume. Reconstructing Eq. (11) leads the following matrix

$$\mathbf{b} = \mathbf{F}\boldsymbol{\alpha} \tag{12}$$

where **b** is an $(N + L) \times 1$ vector that contains the function values of *b* at the collocation points, **F** is an $(N + L) \times (N + L)$ matrix, and **a** is an $(N + L) \times 1$ coefficient vector. For each simple source function f_i , a particular solution φ_i needs to be found and satisfied

$$\nabla^2 \varphi_i + k^2 \varphi_i = f_i \tag{13}$$

One of the key ingredients of the dual reciprocity method is the expansion introduced in Eq. (11). There are virtually an infinite number of ways to choose f_i for use in the expansion. The trouble is that we have to find the associated particular solution φ_i for each choice of f_i . The usual practice is to construct φ_i first and then find f_i from Eq. (13).

In the dual reciprocity method, f_i is chosen usually as [6]

$$f_i = 1 + r_i \tag{14}$$

For the problem in the present paper, we use the following particular solution φ_i [8,12]

$$\varphi_{i} = \begin{cases} -\frac{2}{k^{4}r_{i}} + \frac{1+r_{i}}{k^{2}} + \frac{2\cos(kr_{i})}{k^{4}r_{i}} & r_{i} \neq 0\\ \frac{1}{k^{2}} & r_{i} = 0 \end{cases}$$
(15)

where $r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$.

Substituting Eqs. (11) and (13) into Eq. (7) yields:

$$C(P)\Phi(P) = \int_{S} \left(G\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial G}{\partial n}\right) dS - \sum_{i=1}^{N+L} \alpha_{i} \int_{V} G(\nabla^{2}\varphi_{i} + k^{2}\varphi_{i}) dV$$
$$= \int_{S} \left(G\frac{\partial\Phi}{\partial n} - \Phi\frac{\partial G}{\partial n}\right) dS - \sum_{i=1}^{N+L} \alpha_{i} \left\{C(P)\varphi_{i}(P) - \int_{S} \left(G\frac{\partial\varphi_{i}}{\partial n} - \varphi_{i}\frac{\partial G}{\partial n}\right) dS\right\}$$
(16)

By using discretization and numerical integration for Eq. (16), and combining Eq. (12), the following algebraic system of equations in matrix form may be obtained

$$\mathbf{H}\boldsymbol{\Phi} - \mathbf{G}\frac{\partial \boldsymbol{\Phi}}{\partial n} = \mathbf{R}\boldsymbol{\alpha} = \mathbf{R}\mathbf{F}^{-1}\mathbf{b}$$
(17)

where **H** and **G** are the BEM coefficient matrices, **R** is a matrix obtained by integrating the known particular integral contained in $\{ \}$ of Eq. (16). From Eq. (6), it may be seen that *b* is a function of the first- and second-order derivatives of Φ . The derivatives of Φ can be obtained by first introducing a set of global interpolating functions for Φ and then differentiating the interpolating functions, therefore the values of *b* at each point may be determined. We choose

$$\Phi = \mathbf{E}\boldsymbol{\beta} \tag{18}$$

where $\mathbf{E} = [f_{1ij}]_{(N+L)\times(N+L)}$ is the coefficient matrix formed by the global interpolating functions f_{1ij} , β is the undetermined coefficients for each collocation points. Eventually, the nodal values of $b(\Phi)$ at the collocation points can be written as:

$$\mathbf{b} = \mathbf{B}\boldsymbol{\Phi} \tag{19}$$

where **B** is a coefficient matrix that depends on the choice of **E**. Substituting Eq. (19) into Eq. (17) gives:

$$\mathbf{H}\boldsymbol{\Phi} - \mathbf{G}\frac{\partial \mathbf{\Phi}}{\partial n} = \mathbf{R}\boldsymbol{\alpha} = \mathbf{R}\mathbf{F}^{-1}\mathbf{B}\mathbf{\Phi}$$
(20)

Combining the boundary conditions, Eq. (20) may be solved and the unknown variables are obtained.

The accuracy of the dual reciprocity method will also depend on the choice of f_1 used in the interpolating and differentiating Φ . The global interpolating functions contained in **E** should be as simple as possible because differentiation of complicated functions may result in peculiar behaviors. Here we choose

$$f_{1ij} = 1 + r_{ij}^2 + r_{ij}^3 \tag{21}$$

which is suitable for the problems contained derivatives in the source term $b(\Phi)$ [13], and $r_{ij} = \sqrt{(x_i - x_i)^2 + (y_i - y_i)^2 + (z_i - z_i)^2}$.

4. SUBSTRUCTURE APPROACH

Figure 1 shows an acoustic system composed of two substructures with a common interface, the relationships between sound pressures and normal particle velocities may be expressed as

$$\begin{bmatrix} p_i \\ p_s \end{bmatrix} = \begin{bmatrix} Z_{11}^1 & Z_{12}^1 \\ Z_{21}^1 & Z_{22}^1 \end{bmatrix} \begin{bmatrix} u_i \\ u_s \end{bmatrix}$$
(22)

$$\begin{bmatrix} p_s \\ p_o \end{bmatrix} = \begin{bmatrix} Z_{11}^2 & Z_{12}^2 \\ Z_{21}^2 & Z_{22}^2 \end{bmatrix} \begin{bmatrix} u_s \\ u_o \end{bmatrix}$$
(23)

where, p_i and u_i , p_s and u_s , p_o and u_o are the sound pressures and particle velocities on inlet of substructure 1, common interface and outlet of substructure 2, respectively, Z_{ij}^1 and Z_{ij}^2 are the elements of impedance matrices for the substructures 1 and 2, respectively.

p_i	Z_{11}^{1}	$Z_{12}^1 p_s$	Z ₁₁ ²	Z_{12}^2	р。
u_i	Z_{21}^{1}	$Z_{22}^{1} u_{s}$	Z 2 21	2 Z_{22}^{2}	и,

Figure 1. Two substructures with a common interface.

Using the continuity conditions of sound pressure and particle velocity on the common interface, the following relationship of sound pressures and particle velocities on the inlet and outlet of the system may be given

$$\begin{bmatrix} p_i \\ p_o \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} u_i \\ u_o \end{bmatrix}$$
(24)

where $Z_{11} = Z_{11}^1 - Z_{12}^1 (Z_{11}^2 + Z_{22}^1)^{-1} Z_{21}^1$, $Z_{12} = Z_{12}^1 - (Z_{11}^2 + Z_{22}^1)^{-1} Z_{12}^2$, $Z_{21} = Z_{21}^2 (Z_{11}^2 + Z_{22}^1)^{-1} Z_{21}^1$, $Z_{22} = Z_{22}^2 - Z_{21}^2 (Z_{11}^2 + Z_{22}^1)^{-1} Z_{12}^2$.

For the acoustic system with multiple substructures, the similar method may be used to get the overall impedance matrix for the entire system. When the plane wave conditions are satisfied on the inlet and outlet of the silencer, the four-pole parameters may be obtained and expressed as

$$\begin{bmatrix} p_i \\ \rho_0 c_0 u_i \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_o \\ \rho_0 c_0 u_o \end{bmatrix}$$
(25)

where p_i , u_i , p_o and u_o are the sound pressures and particle velocities on the inlet and outlet of the silencer, respectively, ρ_0 and c_0 are the medium density and sound speed, $A = Z_{11}/Z_{21}$, $B = (Z_{12} - Z_{11}Z_{22}/Z_{21})/(\rho_0 c_0)$, $C = \rho_0 c_0/Z_{21}$, and $D = -Z_{22}/Z_{21}$.

Thereby, the transmission loss of a silencer is given by [3]

$$TL = 20\log\left|\frac{1}{2}(A+B+C+D)\right| + 10\log\frac{S_i}{S_o}$$
(26)

where S_i and S_o are the cross-sectional areas of inlet and outlet of the silencer, respectively.

5. RUSULTS AND DISCUSSION

The double expansion chamber silencer with inter-connecting tubes, as shown in Figure 2, is considered to investigate the effect of three-dimensional flow on the acoustic attenuation performance of the silencer, and to examine the efficiency of the substructure DRBEM.

In order to apply the substructure DRBEM, the silencer is divided into 5 substructures: inlet tube I , expansion chamber II , inter-connecting tube III, expansion chamber IV and outlet tube V. Figure 3 shows the flow velocity field inside the silencer for the case of flow Mach number M=0.3 at the inlet of the silencer. It may be seen that the flow field inside the silencer is three-dimensional.

Figure 4 compares the transmission loss predictions for the silencer with different flow Mach numbers. It may be seen that, the effect of flow on the acoustic attenuation performance of the silencer is marginal at lower frequency, and is obvious at higher frequency. As the increase of flow Mach number, change of flow field inside the silencer will be more complex, and the gradients of velocity are getting bigger, therefore transmission loss of the silencer is changed. It is clear that the effect of three-dimensional flow on the acoustic attenuation characteristics of silencers may not be ignored.

Figure 5 compares the computational time by DRBEM and the substructure DRBEM. It is clear that the substructure approach may save the computation time and increase the computational speed significantly.



Figure 2. Schematic diagram of the double expansion chamber silencer with inter-connecting tubes (dimensions in mm).



Figure 3. Distribution of flow field inside the double expansion silencer with inter-connecting tubes with M=0.3 at the inlet (units in m/s).



Figure 4. Transmission loss of double expansion chamber silencer with inter-connecting tubes.



Figure 5. Comparison of computation time spent in DRBEM and substructure DRBEM.

6. CONCLUSIONS

The substructure dual reciprocity boundary element method is developed to predict the acoustic attenuation characteristics of silencers with complex three-dimensional flow. Compared to the traditional boundary element method, the dual reciprocity boundary element method considers the second orders of Mach number in the governing equation, thus it is suitable for solving the sound propagation problems in the higher Mach number subsonic flow. Numerical results demonstrated that the effect of complex three-dimensional flow on the acoustic attenuation performance of silencers is not negligible. The substructure approach may reduce the computational works and then save the computation time of DRBEM.

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