



A REVIEW OF SELECTED EXPERIMENTAL INVESTIGATIONS OF CURLE'S THEORY

David Alan Bies¹ and Alexei Zinoviev²

¹103 Callaghan St, Mooroobool, Cairns, QLD 4870, Australia. ²School of Mechanical Engineering, University of Adelaide, North Tce, Adelaide, SA 5005, Australia. <u>dabies@sa.chariot.net.au</u>

Abstract

Curle[1] published an article, widely known as Curle's theory, in which he sought to extend the theory of Sir James Lighthill[2] on aerodynamic noise generation to include the effects of rigid boundaries. His theory has been accepted as mathematically correct and has been widely referenced, but it remains without experimental verification. A review of five selected articles published over a span of 35 years reporting experimental investigations of Curle's theory will be presented. The review will show that, in cases where the source is compact, Curle's theory predicts radiated sound power about 5 dB too high, while in non-compact cases Curle's theory predicts radiated sound power about 5 dB too low. These observations are readily explained and the explanation suggests why Curle's theory has not been experimentally verified but, more importantly, what may require further consideration. In the cases investigated Curle's failure to consider source structure is shown to be responsible for the observed disparities. The disparate results may be explained if in the case of non-compact wave numbers the dominant noise source is a surface source distribution proportional to the square of the dimensionless wave number, while at compact wave numbers the latter source tends to insignificance and a less efficient leading edge dipole independent of the dimensionless wave number is then dominant.

1. INTRODUCTION

One of the most frequently used methods for the prediction of sound radiated by a rigid object in a fluid flow is based on a theory proposed by Curle[1] in which he considered aerodynamic noise generation by a stationary rigid body in a turbulent air stream. He concluded that the radiated sound is dipole in nature and that the radiated sound power is proportional to the square of the force exerted by the rigid body on the air stream. Curle's theory has been accepted as mathematically correct by the authors of several volumes on aerodynamic noise published in the last thirty years[3, 4] and has provided the basis for various schemes of noise level prediction and control.

However, numerous attempts to experimentally verify Curle's theory over the intervening years have been unable to confirm his predictions. In a recent article Zinoviev and Bies[5]

identified an error in Curle's theory suggesting a reason for the lack of experimental confirmation. However, as the point raised by the authors remains controversial a very different approach is taken here. In this article some experimental works chosen for their inter relationships will be investigated. The aim of the investigation will be to clarify the problems with Curle's solution.

In the cases considered here narrow band noise of tonal quality is investigated in terms of power ratios defined as the measured sound power divided by Curle's predicted sound power expressed in decibels[6]. The functional dependence of power ratio on the dimensionless wave number, defined as the product of the wave number and the test object stream wise length, is investigated. In this dimensionless format all power ratios are predicted to be identically zero decibels. If results were plotted in a Cartesian coordinate system with power ratios on the ordinate and wave numbers on the abscissa, all compact source determinations would cluster close to the origin and all non-compact source determinations would follow appropriately at zero decibels on the abscissa. As will be shown, however, all power ratios are either +5 dB or -5 dB. These evident disparities are then explained and an indication of how Curle's theory might be amended is given.

2. COMPACT SOURCE INVESTIGATIONS

Clark and Ribner[7] investigated noise emitted by an airfoil in a low flow speed jet using a correlation technique relating measured fluctuating lift to measured sound pressure and determined a pressure level of -2.7 dB, lower than predicted by Curle. At about the same time Heller and Widnall[6] reported that they could not verify Curle's prediction and that they could not explain their failure.

Later Bies[8] investigated noise emitted by a circular saw rotating at several speeds and confirmed dipole radiation as predicted by Curle. Using pressure measurements obtained by means of imbedded microphones on opposite sides of one tooth he calculated the predicted sound pressures on axis. When the predicted pressure levels were compared with the measured pressure levels they were all found to be -2.5 dB, lower than predicted. When all of these pressure levels are converted to power ratios they become -5.4 dB and -5 dB showing that Curle predicts power levels for a compact source about *5 dB higher than measured*.

3. NON-COMPACT SOURCE INVESTIGATION

Using a blow-down test facility, scaled simulation of the gullet spacing and blade thickness configuration of the rotating circular saw investigated earlier and a small steel block as a tooth immersed in the wake of a vortex generator of the same thickness provided Bies[9] with a tone 20 dB above the background noise of the jet for comparison with Curle's prediction at non-compact wave numbers. Replacement of a term neglected by Curle in determining his compact source limit allowed experimental investigation of Curle's theory over an extended dimensionless wave number range from about 3.2 to about 0.4. In this case, experiment produced a cluster of data about a horizontal line at 5 dB in the dimensionless wave number range from 3.2 to about 1, showing that in this range of non-compact wave numbers Curle predicts levels 5 *dB lower than measured*. At dimensionless wave numbers less than 1 the data continued downward until lost in the noise of the jet at about 0.4 where investigation was discontinued as the emitted tone could no longer be distinguished above the noise of the jet[9].

4. TWO SIMULTANEOUS SOURCES

The disparate results reported above may be resolved if in the cases investigated there are two simultaneous noise sources of which one or the other is dominant. In the case of the non-compact source a dominant surface source distribution of radiation efficiency, proportional to the square of the dimensionless wave number, produces the observed noise. Consequently as the wave number decreases the radiation efficiency of the dominant source also decreases until it is negligible.

In the case of the compact source the source is a less efficient leading edge dipole. Here it is observed that a leading edge dipole, while less efficient, is independent of the wave number as it only requires an edge of zero thickness. *Consequently, in the case of Curle's compact source the leading edge dipole, which will be proposed, may be the only source which is possible in the compact limit.*

4.1 Driven monopole surface distributions (non-compact)

In the case considered here it is assumed that the source is distributions of surface sources on opposites sides of the rigid radiating steel block driven by successive vortices in the wake in which the block is immersed. In the case of distributed surface sources the radiation efficiency is proportional to the square of the ratio of the source dimension to the radiated wavelength dimension. Consequently, the radiation efficiency in the compact limit tends to zero so that the surface source in the limit radiates no sound.

For example, in the investigation reported by Bies[9] the tone radiated by the test block was initially 20 dB above the background noise of the jet at dimensionless wave numbers in the range from 3.2 to 1.0. However, at wave numbers less than 1.0 the measured sound pressure levels began to decrease until at dimensionless wave numbers less than 0.4 the tone could not be detected above the background noise of the jet.

These observations are consistent with the proposed model in which driven opposite surface distributions are the source. In the investigation reported[9] all of the data clustered about a line at 5 dB in the range of large wave numbers from 3.2 to 1.0 but gradually decreased as the wave number decreased until at about 0.4 the test tone could no longer be detected in the noise of the jet. For example, as the wave number becomes small the radiation efficiency of the source tends to zero, as observed[9].

4.2 Fluid flow and noise generation

In connection with the saw noise investigation mentioned above Martin and Bies conducted flow visualization tests using a water channel[10]. Flow about two blocks in tandem allowed investigation, which showed vortices of diameter about equal to the upstream generator thickness, which impinged on the downstream block. The latter investigation suggested the experimental arrangement used in the non-compact source investigation and the proposed leading edge dipole model, which will be discussed here.

When the gap width between the trailing edge of the vortex generator and the leading edge of the downstream test block is of the order of the generator thickness a trapped stationary vortex is formed in the gap. When the gap width is increased a vortex street is formed in the gap. When the gap width is greater than about three times the generator thickness the vortices in the gap tend to be washed out of the gap into the passing stream.

A gap width of three times the generator thickness, as illustrated in Figure 1, was chosen as optimal for the purpose of noise generation. As the original purpose was to investigate saw

noise generation[8] the vortex generator, as well as the noise-generating block are coplanar and of equal thickness. *The crucial observation that the vortex is suddenly accelerated across the leading edge of the block explains the origin of the aerodynamic noise, which is proportional to the time rate of change of acceleration.*

4.3 Leading edge dipole (compact source)

It has been proposed that there are two sources, of which the one dominant at large wave numbers is a surface source distribution (which is wave number dependent and tends to zero as the wave number diminishes), while at small wave numbers the other, Curle's compact source, is a leading edge dipole, which is independent of wave number and thus becomes dominant at low wave numbers. Low wave numbers could not be investigated using the apparatus of **Section 4.1** as the emitted tone could not be distinguished above the noise of the jet. Low wave number investigations were carried out at very much lower flow speeds and much less noise (see **Section 2**).

4.4 Distributed noise source (non-compact source)

In the case of large dimensionless wave numbers the arrangement of the test apparatus has been described in detail[11]. The design of the latter apparatus was such that an upstream plate generated a trailing vortex street in the gap between the generator and the downstream block in which dimensions were chosen to scale to simulate the previously investigated rotating circular saw. In the presence of the stream (from left to right in Figure 1) vortices are successively produced which are carried downstream in the gap to impinge upon the leading edge of the block.

A fully developed vortex street is formed initially in the gap, as illustrated in Figure 1A where the small arrows indicate the directions of rotation of the vortices (shown as circles in the figure). One-quarter cycle later, as illustrated in Figure 1B, the leading vortex is deflected upward out of the gap as a new vortex is formed at the trailing edge of the generator. One half cycle later, as illustrated in Figure 1C, the street is again fully formed but the vortices now rotate oppositely, as shown in the figure. Three quarters cycle later, as illustrated in Figure 1D, the lead vortex is deflected downward out of the gap. The cycle is completed one quarter cycle later when the formation is again that of Figure 1A.

Water channel model studies[10] showed that when a vortex impinges on the leading edge of the block it is suddenly thrust out into the passing stream rolling up the leading edge of the block in part pushed by the passing stream as it emerges from the gap. The sudden lateral thrust outward into the stream produces a pressure pulse and a reaction force on the test block while the rolling motion of the vortex, due to the finite viscosity of the fluid, also produces a reaction force on the block but contributes nothing to the pressure pulse.

It is clear that the total reaction force, here identified as "Curle's force," will be greater than the force, which accounts for the radiated sound. Consequently, Curle's theory predicts more noise than will be observed according to this model (in agreement with observations reported earlier in **Section 2**).

When a vortex impinges upon the downstream leading edge of the block the rotating vortex rolls alternately up or down the leading edge and out into the air stream. It should be noted that the associated pressure pulse is formed while the vortex is being accelerated across the leading edge of the block. This sequence of events is illustrated in Figure 1. It should also be noted that although the events described above take place on opposite edges of the model the thickness of the model is a very small fraction of the wavelength of the sound, which is radiated so that any associated phase lag may be neglected.

It may be observed that while one vortex is swept out into the stream and the next vortex arrives at the opposite edge of the block, little noise is produced by these events. Consequently, a great simplification is possible when it is observed that the noise generated by the actual sequence of events may be simulated with acceptable accuracy by consideration of a virtual sphere which rolls back and fourth over the leading edge of the block during each cycle.

It should also be noted that where the proposed model is continuous and the sequence of events is discontinuous during this part of the cycle, for example during ejection of a vortex on one edge of the block and arrival on the other edge of the block of the next vortex, neither the actual source nor the proposed virtual source contributes significantly to the radiated noise. Consequently, it is considered quite reasonable to model the source of radiated noise produced by the actual sequence of events (see Figure 1) as the same as would result from a virtual sphere rolling back and fourth across the leading edge.

Here it is proposed to approximate the cyclic impingement of successive vortices upon the upper and lower edges of a small steel block as a virtual sphere at the leading edge of the block which oscillates back and fourth between leading edges. The oscillating sphere is a dipole source with axis parallel to the leading edge and normal to the direction of flow, which radiates sound as observed[8]. The assumption is implicit that there will be many such spheres on the leading edge contributing to the observed noise.

4.5 A surface source distribution and boundary conditions

A small multi-pole source on a rigid boundary is analyzed in terms of a series containing the zero-eth (monopole) and first order (dipole) terms written in Legendre and Spherical Bessel wave functions[12]. The latter equations are exact solutions of the wave equation in spherical coordinates in unbounded space and may be considered as definitions of what is meant by the terms monopole and dipole.

Two boundary conditions must be satisfied. The first boundary condition requiring that there be no net flow through the boundary at the point of contact of the spherical source and rigid surface will be considered first. Satisfaction of this condition determines the relative amplitudes of the monopole and dipole terms.

A surface source distribution has been proposed based upon the observation that in a viscous boundary layer close to a rigid surface, where viscosity controls, many little nodules of circulating fluid called "vortices" will be formed, which cover the surface. An incident vortex rolling across the rigid surface will compress the nodules in phase, and they in turn will respond collectively as the distributed source producing the observed radiated sound. In the case of a leading edge source the discussion will be similarly based on a typical source but in this case the source distribution will be confined to the leading edge.

Each small nodule will be constrained in exactly the same way by the rigid surface so that consideration may be given to a single source on the rigid surface as representative of all surface sources. In the analysis presented here attention will be given to the radiation of a single small sphere as sufficiently representative of a typical nodule of vorticity in the viscous sub layer close to the rigid surface[13, 14]. The typical viscous boundary layer source will be constrained by the rigid surface upon which it rests to respond as a multipole combination monopole dipole. For example, expansion of the monopole into the surface is compensated by vertical contraction of the associated dipole. Similarly expansion of the dipole is compensated by contraction of the monopole. Thus the proposed model satisfies the condition that there can be *no fluid flow* into the rigid surface.

The second boundary condition is that a monopole near a rigid boundary will radiate directly and also by reflection at the rigid surface into the adjacent half space. A great simplification can be made by observing that in the case of the first boundary condition the rigid boundary need be no greater than the assumed point of contact so the solution may be written as though it were in free space. When all such sources are assembled the effect will be that all sound will be confined to the adjacent half space. A small secondary effect will next be considered. There will be no other effects to be considered.

The acoustic radiation of the multipole source into the rigid surface upon which the source rests will be reflected into the bounding half space opposite the rigid surface. As the radiated sound is a narrow band of noise it is proposed that the reflection will be incoherent. It is of interest to note that reflection of the monopole source by the rigid surface will have no effect upon the radiated sound power. However, the dipole and its reflection will form a linear quadra pole of much reduced radiation efficiency.

It will be shown that for a very small source, far field radiation is controlled by the monopole term, while the force exerted by acoustic pressure on the rigid surface is controlled by the very much smaller dipole term. Consequently, when Curle's recommendation that the force exerted by the rigid body on the passing air stream is used to predicted the radiated sound the predicted sound levels will be lower than the measured levels, as observed in the experiment reported here.

The acoustic pressure p at distance r radiated by a small spherical multipole source on a small rigid boundary may be represented as a series of exact solutions of the spherical wave equation[12]. In the case considered here the series will contain the zeroeth (monopole) and first order (dipole) terms of the Legendre and Spherical Bessel wave functions as follows.

$$p(r,t) = \frac{1}{\rho c} \Big\{ A_0 \Big[j_0(kr) + i n_0(kr) \Big] + A_1 \cos \theta \Big[j_1(kr) + i n_1(kr) \Big] \Big\} e^{-i\omega t}$$
(1)

Introducing the following relations, Morse[12]: Equation (2) and Equation (3), allows Equation (1) to be rewritten as Equation (4) below.

$$j_{0}(kr) + in_{0}(kr) = \frac{\sin(kr)}{kr} - i\frac{\cos(kr)}{kr}$$

$$j_{0}(kr) + in_{0}(kr) = -i\frac{e^{ikr}}{kr}$$
(2)

$$j_{1}(kr) + in_{1}(kr) = -\frac{e^{ikr}}{kr} - i\frac{e^{ikr}}{(kr)^{2}}$$

$$j_{1}(kr) + in_{1}(kr) = -\frac{(kr+i)}{(kr)^{2}}e^{ikr}$$
(3)

$$p(r,t) = -\left\{A_0\left[\frac{i}{kr}\right] + A_1 \cos\theta\left[\frac{(kr+i)}{(kr)^2}\right]\right\} e^{-i(\omega t - kr)}$$
(4)

Morse[12] gives as the related exact expression for the radial velocity, $u(\alpha, t)$, at the surface of the spherical source of small radius, a, as a function of time, t, the following equation.

$$u(a,t) = \frac{1}{\rho c} \left\{ A_0 \left(\frac{1}{(ka)^2} \right) + A_1 \left(\frac{2}{(ka)^3} \right) \cos \theta \right\} \frac{e^{-i\omega t}}{\rho c}$$
(5)

The boundary condition that the surface upon which the source rests is rigid requires that for $\theta = \pi$ the radial velocity $u_a = 0$. In this case Equation (5) takes the following form where $k\alpha$ may be assumed to be very small.

$$A_0 \frac{ka}{2} = A_1 \tag{6}$$

Equation (6) provides the relation between the amplitude, A_0 , of the monopole term and the amplitude, A_1 , of the dipole term. Consideration of Equation (6) shows that as α is assumed to be very small $A_0 >> A_1$. Taking into account the latter observation review of Equation (1) shows that the monopole term determines the sound radiated. This conclusion explains why Curle predicts levels, based upon the measured force exerted on the air stream by the dipole term, that are less than those measured.

It has been shown that the amplitude A_1 of the dipole term is very small compared with amplitude A_0 of the monopole term. It is to be noted that the monopole center of mass is stationary while the center of mass of the dipole accelerates cyclically back and forth along the dipole axis normal to the radiating block. This observation leads to the important conclusion that while the dipole does contribute, the monopole contributes nothing to the force (Curle's force) acting upon the radiating block. On the other hand, as the monopole is a much more efficient sound radiator than the dipole, it may also be concluded that the monopole contributes the radiated sound while the dipole's contribution is negligible. These observations explain why Curle predicts less sound than is observed in the case of the noncompact source.

5. CONCLUSION

A leading edge dipole has been proposed as possibly the only solution which satisfies Curle's compact limit. However, Curle's conclusion that the radiated sound is a function of the force exerted on the passing air stream by a rigid object is only partly true. As has been shown, the total force exerted on the passing air stream is composed of a component which produces translation and sound, and a component which produces rotation but no sound. The results suggest that a small empirical correction may suffice in this case to correct this detail.

Reference to Figure (1) suggests why the configuration was optimal for noise production and suggests an intriguing possibility for noise control and a need for further investigation.

In the more general case of a non-compact source the review provided here suggests that there may be many examples of which only one has been experimentally investigated and an explanation provided. Clearly, all solutions may be expected to be dependent upon the mechanics of the source, to which Curle has given no consideration. However, Curle seems to have provided a good beginning for further investigations



Figure 1. Four states each separated by one-quarter cycle are shown where airflow is from left to right. Vortices are indicated by circles and associated arrows showing direction of rotation. The figures are only approximately to scale. Figure 1A shows an initial state. Figure 1B shows the lead vortex pushed out into the stream producing a pressure pulse as a new vortex is formed. Figure 1C shows the inverse initial state with all vortices rotating oppositely. Figure 1D shows the lead vortex pushed out into the stream opposite to that shown in Figure 1B. A pressure pulse of opposite phase is produced. The cycle is complete in the final quarter cycle when the configuration is again as in Figure 1A.

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