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FLEXURAL VIBRATION BAND GAPS IN PERIODIC PIPE SYSTEM CONVEYING FLUID

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Abstract
Flexural vibration in the periodic pipe system conveying fluid is studied in this paper. Using the transfer matrix method, the complex band structure of the flexural wave is calculated to investigate the gap frequency range and the vibration reduction in band gap. And the complete flexural vibration band gaps exist in the piping system with fluid loading. The effect of the fluid on the gaps is considered. The existence of flexural vibration gaps in periodic pipe with fluid loading gives a new idea in vibration control of pipe.

1. INTRODUCTION

In last decade, the propagation of elastic or acoustic waves in periodic composites, so called phononic crystals (PCs), has received much attention [1-3]. This is of interest for applications such as frequency filters, vibrationless environments for high-precision mechanical systems or design of new transducers.

The vibration propagation in periodic structures was researched some time ago [4-5]. The theory to predicting the vibration response of periodic structures has been applied primarily to analysis the periodic structures as pass band and stop band. Recently, with the theory of PCs, the Vibration band gaps including longitudinal vibration, flexural vibration and so on, in periodic beams have been studied both theoretically and experimentally [6-9].

Vibration analysis of piping systems conveying fluid has received considerable attention due to wide application to areas such as designing heat exchanger tubes, main steam pipes and hot/cold leg pipes in nuclear steam supply systems, oil pipeline, pump discharge lines, marine risers and so forth [10]. So the vibration analysis of pipe had been studied early [10-13]. The free wave propagation in the periodically supported, infinite piping system conveying fluid was studied [11]. The results of Ref.[11] show that if the dominant frequency contents in the excitation loads are known, a proper design of periodic supports for reducing the vibration in those frequency band gaps is possible.

In this paper, we investigate the flexural vibration band gaps in the periodic pipe system conveying fluid. Using the transfer matrix (TM) method, the complex band structure of the flexural wave is calculated to investigate the gap frequency range and the vibration reduction in band gap.

2. THEORY OF TRANSFER MATRIX

Figure1 shows a periodic binary composite pipe system. The system consists of an infinite repetition of alternating pipe A with length a_1 and pipe B with length a_2. Thus the PCs pipe's lattice constant is $a=a_1+a_2$. Pipe A and pipe B can be made up of different material parameters or different geometrical parameters, illustrated as figure1(a) and figure1(b).

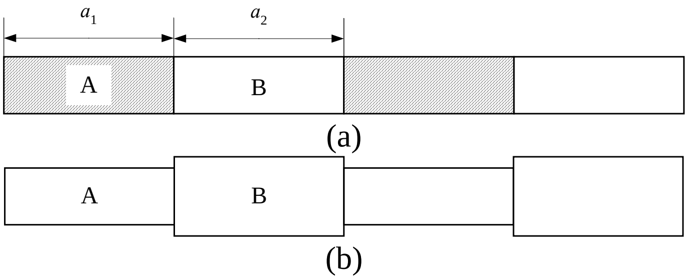

Figure1. The sketch map of periodic binary pipe. (a) pipe wall with periodic material parameters; (b) pipe wall with periodic geometrical parameters.

For the Euler-type pipe conveying fluids at a constant velocity v, if gravitational forces, internal damping, externally imposed tension and pressurization effects are neglected, the well-known governing equation of flexural vibration becomes [12,13]

$$EI\frac{\partial^4 w}{\partial x^4}+m_f v^2\frac{\partial^2 w}{\partial x^2}+2m_f v\frac{\partial^2 w}{\partial x\partial t}+(m_f+m_p)\frac{\partial^2 w}{\partial t^2}=0 \tag{1}$$

where w is the flexural displacement, EI is the flexural rigidity of the pipe, m_f and m_p are fluid and pipe masses per unit length, v and t are the constant uniform fluid velocity and time respectively.

For the Euler pipe without fluid loading, the motion equation is given as

$$EI\frac{\partial^4 w}{\partial x^4}+m_p\frac{\partial^2 w}{\partial t^2}=0 \tag{2}$$

For a harmonic traveling wave $w(x,t)=We^{i(\omega t+kx)}$, one can get the dispersion relation of equation(1) as

$$EIk^4-m_f v^2k^2+2m_f v\omega k-(m_f+m_p)\omega^2=0 \tag{3}$$

For a given ω, the wavenumber roots of equation(3) include two real roots and a conjugate pair of complex roots, signed as [13] $k_d,-k_u,k_R\pm ik_I$. The positive and negative real wavenumbers describe the propagating waves in positive and negative direction. And the conjugate root pair describes the near-field waves [14] (non-propagating, spatially decaying). The near-field wave has an imaginary component for all frequencies and it arises from a neighboring boundary, discontinuity or applied force. Also the wavenumbers depend on the frequency ω and flow speed v [13].

The harmonic solution of equation(1) is

$$w(x,t)=e^{i\omega t}(W_1e^{ik_1x}+W_2e^{ik_2x}+W_3e^{ik_3x}+W_4e^{ik_4x}) \tag{4}$$

where the wave number k_i (i=1,2,3,4) is given by $k_1=k_d$, $k_2=-k_u$, $k_3=k_R+ik_I$, $k_4=k_R-ik_I$.

And the harmonic solution of equation(2) for the pipe without fluid loading is [15]

$$w(x,t) = e^{i\omega t}(a^{+}e^{-ikx} + a_N^{+}e^{-kx} + a^{-}e^{ikx} + a_N^{-}e^{ikx}) \tag{5}$$

where subscript N denotes the near-field wave component. And the wave number k is given by

$$k = \sqrt[4]{m_p\omega^2 / EI} \tag{6}$$

The continuities of displacement, slop, bending moment and shear force at the interfaces between cell n-1 and n, i.e. x=na give:

$$w_{n,A}(0) = w_{n-1,B}(a) \tag{7a}$$

$$w'_{n,A}(0) = w'_{n-1,B}(a) \tag{7b}$$

$$E_A I_A w''_{n,A}(0) = E_B I_B w''_{n-1,B}(a) \tag{7c}$$

$$E_A I_A w'''_{n,A}(0) = E_B I_B w'''_{n-1,B}(a) \tag{7d}$$

One can obtain the matrix form of equation(7)

$$\mathbf{K}\mathbf{W}_{n,A} = \mathbf{H}\mathbf{W}_{n-1,B} \tag{8}$$

where $\mathbf{W} = [W_1, W_2, W_3, W_4]^T$.

The continuities at the interfaces between pipe A and pipe B in cell n, i.e. x=na +a_1, give:

$$w_{n,A}(a_1) = w_{n,B}(a_1) \tag{9a}$$

$$w'_{n,A}(a_1) = w'_{n,B}(a_1) \tag{9b}$$

$$E_A I_A w''_{n,A}(a_1) = E_B I_B w''_{n,B}(a_1) \tag{9c}$$

$$E_A I_A w'''_{n,A}(a_1) = E_B I_B w'''_{n,B}(a_1) \tag{9d}$$

The matrix form of equation(9) can be written as

$$\mathbf{K}_1\mathbf{W}_{n,A} = \mathbf{H}_1\mathbf{W}_{n,B} \tag{10}$$

Basing on the equations(8) and (10), the relation between the nth cell and (n-1)th cell is given

$$\mathbf{W}_{n,B} = \mathbf{T}\mathbf{W}_{n-1,B} \tag{11}$$

where $\mathbf{T} = \mathbf{H}_1^{-1}\mathbf{K}_1\mathbf{K}^{-1}\mathbf{H}$ is the transfer matrix.

Due to the periodicity of the infinite structure in the x direction, the vector $\mathbf{W}_n$ must satisfy the Bloch theorem [16]

$$\mathbf{W}_n = e^{iqa}\mathbf{W}_{n-1} \tag{12}$$

where q is the wave vector in the x direction. For convenience, we write all the one-dimension vectors as scalar form in this paper.

It follows that the eigenvalues of the infinite periodic pipe structures with fluid loading are the roots of the determinant

$$\left|\mathbf{T} - e^{iqa}\mathbf{I}\right| = 0 \tag{13}$$

where $\mathbf{I}$ is the 4×4 unit matrix. For given ω, equation(13) gives the values of q. Depending on whether q is real or has an imaginary part, the corresponding wave propagates through the beam (pass band) or is damped (band gap).

Analogously, one can get the eigenvalues of the infinite periodic pipe structures without fluid loading basing on the equation(2).

3. RESULTS AND DISCUSSION

For the periodical pipe with different wall material parameters illustrated as Figure1(a). As an example, we calculated the band structure of the pipe A being made of epoxy and pipe B being made of steel. The elastic parameters employed in the calculations were ρ_A =1180kg/m^3, $E_A = 4.35\times10^9$ Pa for epoxy; and ρ_B =7780kg/m^3, $E_B = 2.106\times10^{11}$ Pa for steel. And the inner and outer radii of the pipe are chosen as $r_{\mathrm{i}} = 0.09$ m, $r_{\mathrm{o}} = 0.1$ m. The lattice constant is a=2m, and a_1=a_2=1m. And the flow speed v=50m/s.

Figure2 illustrates the complex band structure. The real wave vector is illustrated in Figure2(a), and the absolute value of the imaginary part of complex wave vector is illustrated in Figure2(b). The shadowed region in Figure2(a) indicates the complete band gap, settling between 27Hz-40Hz, 112Hz-218Hz and 316Hz-497Hz. As for the two different real wave number k_1, k_2, there are two branches for a given frequency ω in Figure2(a).

During the gap ranges, wave vectors k_1 and k_2 have the imaginary parts [17]. They are illustrated as continues lines in Figure2(b), which can be used to describe the attenuation properties in band gaps. But from Figure2(b), one can find there is an imaginary wave vector (dashed line) within the frequency range of pass band. This is due to the near-field wave component k_3 and k_4. The values of k_3 and k_4 have imaginary part for all frequencies.

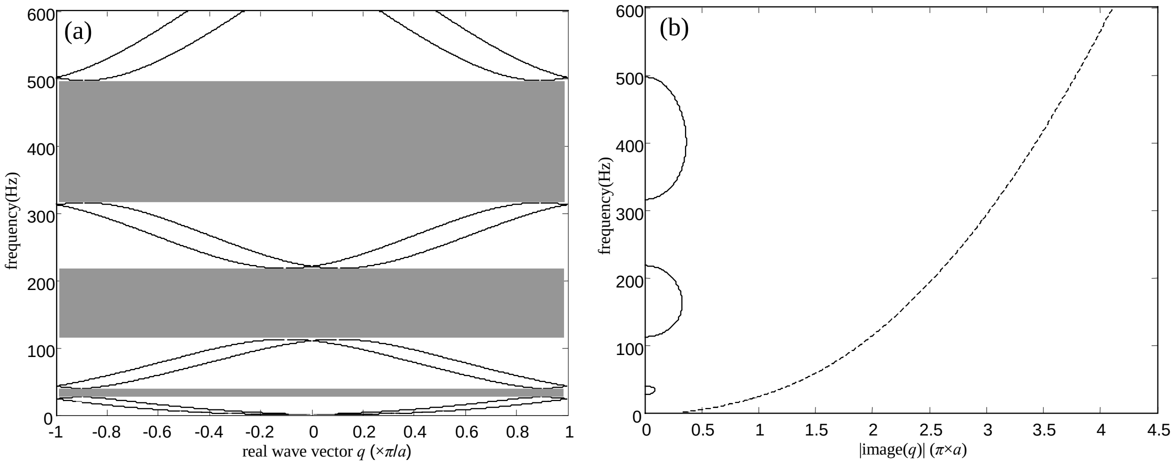

Figure2 The complex band structure of the periodic material pipe with fluid loading, internal fluid velocity v=50m/s

As comparing, we also calculate the complex band structure of the pipe without fluid loading illustrated in Figure3. The material parameters and the geometrical parameters are same with those in Figure2. The first two gap ranges are 41Hz-51Hz and 165Hz-425Hz. Comparing Figure2 and Figure3, one can find the effect of the fluid load make the gap frequency lower.

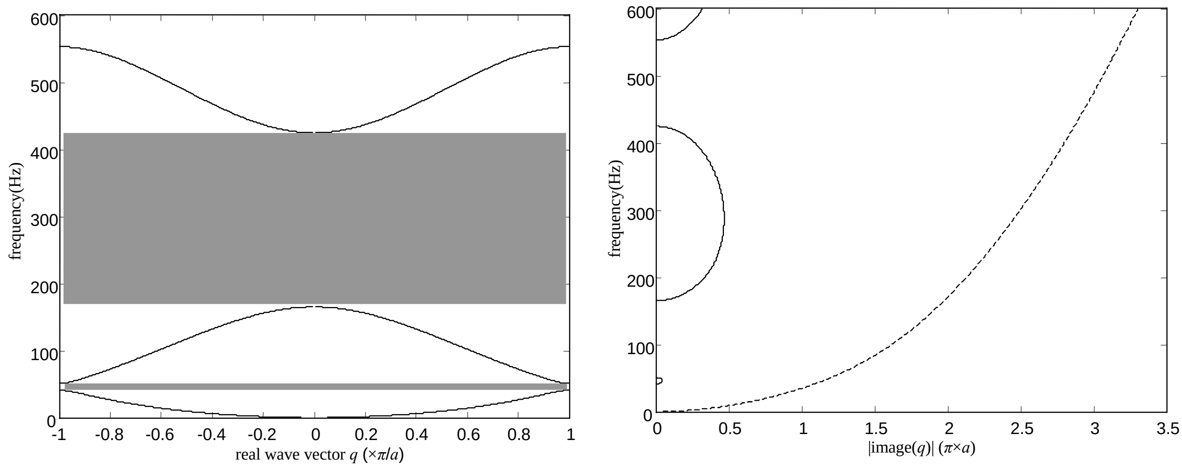

Figure3 The complex band structure of the periodic material pipe without fluid loading

For the different internal fluid velocity v, the wavenumbers will change [13]. So we should consider the effect of the internal fluid velocity v on the band gaps. In Figure4, the band structure of the pipe with fluid loading for velocity v=10m/s and v=100m/s. For v=10m/s, the first three gap ranges are24Hz-38.6Hz, 110Hz-216Hz and 314Hz-495Hz, and for v=100m/s, the gap ranges are 33.5Hz-43Hz, 119Hz-221Hz and 322Hz-502Hz. We can find the gap frequencies become higher with faster velocity v.

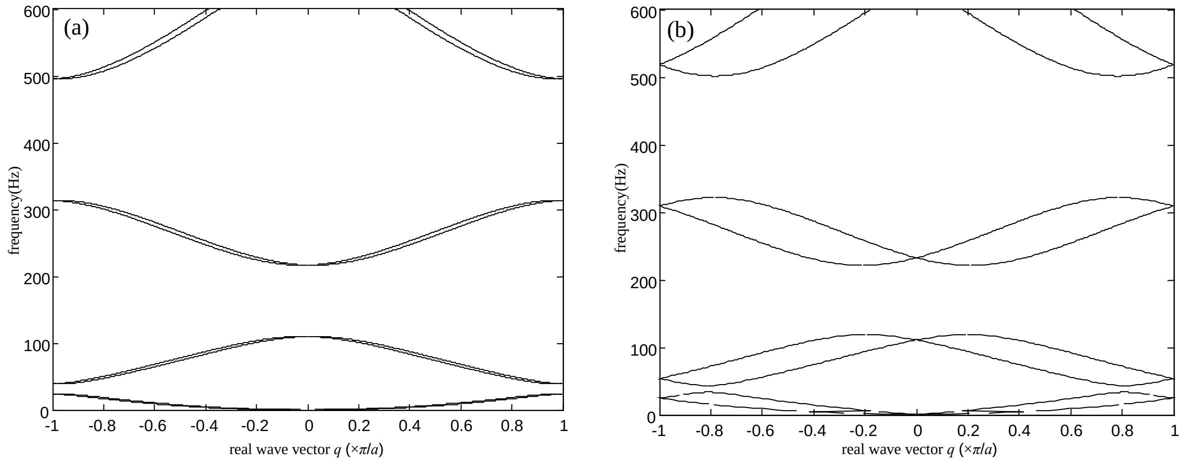

Figure4 The band structure of with different internal fluid velocity, (a) v=150m/s, (b)v=100 m/s

For the pipe wall with periodic material parameters, it will be not applicable in engineering. Now we consider the pipe wall with periodic geometrical parameters shown in figure1(b). In the calculation, the whole pipe wall is made up of steel. And the inner and outer radii of the pipe are chosen as $r_{\mathrm{iA}} = 0.09$ m, $r_{\mathrm{oA}} = 0.1$ m for section A and $r_{\mathrm{iB}} = 0.09$ m, $r_{\mathrm{oB}} = 0.12$ m. The lattice constant is a=2m, and a_1=a_2=1m. And the flow speed v=50m/s. Figure5 illustrates the complex band structure. The band gap ranges are 120Hz-179Hz and 556Hz-667Hz. And for the periodic pipe without fluid loading, the gap ranges are 148Hz-222Hz and 691Hz-830Hz as illustrated in figure6. The effect of the fluid loading also makes the gap frequency lower.

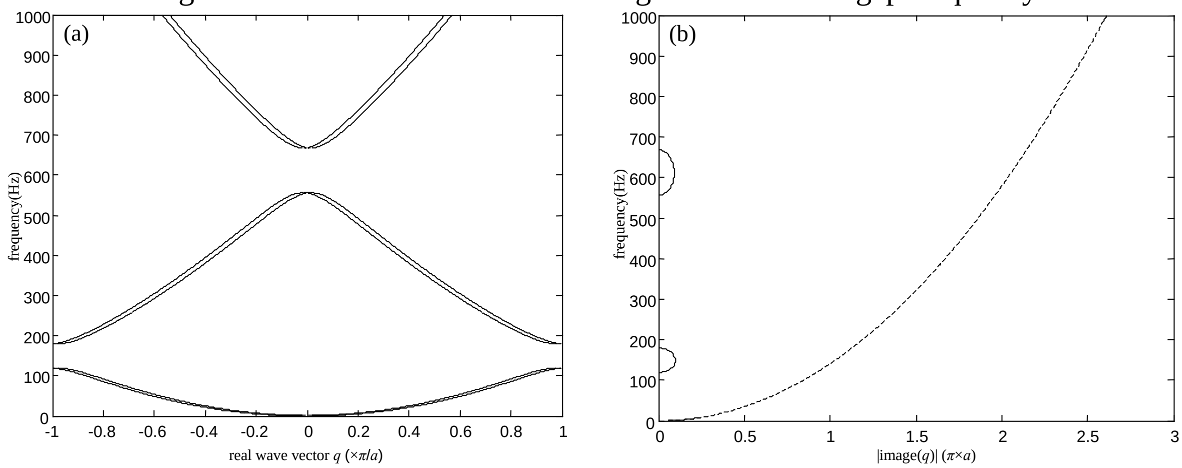

Figure5 The complex band structure of the periodic geometrical pipe with fluid loading, internal fluid velocity v=50m/s

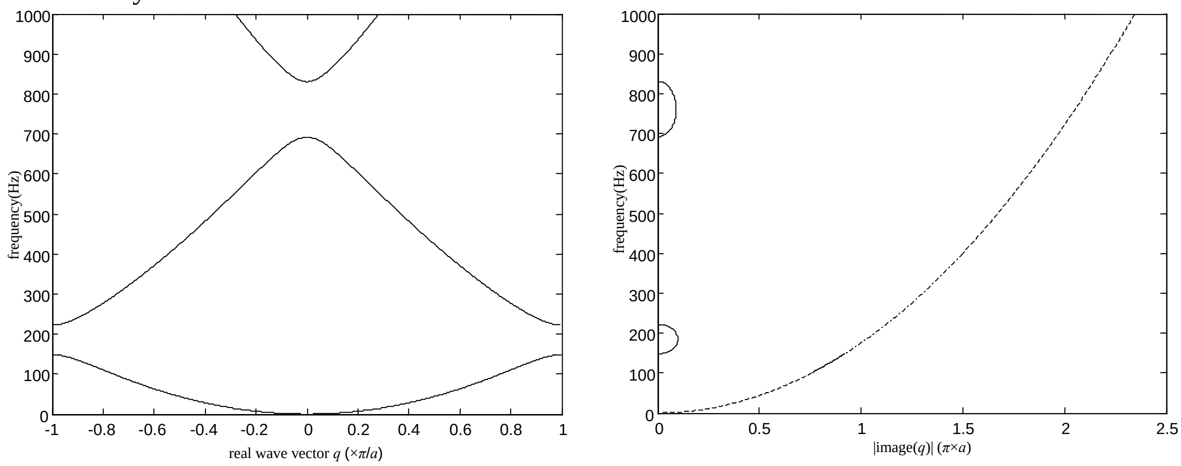

Figure6 The complex band structure of the periodic geometrical pipe without fluid loading

4. CONCLUSIONS

In conclusion, the flexural vibration for a periodic pipe system with fluid loading is studied theoretically in this paper. The transfer matrix method is provided to calculate the complex band structure of the periodic pipe.

By comparing the calculated results of the pipe with fluid loading to those without fluid loading, we find that find the effect of the fluid load make the gap frequency lower. Also, we can find the gap frequencies dependent on the internal fluid velocity. The gap frequencies become higher with faster velocity *v*.

The existence of flexural vibration gaps in periodic pipe with fluid loading gives a new idea in vibration control of pipe. The findings will be significant in the application of band gaps.

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