



# DIRECTIONAL PROPAGATION CHARACTERISTICS OF FLEXURAL WAVE IN TWO-DIMENSIONAL PERIODIC GRID-LIKE PLATE STRUCTURES

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#### Abstract

The directional propagation characteristics of elastic wave during pass bands in two-dimensional periodic grid-like thin plate are analysed by using the Plane Wave Expansion method to yield the phase constant surface. The directions and regions of wave propagation in the periodic grid-like structure for certain frequencies during pass bands are predicted with the iso-frequency contour lines of the phase constant surface, which are then validated with the harmonic responses of a finite two-dimensional such periodic structure with  $15 \times 15$  unit cells. These results are useful for controlling the wave propagation in the pass bands of periodic structure.

## **1. INTRODUCTION**

The propagation of elastic wave in periodic composites called phononic crystals (PCs) has received a great deal of attentions. Particular interests are focused on the characteristics of the so-called phononic band gaps in which elastic waves are all forbidden. The study on PCs materials and structures is driven partly by potential applications such as elastic filters, vibrationless environments for high-precision systems, transducer improvements, as well as pure physical concerns with the Anderson localization of sound and vibration. Except the band gaps, the remaining frequency ranges are called pass bands, where, as the terminology implies, waves are allowed to propagate through the composites [1-10].

PCs are periodic structures, and they have inherent and inevitable relations with periodic structures widely used in traditional engineering, such as honeycomb, grid, periodic stiffened beams and plates, etc. The application of PCs in the field of vibration reduction will be promoted if designs of periodic structures are improved with PCs related theory and band gaps are utilized to realize vibration attenuation and noise reduction of such structures.

Phase constant surface arises from the analysis of plane wave motion through the structure, which is often used in analysis of dynamic response of periodic cellular structure [11-14]. The phase constant surfaces are used to identify optimal design configurations for cellular grids featuring desired transmissibility levels in specified directions and enhanced vibration isolation capabilities [19].

In this paper, the characteristics of flexural wave propagation in two-dimensional periodic grid-like thin plate by using Plane Wave Expansion method to yield phase constant surface. The iso-frequency contour lines of the phase constant surfaces are obtained to predict the directions and regions of flexural waves propagation in the plate structure for the assigned frequency values. The harmonic response of the finite periodic cells was calculated by the Finite Element method, which confirms the predictions from the analysis of the phase constant surface and demonstrates the directions and regions of wave propagation in the thin plate structure.

## 2. CONSTANT SURFACE AND BAND STRUCTURE

Figure1 shows the cross section of representative two-dimensional periodic grid-like plate structure, where the *z* direction is vertical to the *xOy* plane. The black part is material *A* and the white part is material *B*, the rest part in one cell is vacuum. The periodicity of the square lattice is a=2(l+d). The thickness of the structure is h. If the thickness of the plate  $h \ll a$ , we can treat the structure as a thin plate.



Figure1 (a) The cross section of representative two-dimensional periodic grid-like plate structure and (b) one unit cell

The wave equation of flexural wave for thin plates of thickness *h* is known to be [10]

$$-\rho h \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2}{\partial x^2} \left[ D \left( \frac{\partial^2 w}{\partial x^2} + \sigma \frac{\partial^2 w}{\partial y^2} \right) \right] + 2 \frac{\partial^2}{\partial x \partial y} \left[ D (1 - \sigma) \frac{\partial^2 w}{\partial x \partial y} \right] + \frac{\partial^2}{\partial y^2} \left[ D \left( \frac{\partial^2 w}{\partial y^2} + \sigma \frac{\partial^2 w}{\partial x^2} \right) \right]$$
(1)

where w is the flexural displacement in the z direction,  $D = Eh^3/12(1-\sigma^2)$  is the flexural rigidity.

The Equation (1) can be rewritten as

$$-\alpha \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2}{\partial x^2} \left( D \frac{\partial^2 w}{\partial x^2} + \beta \frac{\partial^2 w}{\partial y^2} \right) + 2 \frac{\partial^2}{\partial x \partial y} \left( \gamma \frac{\partial^2 w}{\partial x \partial y} \right) + \frac{\partial^2}{\partial y^2} \left( D \frac{\partial^2 w}{\partial y^2} + \beta \frac{\partial^2 w}{\partial x^2} \right)$$
(2)

where  $\alpha = \rho h$ ,  $\beta = D\sigma$ ,  $\gamma = D(1-\sigma)$ .

Following the Plane Wave Expansion (PWE) methodology, Equation (2) can be derived in the form of a standard eigenvalue problem at hand:

where **k** is Bloch wave vector restricted within the first Brillouin zone, and **G**'' and **G**' are reciprocal vectors, and D,  $\alpha$ ,  $\beta$  and  $\gamma$  are periodic function of  $\mathbf{r} = (x, y), \alpha(\mathbf{G}), \beta(\mathbf{G}), \gamma(\mathbf{G})$  and  $D(\mathbf{G})$  are the Fourier coefficient.

Furthermore, based on the Equation (3), for all the wave vectors  $\mathbf{k} = (k_x, k_y)$  in the first Brillouin zone, we can derive a series of functions  $\omega = f(k_x, k_y)$ , which are denoted as phase constant surfaces [14]. The phase constant surfaces can provide rich information about the wave propagation characteristics of the considered domain for the periodic grid-like thin plate.

The group velocity of wave motion in a two dimensional periodic structure is given [14]

$$c_{gx} = \frac{\partial \omega}{\partial k_x}, c_{gy} = \frac{\partial \omega}{\partial k_y}$$
(4)

where  $c_{gx}$  and  $c_{gy}$  denote the group velocity components along the x and y direction respectively. For the undamped structures, the group velocity is equal to the velocity of propagation of the vibrational energy. Based on Equation (4), one can find that the direction of wave propagation in a 2D periodic grid-like thin plate can be estimated by taking the normal to the iso-frequency contour lines of the phase constant surfaces. So the direction of wave propagation at a given frequency can be determined through iso-frequency contour lines of the phase constant surfaces.

The grid-like thin plate composed of square lattice of Steel and Lucite is considered here. The structure parameters employed in the calculations are a=20 mm, l=5 mm, d=5 mm, h=0.4 mm. The material parameters used are  $\rho_{Lu} = 1180 \text{ kg} \cdot \text{m}^{-3}$ ,  $E_{Lu} = 4.35 \times 10^9 \text{ Pa}$ ,  $\mu_{Lu} = 1.59 \times 10^9 \text{ and}$ Poisson ratio  $\sigma = 0.37$  for Lucite;  $\rho_{Fe} = 7780 \text{ kg} \cdot \text{m}^{-3}$ ,  $E_{Fe} = 21.06 \times 10^{10} \text{ Pa}$ ,  $\mu_{Fe} = 8.1 \times 10^{10} \text{ and}$ Poisson ratio  $\sigma = 0.3$  for Steel.

Figure 2(a) is illustrated the calculated three-dimensional (3D) representation of the phase constant surface of the 2D grid-like thin plate. One can finds a gap between the third and fourth phase constant surface with frequency range from 5600 Hz to 6500 Hz. This indicates that there are no associated frequency values within the frequency range, so the elastic wave cannot propagate through the grid-like thin plate in the gap range. Figure 3(b) shows the band structure of the same 2D periodic grid-like thin plate. The frequency range of the first band gap in the band structure is same with that in phase constant surfaces. Therefore, the band structures of PCs can be described intuitively by the phase constant surfaces.



Figure 2(a) 3D phase constant surfaces of 2D grid-like thin plate for  $k_x$  and  $k_y$  from  $-\pi/a$  to  $\pi/a$ , (b) band structure of 2D grid-like thin plate.

### **3. DIRECTIONS OF WAVE PROGATION**

For 2D periodic grid-like thin plate, the characteristics of the wave propagation may be visualized in either one of the following two ways: the energy flow is along the line of the steepest ascent on the phase constant surface, or equivalently, normal to the  $\omega$ =constant lines on a contour plot of the phase constant surface. This property can be utilized to identify the region the vibration waves cannot propagate through at certain frequencies.

As examples, the contour lines of 400 Hz and 4500 Hz are analyzed for determining the propagation direction of the flexural wave in the grid-like thin plate. For a given point on the contour, let  $\theta$  be the angle between the direction of the normal and the positive direction of *x*-axis, then  $\theta$  varies within the range of 0°~360°. *N* points, which evenly distribute on the contour, are chosen, and *N*  $\theta$  values are obtained. The numbers of  $\theta$  values are counted within each unit of 5° range for the full range of 0°~360°, yielding total 72 numbers, which are plotted as a function of angles and gives a polar plot.

The contour line with directional vector for frequency value of 400Hz is shown in Figure 3 (a) and the corresponding polar plot of the directional propagation is shown in Figure 3(b). The numbers 5, 10, 15 and 20 along the direction from the center out in the polar plot is the scale for the number of  $\theta$  values located in a specified 5° range. Figure 3(b) indicates that the main directions of the flexural wave propagation of 400 Hz are along the lines of  $\pm 45^{\circ}$ .



Figure3 (a) The contour line with directional vector for frequency value of 400Hz. (b) The polar

plot of the directional propagation of flexural wave of 400Hz.

The contour line with directional vector for frequency value of 4500Hz and the corresponding polar plot is shown in Figure 4. From the Figure 4(b), the directions of wave propagation vary between  $0^{\circ}$ ~360°, and no prominent direction of wave propagation is found.



Figure4 (a) The contour line with directional vector for frequency value of 4500Hz. (b) The directions of wave propagation for frequency value of 4500Hz.

## 4. RESPONSE TO HARMONIC LOADING

The predictions from the analysis of phase constant surface and its contour lines can be readily verified through computation of the harmonic response of a finite structure of 2D periodic grid-like thin plate. Here the harmonic response of a finite periodic with 15×15unit cells is calculated with the finite element (FE) method. An out of plane point excitation is applied at the center of the structure. The harmonic response for frequency of 400Hz is shown in Figure 5(a), which indicates that the main directions of propagation of the flexural wave of 400Hz through the structure are along  $\pm 45^{\circ}$ . As for 4500Hz, the direction of wave propagation covers all direction illustrated in Figure 5(b). The results calculated by the FE method confirm the results predicted by analysis the contour lines of the phase constant surface.



Figure 5 (a) The harmonic response for the flexural wave of frequency of 400Hz, (b) of 4500Hz.

### **4. CONCLUSIONS**

In conclusion, the propagation characteristics of flexural waves in 2D periodic grid-like thin plate are analyzed with the PWE method to yield phase constant surfaces. By taking the normal to the iso-frequency contour lines of the phase constant surfaces and evaluating the distribution of the angle of the normal, high directivity of flexural wave propagation for some frequency values is found. Harmonic response of a finite structure calculated by the FE method confirms the PWE method predictions. At some given frequency, the directions of flexural wave can propagate through the 2D periodic grid-like thin plate along  $\pm 45^{\circ}$ . These results are useful for controlling the wave propagation in the pass bands of periodic structure.

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