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AN ASYMPTOTIC APPROACH FOR THE COUPLED DISPERSION CHARACTERISTICS OF A FLUID-FILLED CYLINDRICAL SHELL

Abhijit Sarkar and Venkata R. Sonti

Facility for Research in Technical Acoustics
Department of Mech. Engg., Indian Institute of Science,
Bangalore - 560012, India

sonti@mecheng.iisc.ernet.in

Abstract

The coupled wavenumbers in the axisymmetric mode of a fluid-filled cylindrical shell are studied. The coupled dispersion equation of the system is rewritten in the form of the uncoupled dispersion equation of the structure and the acoustic domain, with an added fluid-loading term (ϵ) due to the coupling. Using the smallness of poisson's ratio, a double-asymptotic solution to this equation is found for large and small values of ϵ. Analytical expressions are derived for the coupled wavenumbers. Different asymptotic expansions are used for different frequency ranges with continuous transitions occurring between them. The wavenumber solutions are continuously tracked as ϵ varies from small to large values. A general trend observed is that a given wavenumber branch transits from a rigid-walled solution to a pressure-release solution with increasing ϵ. Also, it is found that at any frequency where two wavenumbers intersect in the uncoupled analysis, there is no more an intersection in the coupled case, but a gap is created at that frequency. Only the axisymmetric mode is considered, however the findings can be extended to the higher order modes.

1. INTRODUCTION

A classical problem in structural-acoustics concerns the wavenumber characteristics of a fluid-filled flexible cylindrical shell of infinite length [1]. The fluid-structure coupled wavenumbers can be found numerically as solutions to the coupled dispersion equation for a particular set of system parameters [2]. However, these numerical solutions do not clearly bring out the physics, such as transition of the solutions as a function of the coupling parameter. Asymptotic analysis can be efficiently used to continuously track wavenumber solutions as a function of a parameter (ϵ) varying from small to large values. This provides additional insights into the coupling effect along with analytical formulas for the solution. Asymptotic analysis has been used for finding the coupled wavenumbers of a structure in contact with of an unbounded acoustic medium [4]. However, no such analogous studies have come to our notice for the case of flexible acoustic ducts. In this direction a work on planar geometry has been completed [5]. In this study, the axisymmetric vibrational mode of an infinite fluid-filled cylindrical shell is considered which is more complex due to the curvature effects.

2. FORMULATION

Using Donell-Mushtari theory for cylindrical shells [2, 3], the governing equation for the *in vacuo* free vibration of an infinite cylindrical shell of radius a, thickness h, at a circular frequency ω is given by

$$\overbrace{\begin{bmatrix} -\Omega^2+\kappa^2+\frac{1-\nu}{2}n^2 & \frac{1}{2}(1+\nu)n\kappa & \nu\kappa \\ \frac{1}{2}(1+\nu)n\kappa & -\Omega^2+\frac{1-\nu}{2}\kappa^2+n^2 & n^2 \\ \nu\kappa & n^2 & -\Omega^2+1+\beta^2\left(\kappa^2+n^2\right)^2 \end{bmatrix}}^{\mathbf{L}} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (1)$$

where u, v, w are the vibrational amplitudes in the axial (x), circumferential (θ) and radial (r) directions, respectively. n is the circumferential mode of vibration, κ=$k_x a$ is the non-dimensional wavenumber in the axial direction (k_x being the corresponding dimensional quantity), $\Omega = \omega a/c_L$ is the non-dimensional frequency and β^2=$h^2/(12a^2)$. Also, ρ_s is the shell-density, ν is the poisson's ratio and c_L is the extensional phase speed of the shell material. The square matrix in the above equation shall be denoted by **L**.

For the cylindrical shell filled with an acoustic fluid of density ρ_f and sonic velocity c_f, the third diagonal term is modified by a fluid loading term as follows [2]

$$L_{33}=-\Omega^2+1+\beta^2\left(\kappa^2+n^2\right)^2-\frac{\Omega^2\rho_f J_n(k_s^r a)}{\rho_s k_s^r h J_n'(k_s^r a)}, \quad (2)$$

where $\kappa^2+\left(k_s^r a\right)^2=\Omega^2\left(c_L/c_f\right)^2$. J_n denotes the n^{th} order Bessel function of the first kind and $'$ denotes differentiation with respect to the argument of the function.

It is apparent from the non-diagonal form of $\boldsymbol{L}$ that the essential complication introduced by the shell curvature is to couple the dynamics in the three perpendicular directions. The radial and the circumferential directions are coupled because of the curvature. The axial vibrations in turn get coupled to the radial vibrations due to the poisson's effect [3].

From now on, we shall consider the axisymmetric circumferential mode only (n = 0). This mode of vibration is completely due to the extensional nature of vibration and hence the torsional vibration is completely uncoupled from the radial and axial vibrations. This is also seen from the form of $\boldsymbol{L}$ having L_{21}=L_{12}=L_{23}=L_{32}=0. As is clear from the nature of L_{13}=L_{31}=$\nu\kappa$, the coupling between the radial and the axial directions is due to the poisson's effect.

Thus, a non-trivial solution to equation (1) is obtained when $\kappa = \sqrt{2/(1-\nu)}\Omega$. The corresponding solution u=w=0 and $v\neq 0$, represents a torsional wave, traveling at a speed ω/k = $c_L\sqrt{(1-\nu)/2}$. With the approximation 1-$\nu^2\approx 1$, we have c_T = $\sqrt{G/\rho_s}$, where G is the shear modulus of elasticity of the shell material. As elaborated earlier, fluid-loading affects the L_{33} term only and thus, the torsional dynamics discussed above remains unaffected. In the remainder of this article, we shall exclude this mode from further discussion.

The coupled axial and radial motion may be represented by a reduced set of equations as follows

$$\begin{bmatrix} L_{11} & L_{13} \\ L_{31} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \quad (3)$$

where L_{11}, L_{13}=L_{31} are given by equation (1) and L_{33} is given by equations (1) and (2) for the *in vacuo* and the fluid-loaded cases, respectively. To obtain a non-trivial solution to the equation above, the determinant needs to be equated to zero. This leads to the dispersion equations for the *in vacuo* and the fluid-loaded cases which can be solved numerically [2]. As elaborated earlier, such numerical solutions do not clearly give the physical insights into the solutions. We shall use asymptotics to arrive at the wavenumber-frequency characteristics for both the *in vacuo* and the coupled case.

3. UNCOUPLED ANALYSIS

In this section, we shall find the acoustic wavenumber when the cylinder is rigid-walled and also the *in vacuo* structural wavenumber. These solutions shall be referred to as the uncoupled acoustic and the uncoupled structural wavenumbers, respectively.

3.1. Uncoupled Acoustic Wavenumber

The wavenumber-frequency characteristics of a cylindrical acoustic waveguide have been derived in detail in [6]. Here, we present the main results.

For a cylindrical waveguide with rigid-walled or pressure-release boundary condition, the acoustic pressure is represented by a traveling wave in the axial direction (x) and by a Bessel function in the radial direction (r). For the axisymmetric case, the radial mode is of the form $J_0(\lambda r/a)$, where λ is given in Figure 1. The wavenumber of the x-directional traveling wave is $k_x = \sqrt{\omega^2/c_f^2 - \lambda^2/a^2}$.

Mode	Rigid walled	Pressure release
Plane wave	0	not applicable
First cut-on	3.832	2.405
Second cut-on	7.016	5.52

Figure 1. λ values of a cylindrical acoustic duct under different boundary conditions.

3.2. Uncoupled Structural Wavenumber

The determinant of the reduced matrix in equation (3) is as follows

$$\overbrace{\left(\kappa^2-\Omega^2\right)}^{L}\underbrace{\left(\kappa^4-\frac{\Omega^2-1}{\beta^2}\right)}_{B}-\frac{\nu^2}{\beta^2}\kappa^2=0. \tag{4}$$

With ν=0, the term L has roots $\kappa = \pm\Omega$ which implies $k_x = \omega/c_L$. This is the longitudinal wave in the x-direction propagating at the extensional wave-speed (c_L). The term B in the equation (4) has the following roots $\kappa = \pm\sqrt[4]{\frac{\Omega^2-1}{\beta^2}},\ \pm i\sqrt[4]{\frac{\Omega^2-1}{\beta^2}}$., which represents the flexural wave in x direction.

With $0<\nu^2\ll 1$, (as is the case in practice), we expect solutions to the dispersion equation (4) to be close to the solutions described above. To obtain these solutions we use a regular perturbation method. Substituting $\kappa = k_0 + \nu^2 k_1$ in equation (4) and performing a series expansion about ν=0, we get

$$\left({k_0}^2-\Omega^2\right)\left({k_0}^4-\frac{\Omega^2-1}{\beta^2}\right)+\left[4\left({k_0}^2-\Omega^2\right){k_0}^3k_1+2\,k_0\,k_1\left({k_0}^4-\frac{\Omega^2-1}{\beta^2}\right)-\frac{{k_0}^2}{\beta^2}\right]\nu^2+\mathcal{O}\left(\nu^4\right)=0. \tag{5}$$

Equating the $\mathcal{O}(1)$ term to zero, we obtain the roots of κ as discussed previously for the case of ν=0. Putting k_0=Ω, in the equation at $\mathcal{O}(\nu^2)$ we get

$$k_1 = \frac{1}{2}\frac{\Omega}{\Omega^4\beta^2-\Omega^2+1} \text{ and } \kappa = \Omega + \frac{1}{2}\frac{\Omega\nu^2}{\Omega^4\beta^2-\Omega^2+1} + \mathcal{O}(\nu^4). \tag{6}$$

It is verified through numerical analysis that the above solution gives the wavenumber corresponding to the dominantly longitudinal wave for all frequencies except $\Omega \approx 1$. This is because the correction term k_1 becomes large for $\Omega \approx 1$. This is typical of the perturbation method and arises due to improper scaling of the asymptotic term (ν^2 in this case) [5].

Similarly, putting $k_0 = \sqrt[4]{(\Omega^2-1)/\beta^2}$, in the $\mathcal{O}(\nu^2)$ term of equation (5) we get

$$k_1 = \frac{1}{4\beta^2\sqrt[4]{\frac{\Omega^2-1}{\beta^2}}\left(\sqrt{\frac{\Omega^2-1}{\beta^2}}-\Omega^2\right)} \text{ and } \kappa = \sqrt[4]{\frac{\Omega^2-1}{\beta^2}} + \frac{\nu^2}{4\beta^2\sqrt[4]{\frac{\Omega^2-1}{\beta^2}}\left(\sqrt{\frac{\Omega^2-1}{\beta^2}}-\Omega^2\right)}. \tag{7}$$

This solution corresponds to the bending wave and also matches with numerical analysis results. This solution also remains accurate for all frequencies except $\Omega \approx 1$.

4. COUPLED ANALYSIS

In this section, we present the coupled analysis of the problem described in section 2 for $\Omega >$ 1. The problem under suitable conditions will be posed as a perturbation to that described in section 3. Then, coupled wavenumber solutions will be found by using a regular perturbation method. With $J_0'(x) = -J_1(x)$, the coupled dispersion relation is

$$\overbrace{\left(-\Omega^2+\kappa^2\right)}^{L}\left[\overbrace{\left(-\Omega^2+1+\beta^2\kappa^4\right)}^{B}\overbrace{J_1(\xi)}^{R}\overbrace{\xi}^{A}+\underbrace{\Omega^2\epsilon J_0(\xi)}_{F}\right]-\underbrace{\nu^2\kappa^2 J_1(\xi)\,\xi}_{P}=0, \tag{8}$$

$$\text{where } \xi = \sqrt{\left(\frac{c_L}{c_f}\right)^2\Omega^2-\kappa^2} \text{ and } \epsilon = \frac{\rho_f a}{\rho_s h}.$$

We now describe the physical relevance of each term in the equation above. As explained in the previous section, the terms L and B equated to zero are the dispersion relations corresponding to the structural *in vacuo* longitudinal and bending waves, respectively. The solution to R=0, represents the acoustic cut-on waves in a rigid-walled cylindrical waveguide. The root of A represents the acoustic plane wave. The term P represents the poisson-effect in the structure. We have observed earlier (see 3.2), that this can be taken into account by considering ν as a small asymptotic parameter and solutions can be obtained for $\nu \to 0^+$. The term F represents the effect of fluid-loading. This term is of $\mathcal{O}(\epsilon)$ magnitude. With F and P being asymptotic terms, the following two cases arise :

1. When F and P are zero (*i.e.* ϵ and ν are zero), the roots of equation (8) are the roots of L, B, R and A. If $0<\epsilon\ll 1$, F becomes a small term of magnitude $\mathcal{O}(\epsilon)$. On the other hand, since $\nu<1$ in practice, P representing an $\mathcal{O}(\nu^2)$ term is also small. Thus, inclusion of the asymptotic terms F and P in equation (8), shall give solutions which are perturbations of the roots of L, B, R and A.
2. When $P \to 0$ but $F \to \infty$ (*i.e.* $\epsilon \to \infty$ and $\nu \to 0$), the solution of equation (8) approaches the roots of $J_0(\xi)$=0. This root represents the wavenumber for the pressure-release acoustic duct. In addition, there is another solution which approaches the root of L, which will not be discussed here for brevity.

We will arrive at the asymptotic solution of the coupled dispersion equation (8) by considering the cases of $0<\epsilon\ll 1$ and $1\ll\epsilon<\infty$ separately.

4.1. Large ϵ (near pressure-release acoustic mode)

To model the effect of large ϵ, we make a transformation $\epsilon'=1/\epsilon$, where $0<\epsilon'\ll 1$ in equation (8). This results in the following equation

$$\left(-\Omega^2+\kappa^2\right)\left[\epsilon'\left(-\Omega^2+1+\beta^2\kappa^4\right)J_1(\xi)\xi+\Omega^2 J_0(\xi)\right]-\epsilon'\nu^2\kappa^2 J_1(\xi)=0. \tag{9}$$

A form of the solution $k=k_0+b_1\nu^2+a_1\epsilon'$ is subsituted in equation (9) and a double series expansion about ϵ' and ν is performed. Balancing terms at $\mathcal{O}(1)$ gives the equation for k_0

$$(-\Omega^2+k_0^2)\Omega^2 J_0\left(\sqrt{\left(\frac{c_L\Omega}{c_f}\right)^2-k_0^2}\right)=0. \tag{10}$$

One solution for k_0 is the *in vacuo* longitudinal wavenumber (which will not be discussed further). The second solution is the pressure-release acoustic wavenumber, for which a perturbed solution shall be found under the coupling effect. k_0 is given by the following equation

$$\left(\frac{c_L\Omega}{c_f}\right)^2-k_0^2=2.405^2. \tag{11}$$

At $\mathcal{O}(\epsilon')$ of the double series expansion (equation(9)), we obtain

$$(-\Omega^2+k_0^2)\left[(-\Omega^2+1+\beta^2 k_0{}^4)J_1(\xi_0)\sqrt{\xi_0}+k_0 a_1\Omega^2 J_1(\xi_0)\Big/\sqrt{\left(\frac{c_L\Omega}{c_f}\right)^2-k_0^2}\right]=0, \tag{12}$$

where, k_0 is the root of equation (11) and $\xi_0=\sqrt{\left(\frac{c_L}{c_f}\right)^2\Omega^2-{k_0}^2}$.

The above equation may be solved to obtain a_1. Similarly, balancing terms at $\mathcal{O}(\nu^2)$, we obtain an equation for b_1. b_1 is found to be zero. Thus, the asymptotic solution has no $\mathcal{O}(\nu^2)$ term. This is expected as in equation (9) the ν^2 term comes along with ϵ'.

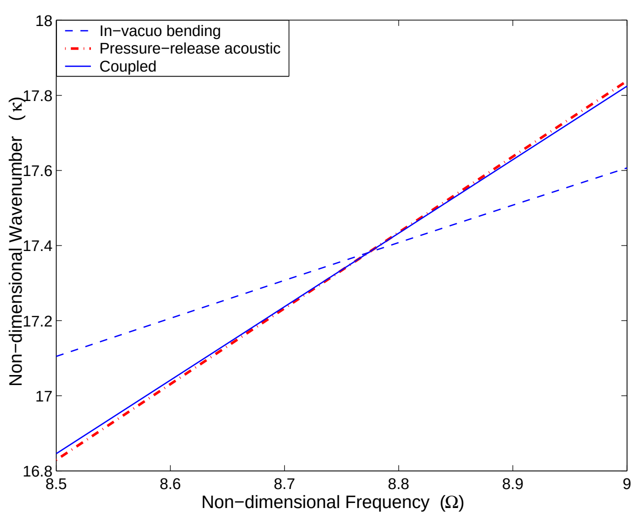

Figure 2. Wavenumber solution for for large ϵ. The parameters chosen are $h/a=0.1$, $c_L/c_f=2$, $\epsilon=1.25$ and $\nu=0.25$.

Other than frequencies near the cut-on frequency for the first pressure-release mode, the correction factor a_1 remains small for all frequencies. The asymptotic solution for this range is validated numerically for a suitable choice of system parameters. The coupled wavenumbers are plotted in figure (2) for $h/a=0.1$, $c_L/c_f=2$, $\epsilon=1.25$ ($\epsilon'=0.8$) and $\nu=0.25$ along with the *in vacuo* bending wavenumber (equation (7)) and the first cut-on wavenumber of the pressure-release acoustic duct. As is seen from the plot, the coupled wavenumber is a perturbation to the pressure-release acoustic wavenumber. Note, the nature of perturbation (*i.e.* positive or negative) changes at the frequency where the *in vacuo* bending wavenumber intersects the acoustic pressure-release wavenumber. Below (or above) this frequency, the coupled wavenumber is greater (or lesser) than the acoustic pressure-release wavenumber.

4.2. Small ϵ (near bending wave and near acoustic plane wave)

As for the case of large ϵ we assume, a double asymptotic expansion method in the form $\kappa = k_0 + a_1\epsilon + b_1\nu^2$. The procedure to obtain a_1 and b_1 is the same as discussed previously and will not be repeated here. The final results are discussed in the following.

Away from coincidence: The possible solutions for k_0 ($\mathcal{O}(1)$ solutions to the expansion) are indicated in Table 1 along with the physical relevance of the solution. Here we shall be interested in finding the perturbations corresponding to the bending wave and the acoustic plane wave. The procedure is similar in case of the other solutions. For the bending wave, we have

$\mathbf{k_0}$	Ω	$\sqrt[4]{\frac{\Omega^2-1}{\beta^2}}$	$\frac{c_L}{c_f}\Omega$	Root of $J_0(\sqrt{(c_L\Omega/c_f)^2 - k^2})$
Physical Description	Longitudinal wave	Bending wave	Acoustic plane wave	Rigid-walled acoustic duct cut-on

Table 1. $\mathcal{O}(1)$ solution for coupled wavenumbers with small fluid-loading.

$$b_1 = -\frac{1}{4}\frac{\beta}{\sqrt[4]{(\Omega^2-1)\,\beta^2}\left(\Omega^2\beta^2 - \sqrt{(\Omega^2-1)\,\beta^2}\right)},\ a_1 = -\frac{\Omega^2\beta J_0\left(\Theta\right)}{4\Theta J_1\left(\Theta\right)\left(\left(\Omega^2-1\right)\beta^2\right)^{3/4}}, \quad (13)$$

where $\Theta = \sqrt{\frac{(c_L/c_f)^2\Omega^2\beta^2 - \sqrt{(\Omega^2-1)\beta^2}}{\beta^2}}$.

For the acoustic plane wave, we have $b_1 = 0$ and $a_1 = \dfrac{\Omega}{2\left[-\Omega^2 + 1 + \left(\frac{c_L\Omega}{c_f}\right)^4\beta^2\right]}$. (14)

Near coincidence: The correction factor a_1 obtained for both the acoustic plane wave and the flexural wave become large at frequencies near coincidence, where the wavenumber of the *in vacuo* flexural wave equals the wavenumber of the acoustic plane wave. This is obtained by simultaneously solving for the components B and A in equation (8). The coincidence frequency (Ω_c) and the wavenumber at coincidence (κ_c) are given by

$$\Omega_c = \sqrt{\frac{1}{2}\frac{1+\sqrt{1-4\,\beta^2(c_L/c_f)^4}}{\beta^2(c_L/c_f)^4}},\ \kappa_c = \frac{c_L\Omega_c}{c_f}.$$

Note, Ω_c decreases with increase in both β and c_L/c_f, but remains more than unity. For cases when $\beta c_L^2/c_f^2 < 1$, we have $\Omega_c \approx \frac{c_f^2}{\beta c_L^2} \gg 1$ and $\kappa_c = \frac{c_f}{\beta c_L}$. Under these conditions for $\Omega \approx \Omega_c$, the term B in equation (8) may be simplified to $-\Omega^2 + \beta^2\kappa^4$. As we are looking for the solution around the coincidence frequency, κ should be such that $-\Omega^2 + \beta^2\kappa^4 \approx 0$ (near the wavenumber of the *in vacuo* flexural wave) and $\kappa \approx c_L\Omega/c_f$ (near the wavenumber of the acoustic plane wave). Under these conditions, the argument of the Bessel functions in equation (8) is small. Note, for small x, $J_0(x) \approx 1$ and $J_1(x) \approx x/2$. Also, as ν^2 is another asymptotic term, it has no effect on a_1 which is the correction due to the asymptotic term ϵ. To model the effect at frequencies near coincidence, we substitute $\Omega = \Omega_c + \epsilon\Psi$ in equation (8) leading to

$$\frac{-\bar{\Omega}^2+\beta^2\kappa^4}{2}\left(\frac{c_L^2\bar{\Omega}^2}{c_f^2}-\kappa^2\right)+\epsilon\bar{\Omega}^2=0,\ (\bar{\Omega}=\frac{c_f^2}{\beta c_L^2}+\epsilon\Psi). \tag{15}$$

To find the coupled wavenumber near acoustic plane-wave, we substitute $\kappa = (c_L/c_f)(\Omega_c+\epsilon\Psi)+a_1\sqrt{\epsilon}+a_2\epsilon$ in the equation above and perform a series expansion about ϵ. Balancing terms at $\mathcal{O}(\epsilon)$ we get a_1=$\pm\frac{1}{2}$. Similarly, to find the coupled wavenumber near the bending wave substitute κ=$\sqrt{(\Omega_c + \epsilon\Psi)/\beta}$+$a_1\sqrt{\epsilon}$+$a_2\epsilon$ and repeat the process of order balance to get a_1=$\pm\frac{1}{2}$.

To choose the appropriate sign of a_1 in the above two cases we use a continuity argument. We have seen for Ω sufficiently far from Ω_c, when $\Omega < \Omega_c$, the correction term corresponding to the wavenumber of the near acoustic-plane wave is negative, and when $\Omega > \Omega_c$, the correction term corresponding to the wavenumber of the near acoustic-plane wave is positive. Thus, a_1=$-1/2$ (or $+1/2$) when $\Omega \approx< \Omega_c$ (when $\Omega \approx> \Omega_c$). Similarly, for the coupled wavenumber near the *in vacuo* bending wavenumber a_1=$1/2$ (or $-1/2$) when $\Omega \approx< \Omega_c$ (or $\Omega \approx> \Omega_c$). Thus, the perturbed acoustic branch below Ω_c continues as the perturbed structural branch beyond Ω_c and *vice-versa*. Also, each branch encounters a jump at Ω_c.

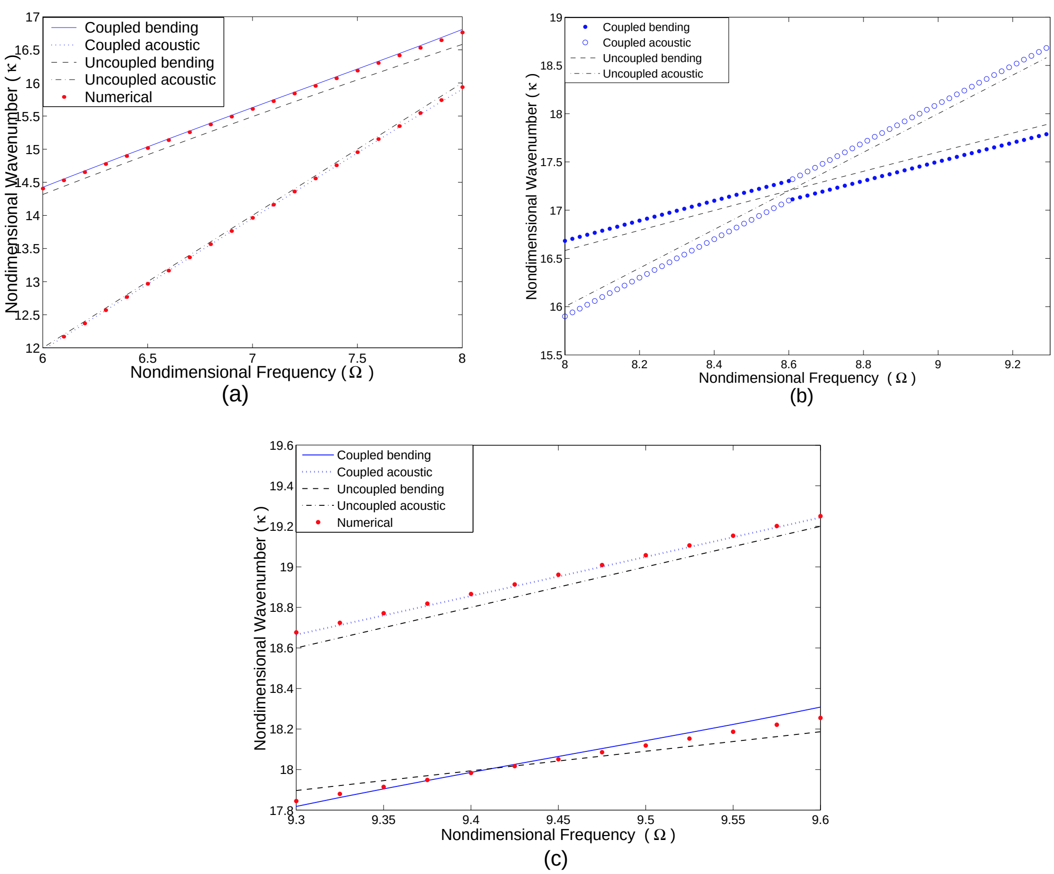

Figure 3. Wavenumber solution for small ϵ (a) Below coincidence frequency (b) Around coincidence (c) Above coincidence frequency. The parameters chosen are h/a=0.1, c_L/c_f=2, ϵ=0.2 and $\nu = 0.25$.

The asymptotic solution obtained above was validated numerically. Figures (3) (a), (b) and (c) show the results for below coincidence, near coincidence and above coincidence frequency ranges, respectively, for h/a=0.1, c_L/c_f=2, ϵ=0.2 and $\nu = 0.25$. As seen from the figures, a continuous transition is obtained across the frequency ranges. The below coincidence plot in figure (3) starts from Ω=6 onwards, for the sake of clarity. The match is good for frequencies beyond Ω=1. The above coincidence region is plotted till the frequency at which the *in vacuo* bending wavenumber equals the wavenumber of the first rigid-walled acoustic duct cut-on. At this frequency, again a coincidence-like phenomenon happens with the first cut-on mode instead of the plane wave. In this range a_1 as given in equation (13) becomes large. An alternative asymptotic expansion needs to be found for this range.

5. CONCLUSION

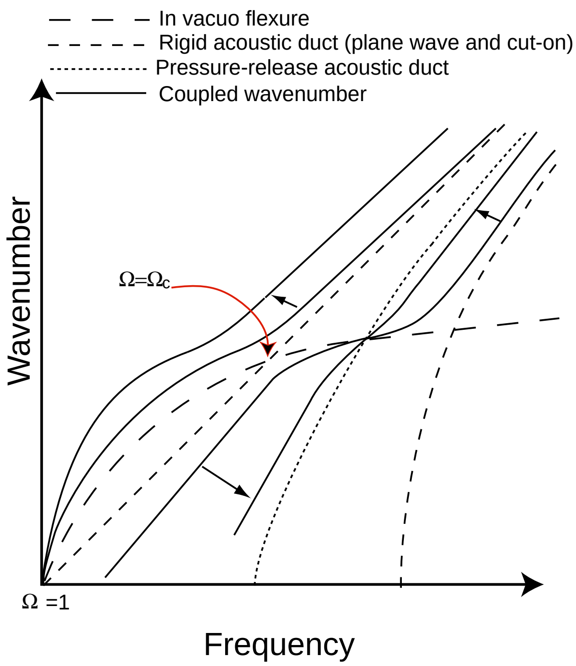

Figure 4. Schematics of the coupled wavenumber solution. Arrows indicate transition of solutions as wavenumber increases.

The relation of the coupled wavenumbers to the *in vacuo* bending wavenumber, and the uncoupled acoustic wavenumbers (planewave and both the cut-on waves) is established using asymptotics. A schematic of the results found is presented in figure (4). For small ϵ, the coupled wavenumbers are perturbations of the *in vacuo* bending wave and the wavenumbers of the rigid-walled acoustic waveguide (including cut-on). At the coincidence frequency, the branches corresponding to the uncoupled flexural wave join with that of the uncoupled acoustic plane wave and *vice versa*. With increasing ϵ the perturbations increase until for large values the coupled wavenumbers can be better identified as perturbations to the pressure-release acoustic duct. However, for all values of ϵ there is a solution of the coupled wavenumber which is greater than the *in vacuo* bending wavenumber and also the wavenumber of the acoustic plane wave. This branch for large ϵ though indicated in the schematic result has not been discussed in the article(it can be found numerically). The derivations presented can be used to continuously track the coupled wavenumber solutions from small to large ϵ values. Even a first order asymptotic expansion matched well with the numerical results.

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