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ACTIVE CONTROL OF INTERIOR NOISE IN A RECTANGULAR CAVITY WITH TWO FLEXIBLE PANELS

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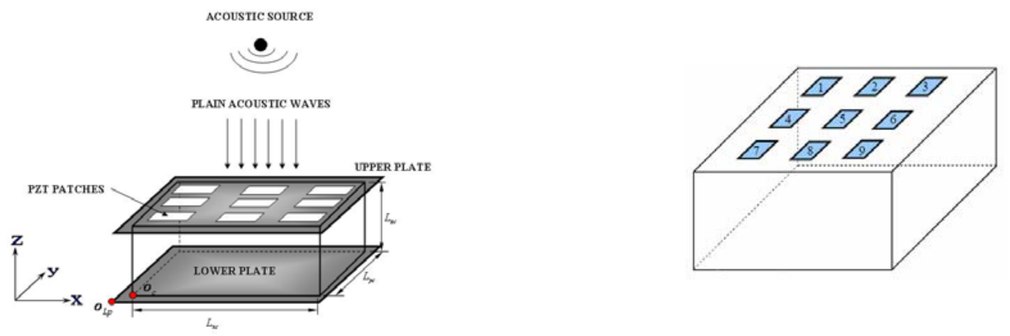
Abstract

In this work, a numerical effort is presented for modeling and control of structure-acoustics coupled systems. Modeling of sound transmission through a panel-cavity-panel system is presented. An approximate series solution is assumed and the solution is obtained using Galarkin's method. The system to be modeled is consisting of a rectangular cavity with two flexible panels, one at the top of the cavity while the other at the bottom and four other fixed boundaries. PZT pair patches are considered to be bonded to the top panel and each pair is assumed to produce a pure moment actuation when an electric drive signal is used to excite them. The flexible panel is exposed to an external pressure excitation due to a planar wave generated by a sound source mounted above the cavity. Displacements at the mid points are calculated for the upper and lower panels. The developed model is controlled using the optimal LQR control law. The numerically obtained time responses from the compensated model are found to be acceptable compared to the uncompensated ones. It is found that the actuation of the upper panel can decrease the vibration of the lower one rather than decreasing the acoustic pressure inside the cavity.

1 INTRODUCTION

Control of noise and vibration is important for many civil, industrial, and defence applications. In Active Structural Acoustical Control (ASAC) **[1]**, this can be considered a modified version of Active noise Control (ANC), one takes advantage of vibrating structural elements as secondary noise sources to cancel the sound fields generated by a primary noise source. Considerable effort has been devoted to the modeling of structural acoustics, in particular, for enclosures with flexible boundaries. The efforts of Dowell and Voss **[2]** and Lyon **[3]** represent some of the early investigations into modeling of vibrations of plates backed by a cavity. Balachandran et al. **[4]** have developed a mechanics-based analytical model to address the interactions between a panel and the sound field inside a rectangular enclosure. In this work, piezoelectric patches bonded to the panel are used as actuators, which are also included in the modelling. Chang and Nicholas **[5]** used Green's functions to study the frequency response of structural–acoustic systems. This approach is suitable for frequency–response analysis, but not convenient for control designs that require time–domain models. Al-Bassyiouni and Balachandran **[6]** taking into consideration the sound radiation from the panel into the external field; this aspect is important for feedforward control schemes. In addition, the case where the panel–enclosure system is located in the near field of the noise source.

The purpose of this work is to model and control the structural-acoustic system. The system to be modelled is consisting of a rectangular cavity with two flexible panels, one at the top and the other at the bottom and four other fixed boundaries. PZT pair patches are considered to be bonded to the top panel (Figure 1), and each pair is assumed to produce a pure moment actuation when an electric drive signal is used to excite them. The flexible panel is exposed to an external pressure excitation due to a planar wave generated by a sound source mounted above the cavity. The ambient values are indicated with the subscript $(\ )_o$.



(a) (b)

Figure 1. (a) Schematic of the panel-cavity-panel system used for the analysis model, (b) Centres locations of the actuator pairs on the plate and locations of the calculated pressure in the cavity

2 MODELING OF THE PANEL-CAVITY-PANEL PROBLEM

2.1 The Panel-Cavity-Panel System

The two governing equations of this system are the conservation of mass equation and the conservation of momentum equation. In three-dimensional space, making use of linear approximations, the wave equation describing the sound field inside the cavity can be obtained as:

$$\nabla^2 P - \frac{1}{c_o{}^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{1}$$

where $P(x, y, z; t)$ is the air pressure inside the cavity, the speed of sound in a medium assuming isentropic flow is defined as **[7]**

$$c = \sqrt{\frac{dP}{d\rho}} \tag{2}$$

The boundary conditions can be stated as:

$$\frac{\partial P}{\partial n}=\begin{cases} 0 & \textit{at rigid boundary} \\ -\rho_o \dfrac{\partial^2 w}{\partial t^2} & \textit{at flexible boundary} \end{cases} \tag{3}$$

where $w(x,y;t)$ is the normal displacement of the flexible boundary, and n is the direction normal to the boundary. The pressure field inside the cavity can be expressed in the series form

$$P(x,y,z;t)=\sum_{i=1}^{\infty}\Phi_i(x,y,z)\,q_i(t)=\sum_{i=1}^{\infty}\psi_i(x)\,\varphi_i(y)\,\Gamma_i(z)\,q_i(t) \tag{4}$$

where $\Phi_i(x,y,z)$ are used to describe the spatial field and $q_i(t)$ are used to describe the associated temporal part of the pressure response. The spatial functions $\psi_i(x)$, $\varphi_i(y)$ and $\Gamma_i(z)$ are assumed to be orthogonal. The cavity governing equations can be derived to have the following form **[12]**

$$\frac{1}{c_o^{\,2}}\frac{\partial^2 q_i}{\partial t^2}+\left[k_i^{\,2}-\left(\overline{\Gamma}_i\frac{d\Gamma_i}{dz}\right)\Bigg|_{o_c}^{L_{zc}}\right]q_i=0 \tag{5}$$

where

$$k_i^{\,2}=\left[\int_{o_c}^{L_{xc}}\left(\frac{d\psi_i}{dx}\right)^2 dx+\int_{o_c}^{L_{yc}}\left(\frac{d\varphi_i}{dy}\right)^2 dy+\int_{o_c}^{L_{zc}}\left(\frac{d\Gamma_i}{dz}\right)^2 dz\right] \tag{6}$$

2.2 The Piezoelectric Actuator-Panel System

The panel–piezo system is treated here as a multi-laminate system that consists of three plies in places where the piezo pair patches are bonded to the panel, and as a single ply panel otherwise. Making use of the assumptions used in earlier studies **[8]**, the panels displacements can be described by

$$D\nabla^4 w_U+\rho_p\,h_p\,\ddot{w}_U=p_{in}-p_{U\,i}-\sum_{i=1}^{k}\frac{(h_p-h_{pzt})\,E_{pzt}\,d_{31}}{(1-\nu)}\nabla^2\chi_{U\,i}V_i(t) \tag{7a}$$

$$D\nabla^4 w_L+\rho_p\,h_p\,\ddot{w}_L=p_{in} \tag{7b}$$

The plate response is assumed in the series

$$w(x,y;t)=\sum_{i=1}^{\infty}\alpha_i(x)\,\beta_i(y)\,\eta_i(t) \tag{8}$$

where the $\eta_i(t)$ are temporal functions and the appropriate expressions for the spatial functions $\alpha_i(x)$ *and* $\beta_i(y)$ are obtained from the work of **[9]** and the subscript $(\ )_U$ and $(\ )_L$ are used to describe the upper and lower panels respectively.

2.3 The Coupled Piezo-Panel-Cavity-Panel System

Making use of the boundary conditions along with Eqs. (4) and (8), and making use of the orthogonality property, we get the following equation:

$$\left.\frac{\partial \Gamma_j}{\partial z}\right|_{z=L_{zc}} q_j(t) = -\rho_o \sum_{i=1}^{\infty} Bx_{ij}^{(c)} By_{ij}^{(c)} \ddot{\eta}_U(t) \qquad \left.\frac{\partial \Gamma_j}{\partial z}\right|_{z=0_o} q_j(t) = -\rho_o \sum_{i=1}^{\infty} Bx_{ij}^{(c)} By_{ij}^{(c)} \ddot{\eta}_L(t) \tag{9}$$

where $Bx_{ij}^{(c)}$ and $By_{ij}^{(c)}$ are given by **[6]**. The spatial functions in Eq. (4) are given by rigid-body cavity modes; that is **[6]**, **[8]**, and **[10]**

$$\psi_i(x) = \frac{A_i}{\sqrt{L_{xc}}}\cos\left(\frac{l_i\, \pi\, x}{L_{xc}}\right), \quad \varphi_i(y) = \frac{A_i}{\sqrt{L_{yc}}}\cos\left(\frac{m_i\, \pi\, y}{L_{yc}}\right), \quad \Gamma_i(z) = \frac{A_i}{\sqrt{L_{zc}}}\cos\left(\frac{n_i\, \pi\, z}{L_{zc}}\right) \tag{10}$$

where the indices $l_i, m_i, and\ n_i$ are associated with the spatial functions of the i^{th} rigid cavity mode, in the $x, y, and\ z$ directions, respectively. Using Eq. (9) into Eqs. (5) and (6) and making use of Eqs. (9) and (10) in Eq. (5), it is found that

$$\left(\frac{1}{c_o{}^2}\right)\ddot{q}_j(t) + \left(\frac{l_j^2\pi^2}{L_{xc}^2} + \frac{m_j^2\pi^2}{L_{zc}^2} + \frac{n_j^2\pi^2}{L_{zc}^2}\right) q(t) + \left(\frac{(-1)^j A_j}{\sqrt{L_{zc}}}\rho_o \sum_{i=1}^{\infty} Bx_{ij}^{(c)} By_{ij}^{(c)}\right)\ddot{\eta}_U(t) - \left(\frac{A_j}{\sqrt{L_{zc}}}\rho_o \sum_{i=1}^{\infty} Bx_{ij}^{(c)} By_{ij}^{(c)}\right)\ddot{\eta}_L(t) = 0 \tag{11}$$

The equations governing the panel modal amplitudes are obtained by making use of Eqs. (4), (7), (8) and (10). After making use of the orthogonality properties and boundary conditions, the equation governing each panel modal amplitude is obtained as:

$$\begin{aligned} \rho_p h_p \ddot{\eta}_{Uj}(t) + D\left[Ix_j + Iy_j\right]\eta_{Uj}(t) + 2D\sum_{i=1}^{\infty} Ix_{ij}\ Iy_{ij}\eta_{Ui}(t) &= \sum_{i=1}^{\infty} \frac{(-1)^j A_i}{\sqrt{L_{zc}}} Bx_{ij}^{(p)} By_{ij}^{(p)}\ q_i(t) \\ -\left[\int_{A_p} \alpha_j \beta_j p_{Ui}^s(x,y)\mathrm{d}A_p\right] p_i^t(t) - \sum_{i=1}^{k}&\left[\int_{A_p} \alpha_j \beta_j \frac{\left(h_p + h_{pzt}\right)E_{pzt} d_{31}}{(1-\nu)} \nabla^2 \chi_U(x_i, y_i)\mathrm{d}A_p\right] V_i(t) \end{aligned} \tag{12a}$$

$$\rho_p h_p \ddot{\eta}_{Lj}(t) + D\left[Ix_j + Iy_j\right]\eta_{Lj}(t) + 2D\sum_{i=1}^{\infty} Ix_{ij}\ Iy_{ij}\eta_{Li}(t) = \sum_{i=1}^{\infty} \frac{A_i}{\sqrt{L_{zc}}} Bx_{ij}^{(p)} By_{ij}^{(p)}\ q_i(t) \tag{12b}$$

where the different spatial integrals are given by **[6].** In Eq. (12), the incident pressure loading can be expressed as the product of spatial and time domain functions. Now, Eqs. (11) and (12) can be represented in matrix from, after truncating the infinite number of modes to the first M panel modes and N acoustic modes, as follows:

$$\begin{bmatrix} M_{pp}^U & 0 & 0 \\ M_{cp}^U & M_{cc} & M_{cp}^L \\ 0 & 0 & M_{pp}^L \end{bmatrix} \begin{pmatrix} \ddot{\eta}_U \\ \ddot{q} \\ \ddot{\eta}_L \end{pmatrix} + \begin{bmatrix} K_{pp}^U & K_{pc}^U & 0 \\ 0 & K_{cc} & 0 \\ 0 & K_{pc}^L & K_{pp}^L \end{bmatrix} \begin{pmatrix} \eta_U \\ q \\ \eta_L \end{pmatrix} = \begin{bmatrix} F_p^U & F_V^U \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} p_i^t \\ V \end{pmatrix} \tag{13}$$

The different quantities in the above equation are given by:

$$M_{pp}^{U} = M_{pp}^{L} = diag\lfloor \rho_p h_p \rfloor \qquad M_{cc}^{U} = M_{cc}^{L} = diag\left[\frac{1}{c_o^2}\right]$$

$$M_{cp}^{U} = \rho_o\left[\frac{(-1)^i A_i}{\sqrt{L_{zc}}} Bx_{ji}^{(c)} By_{ji}^{(c)}\right] \qquad M_{cp}^{L} = -\rho_o\left[\frac{A_i}{\sqrt{L_{zc}}} Bx_{ji}^{(c)} By_{ji}^{(c)}\right]$$

$$K_{pp}^{U} = K_{pp}^{L} = 2D\lfloor Ix_{ji}\ Iy_{ji}\rfloor + D\ diag\lfloor Ix_i + Iy_i\rfloor \qquad K_{cc}^{U} = K_{cc}^{L} = diag\left[\frac{l_j^2\pi^2}{L_{xc}^2} + \frac{m_j^2\pi^2}{L_{zc}^2} + \frac{n_j^2\pi^2}{L_{zc}^2}\right] \tag{14}$$

$$K_{pc}^{U} = \left[-\frac{(-1)^j A_j}{\sqrt{L_{zc}}} Bx_{ji}^{(p)} By_{ji}^{(p)}\right] \qquad K_{pc}^{L} = \left[-\frac{A_j}{\sqrt{L_{zc}}} Bx_{ji}^{(p)} By_{ji}^{(p)}\right]$$

$$F_p^U = \left(-\int_{A_p} \alpha_i\ \beta_i\ p_{Ui}^{s}\ dA_p\right) \qquad F_V^U = \left[-\int_{A_p} \alpha_i\beta_i \frac{(h_p + h_{pzt})E_{pzt}d_{31}}{(1-\nu)} \nabla^2 \chi_U(x_j, y_j)\, \mathrm{d}A_p\right]$$

The matrices M_{cp} *and* K_{pc} describe the structural-acoustic coupling, while the matrices K_{pp} represent the panel stiffness matrix. Equations (20) represent the time-domain model developed for the system shown in Figure 1. The panel displacements $w(x,y;t)$ and the pressure fields inside the cavity $p(x,y,z;t)$ can be obtained from the following relations:

$$\begin{Bmatrix} w_U(x,y;t) \\ p(x,y,z;t) \\ w_L(x,y;t) \end{Bmatrix} = \begin{bmatrix} C_U^{(w)} = [\alpha_i(x)\beta_i(y)] & 0 & 0 \\ 0 & C^{(p)} = [\psi_i(x)\varphi_i(y)\Gamma_i(z)] & 0 \\ 0 & 0 & C_L^{(w)} = [\alpha_i(x)\beta_i(y)] \end{bmatrix} \begin{Bmatrix} \eta_U(t) \\ q(t) \\ \eta_L(t) \end{Bmatrix} \tag{15}$$

3 NUMERICAL RESULTS

Here, the numerical results obtained from the analytical model developed in this chapter are presented. The first few natural frequencies of the uncoupled and coupled system are tabulated Table 1.

Table 1 the undamped natural frequencies (Hz) of the uncoupled and coupled system

Uncoupled System							**Coupled system**
Cavity				Panel			Panels-Cavity-Panel
Mode			Closed Form **[9]**	Mode		Blevins's Formula **[7]**	
1	0	0	281.33	1	1	44.452	42.75
0	0	1	337.6	2	1	76.017	43.85
0	1	0	375.11	1	2	103.61	67.447
1	0	1	439.45	3	1	127.66	68.407
1	1	0	468.89	2	2	132.78	132.98
0	1	1	504.66	3	2	181.7	134.11
							180.24
							181.31
							285.41

The entries of M_{cp} increase the values of the first few acoustic resonance frequencies above their uncoupled values, hence, contributing a "mass reduction" effect. On the other hand, the entries of K_{pc} decrease the values of the low (vibration) resonance frequencies below their uncoupled values, hence, contributing a "stiffness reduction" effect **[6]**.

4 CONTROL APPROACHE

4.1 State Space Design Method

The idea of state space comes from the state-variable method of describing differential equations **[11]**. The differential equations for a dynamic system can be represented in the state-variable-form vector equation

$$\dot{x} = Ax + Bu \tag{16}$$

where the input is u and the output is

$$y = C\,x + D u \tag{17}$$

4.2 Optimal Control Design

An optimal control system seeks to maximize the return from a system at the minimum cost. In general terms, the optimal control problem is to find a control u which causes the system $\dot{x} = g\big(x(t), u(t), t\big)$ to follow an optimal trajectory $x(t)$ that minimizes a performance criterion, or cost function $J = \int_{t_o}^{t} h\big(x(t), u(t), t\big)dt$. The most effective and widely used method to design a full state feedback control for linear systems is the optimal Linear Quadratic Regulator (LQR). The control law that minimizes J is given by the linear state feedback

$$u = -k\,x \tag{18}$$

Figure 2-b describes the assumed closed loop system for control-law design. A simpler form of the performance function introduced by **[11] is**

$$J = \int_{0}^{\infty} \left[\, \rho\, y^2(t) + u^2(t) \,\right] dt \tag{19}$$

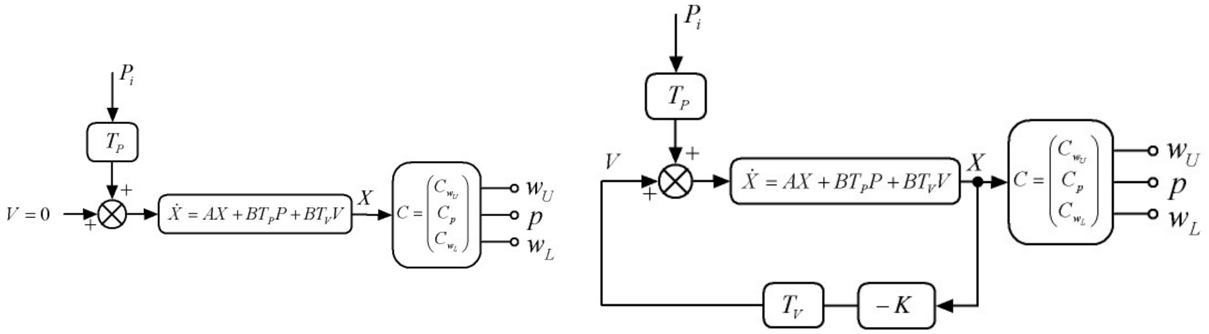
Where ρ is a weighting factor of the designer's choice.

4.3 The Panel-Cavity-Panel System Control Model

The complete equations of motion given in the structure model described in section 2 can be represented as

$$[M]\big(\ddot{X}\big) + [K](X) = [F](U) \tag{20}$$

Since this model is mainly required for the control system design, the model equations are rearranged to fit into a state space form as Eq. (20). The block diagram illustrating the open and closed loop systems is shown in Figure 2. The response of the system depends mainly on the input signals that excite it. For plate-cavity-plate system, the upper plate is excited by the disturbance pressure which may take several shapes. The external disturbance applied to the top of the panel is expected to take one of the following types: (1)Impulse pressure: a sudden change of the ambient pressure above the panel, (2)Damped periodic pressure: will be described by a damped sine wave, and(3) Random pressure: a varying pressure wave in frequency and amplitude in a random fashion.



(a) (b)

Figure 2 (a) Open loop block diagram for the panel-cavity-panel system, (b) Assumed system for optimal control-law design in case of a disturbance input

5 SIMULATION RESULTS

Using the previous analysis, numerical simulation is applied to the system. The objective in the simulation is to reduce the vibration of the upper and lower plate. Figures 3 to 8 illustrate the resulting displacements response for different input types. It is observed that the maximum amplitude and the settling time are highly reduced and the response can be tuned to a certain values by adjusting the weighting factor used in the control model.

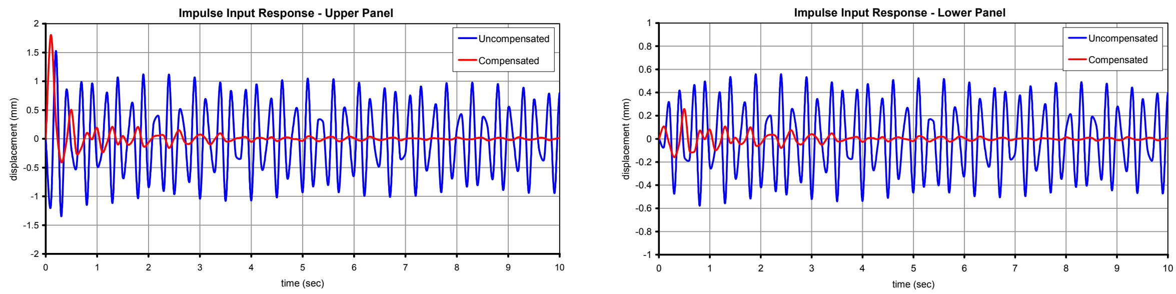


Figure 3 Uncompensated and compensated time responses for the upper and lower panels due to impulse input

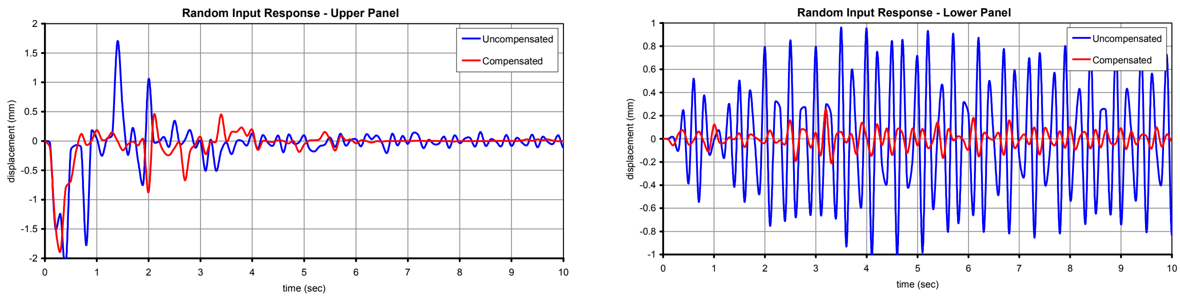


Figure 5 Uncompensated and compensated time responses for the upper and lower panels due to random input

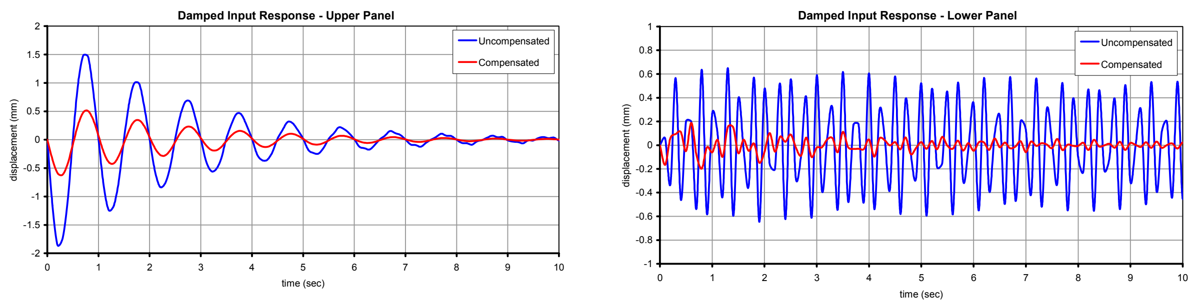


Figure 7 Uncompensated and compensated time responses for the upper and lower panels due to damped input

6 CONCLUSIONS

In this work, model based on time domain state space approach have been developed for an active structural-acoustic control (ASAC) application. It has been demonstrated that “appropriate” choices of the controller design parameter can result in decreasing the settling time required to damp the noise inside the cavity. Comprehensive mechanics-based analytical models have been developed to predict the structural-acoustic interactions in the case of Plate-cavity-plate system, where one plate is placed in the far field of a noise source at the top of the cavity, and the other is placed at the bottom of the cavity. Piezoelectric patches, which are bonded symmetrically to the top and bottom surfaces of the top plate, are used as actuators, and the acoustic pressure is calculated inside the cavity. The developed models have the following advantages: (i) They are capable of predicting the structural-acoustic interactions, and (ii) They take into account the coupling between the plate vibration and the pressure inside the cavity. The control scheme that has been developed throughout this work is Optimal LQR control law.

The numerical results obtained for the coupled system show how the vibration and acoustic fields interact with each other. It was found that the entries of the mass matrix M_{cp} increase the values of the first few acoustic resonance frequencies above their uncoupled values. On the other hand, the entries of the stiffness matrix K_{pc} decrease the values of the low vibration resonance frequencies below their uncoupled values. Using the Optimal Linear Quadratic Regulator (LQR) it is observed that this technique is efficient in reducing the maximum amplitude, and the settling time.

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