ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

THE COUPLING OF A HEARING AID LOUDSPEAKER MEMBRANE TO VISCO-THERMAL AIR LAYERS

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Abstract

Hearing aids and their components are becoming smaller. This presents new problems for the acoustical components, such as the loudspeaker. A circular membrane of a hearing aid loudspeaker is modeled in this paper. Neglecting air influences, the membrane and its suspension behave as a mass spring system. However, under operating conditions, thin layers of air on both sides of the membrane influence its behavior. Air can enter and leave these layers at certain locations on the circular edge of the layer. Since these air layers are thin, visco-thermal effects may have to be taken into account. Therefore, the air layers are not modeled by the wave equation, but by the low reduced frequency model that takes these visco-thermal effects into account. The equations of this model are solved in a polar coordinate system, using a wave-based method. The other acoustical parts of the hearing aid loudspeaker, and the membrane itself are modeled by simple lumped models. The emphasis in this paper is on the coupling of the visco-thermal air layer model to the mechanical model of the membrane. Coupling of the air layer to other acoustical parts by using an impedance as boundary condition for the layer model, is also described. The resulting model is verified by experiments. The model and the measurements match reasonably well, considering the level of approximation with lumped parts.

1. INTRODUCTION

Manufacturers of hearing aids try to meet the demand from their customers for smaller and aesthetically more appealing products. Miniaturization of hearing aids, while maintaining high levels of performance, presents designers with new challenges; for example, the visco-thermal effects of air become highly noticeable in smaller acoustic devices. Many existing modeling techniques do not accurately account for these effects. Therefore, new simulation tools are required to help in the design of particular hearing aid components, such as the loudspeaker. Modeling the hearing aid loudspeaker is the focus of our research.

The hearing aid loudspeaker under consideration, contains a circular membrane that translates between two air layers; see Fig. 1. The membrane is mechanically supported by an airtight suspension. The air on one side of the membrane is ‘pumped’ through a tube into the volume

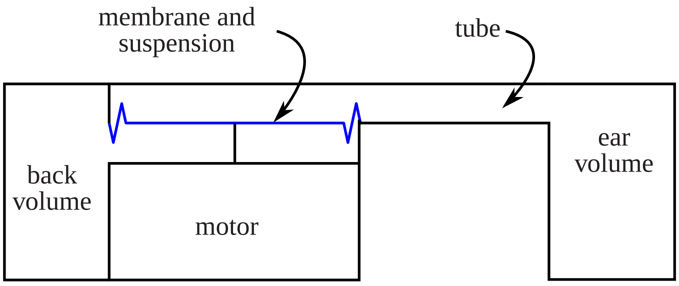

Figure 1. schematic cross-section of a hearing aid loudspeaker. The membrane translates vertically.

of the closed-off hearing canal. The air on the other side of the membrane is connected to a rear volume inside the loudspeaker, which acts as an expansion chamber.

Figure 2 shows a schematic drawing of the model described in this paper. The layer above the membrane is connected to the ear through a single opening. The layer below the membrane has eight openings to the back volume. When the membrane translates, air can enter and leave the layers through these openings. The effect of the ear volume and the back volume are modeled as impedances acting on the openings. The mechanical suspension is modeled as a translation spring and two perpendicular rotation springs. The behavior of the complete system is modeled as a function of the applied excitation force. This model requires a sub-model for each of the air layers and a sub-model of the mechanics of the membrane and its suspension. These sub-models are coupled.

The air layers are modeled by the low reduced frequency (LRF) model; see Beltman [1]. This model takes visco-thermal effects into account, which is essential for *thin* air layers. Wijnant [2] has presented a solution of the LRF model for circular layers, similar to the wave-based method, which will be used in this paper. Desmet [3] shows the strength of wave based methods for acoustics, especially for higher frequencies or larger domains. Nevertheless, the domain used in the model of this paper is very small. The work presented in this paper is one of the results from the master thesis of Bosschaart [4].

2. THEORY

2.1. The mass-spring model of the membrane and its suspension

The membrane is assumed to be *rigid* and has three degrees of freedom (DOFs): the translation u_z and the rotations α_1 and α_2; see Fig. 2 for the positive directions. The suspension has a stiffness for each DOF and is assumed to be infinitely stiff in all other directions. The damping of the membrane suspension is not modeled in this paper. The DOFs are made dimensionless, because the air layer model is also expressed in dimensionless variables. This results in the following harmonic equations for the mechanical system:

$$(-m + \kappa_z)\, u_z - (F_l + F_u) = F_{ex}, \tag{1a}$$

$$(-I_1 + \kappa_{r1})\, \alpha_1 - (M_{1,l} + M_{1,u}) = 0, \tag{1b}$$

$$(-I_2 + \kappa_{r2})\, \alpha_2 - (M_{2,l} + M_{2,u}) = 0, \tag{1c}$$

with the dimensionless variables: m and I_j mass and mass moment of inertia of the membrane; κ_z and κ_{rj} the translation stiffness and rotation stiffness; and F_{ex} the excitation force. The

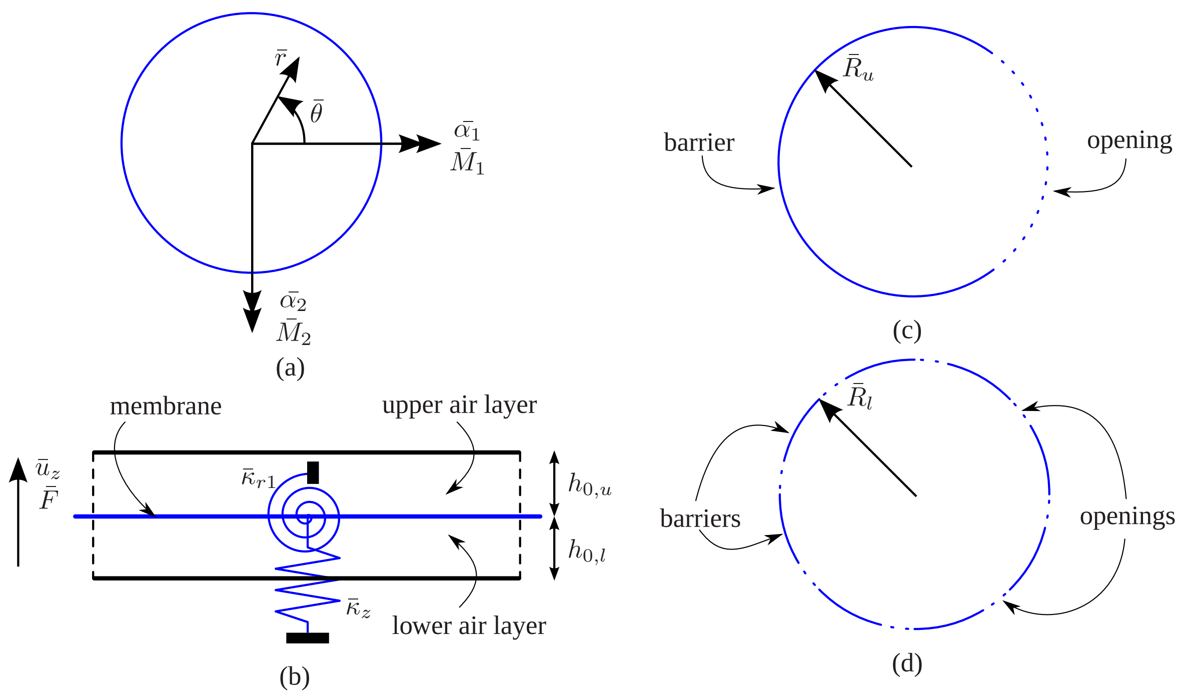

Figure 2. Schematic drawing of the model: (a) top view of the membrane with polar coordinates and positive directions of rotation DOFs and moments; (b) front view of the model with springs of the suspension, thickness of the layers and positive direction of force and translation DOF; (c) geometry of the upper air layer; (d) geometry of the lower air layer.

coupling of the mechanical model to the air layer models is achieved using F_i and $M_{j,i}$, the forces and moments caused by the air layers. The subscripts $_l$ and $_u$ indicate the lower and upper air layer respectively. Expressions for these forces and moments will be derived in subsection 2.3. The variables of the model in this paper are defined in Table 1.

2.2. The low reduced frequency model of the air layers

The air layers are modeled by the low reduced frequency (LRF) model; see Beltman [1]. This LRF model is valid for wave propagation in narrow layers if the flow in the layers is laminar. Furthermore, the layer thickness and the viscous boundary layer thickness must both be small compared to the wavelength. These assumptions in the LRF model result in a uniform pressure across the layer thickness. Therefore, the dimensionless PDE for the pressure perturbation p is two dimensional:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2} - \Gamma^2 p = -\frac{in\Gamma^2 v_s}{k}. \tag{2}$$

The dimensionless variables and coordinates used in this equation are also explained in Table 1. The expressions in this table for B and D are valid for zero slip and isothermal conditions at the surfaces on both sides of the air layer.

The left-hand side of eq. (2) is the Helmholtz equation in polar coordinates with a complex propagation constant. The right-hand side of this equation is the squeeze term or source term. The squeeze velocity v_s is positive in the direction outward from the layer. Its expression will

Table 1. Variables and constants used in the model

Symbol	*Dimensionless variables*	Definition*
F	force	$\bar{F} = \frac{p_0 c_0^2}{\omega^2} F e^{i\omega t}$
M_j	moment around axis j	$\bar{M}_j = \frac{p_0 c_0^3}{\omega^3} M_j e^{i\omega t}$
u_z	translation DOF of membrane	$\bar{u}_z = \frac{c_0}{\omega} u_z e^{i\omega t}$
α_j	rotation j DOF of membrane	$\bar{\alpha}_j = \alpha_j e^{i\omega t}$
m	mass of membrane	$\bar{m} = \frac{p_0 c_0}{\omega^3} m$
I_j	membrane mass moment of inertia j	$\bar{I}_j = \frac{p_0 c_0^3}{\omega^3} I_j$
κ_z	translation stiffness of suspension	$\bar{\kappa}_z = \frac{p_0 c_0}{\omega} \kappa_z$
κ_{rj}	rotation stiffness j of suspension	$\bar{\kappa}_{rj} = \frac{p_0 c_0^3}{\omega^3} \kappa_{rj}$
k	reduced frequency	$k = \frac{\omega h_0}{c_0}$
s	shear wave number	$s = h_0 \sqrt{\frac{\rho_0 \omega}{\mu}}$
p	pressure perturbation	$\bar{p} = p_0(1 + p e^{i\omega t})$
n	isotropic constant	$n = \left(1 + \frac{\gamma - 1}{\gamma} D\right)^{-1}$
B	mean of velocity profile across layer	$B = 2\left(\frac{\cosh(s\sqrt{i}) - 1}{s\sqrt{i}\sinh(s\sqrt{i})}\right) - 1$
D	mean of thermal profile across layer	$D = 2\left(\frac{\cosh(s\sigma\sqrt{i}) - 1}{s\sigma\sqrt{i}\sinh(s\sigma\sqrt{i})}\right) - 1$
v_s	squeeze velocity	$\bar{v}_s = c_0 v_s e^{i\omega t}$
$\hat{v}$	mean particle velocity across layer	$\hat{v} = \frac{1}{h_0}\int_0^{h_0} v \, \mathrm{d}\bar{z}$
v	particle velocity	$\bar{v} = c_0 v e^{i\omega t}$
r	radial coordinate	$\bar{r} = \frac{c_0}{\omega} r$
θ	angular coordinate	$\bar{\theta} = \theta$
Γ	propagation constant	$\Gamma = \sqrt{\frac{\gamma}{nB}}$
R	layer radius	$\bar{R} = \frac{c_0}{\omega} R$
Z	acoustical impedance for layer openings	$\bar{Z} = \frac{p_0}{c_0 h_0} Z$

Symbol	*Constants*	value	Unit
$\bar{F}_{ex}$	excitation force	0.55	[N]
$\bar{m}$	membrane mass	1.1e-6	[kg]
$\bar{I}_j$	membrane mass moment of inertia j	7.96e-4	[$\mathrm{kg\,m^2}$]
$\bar{\kappa}_z$	translation stiffness of suspension	78	[$\mathrm{N\,m^{-1}}$]
$\bar{\kappa}_{rj}$	rotation stiffness j of suspension	6	[$\mathrm{N\,m\,rad^{-1}}$]
$\bar{R}_u$	upper membrane radius	1.21e-3	[m]
$\bar{R}_l$	upper membrane radius	1.19e-3	[m]
p_0	atmospheric pressure	1e5	[$\mathrm{N\,m^{-2}}$]
c_0	speed of sound	343.6	[$\mathrm{m\,s^{-1}}$]
$h_{0,u}$	upper layer thickness	0.243e-3	[m]
$h_{0,l}$	lower layer thickness	0.119e-3	[m]
ρ_0	atmospheric density	1.204	[$\mathrm{kg\,m^{-3}}$]
μ	viscosity	1.837e-5	[$\mathrm{kg\,m^{-1}s^{-1}}$]
σ	sqrt of Prandtl number	0.8449	[–]
γ	ratio of specific heats	1.4022	[–]

Symbol	*Other variables*	Unit
ω	angular frequency	[$\mathrm{rad\,s^{-1}}$]
Θ_B	angle intervals of boundary barriers	[rad]
Θ_O	angle intervals of boundary openings	[rad]
L_{Θ_O}	total length of the openings	[m]

* The barred variables are have dimension; for example, R is the dimensionless equivalent of $\bar{R}$.

be derived in sub-section 2.3. The analytical solution of the PDE is:

$$p = C_0^c \mathrm{I}_0(\Gamma r) + \sum_{m=1}^{\infty} \left[\mathrm{I}_m(\Gamma r)\big(C_m^c \cos(m\theta) + C_m^s \sin(m\theta)\big)\right] + \frac{in}{k} v_s. \tag{3}$$

This form of the solution results after separation of variables; demanding periodicity with period $\theta = 2\pi$; and demanding continuity at $r = 0$. The symbol I_m denotes the modified Bessel functions of the first kind. The particular part of the solution, namely the term containing v_s, has been derived using $\Delta v_s = 0$, which is true only because the membrane is *rigid*. The *mean particle velocities* within the air layer can be calculated from the pressure solution, using:

$$\hat{v}_r = -\frac{iB}{\gamma}\frac{\partial p}{\partial r}, \qquad \hat{v}_\theta = -\frac{iB}{\gamma r}\frac{\partial p}{\partial \theta}. \tag{4}$$

The analytical solution satisfies the PDE, and the constants C_m^c and C_m^s can be calculated such that the solution satisfies the boundary conditions at $r = R$. This calculation is achieved by using a weak formulation of the boundary conditions, with weighing functions cos$(w\theta)$ and sin$(w\theta)$; see Wijnant [2]. The resulting linear system is:

$$\sum_{m=0}^{N} \left[\left(\frac{m}{R} + \frac{\Gamma \mathrm{I}_{m+1}(\Gamma R)}{\mathrm{I}_m(\Gamma R)} + \frac{\gamma}{iBZ}\right) \begin{bmatrix} \int_{\Theta_O} \cos(m\theta)\cos(w\theta)\,\mathrm{d}\theta & \int_{\Theta_O} \sin(m\theta)\cos(w\theta)\,\mathrm{d}\theta \\ \int_{\Theta_O} \cos(m\theta)\sin(w\theta)\,\mathrm{d}\theta & \int_{\Theta_O} \sin(m\theta)\sin(w\theta)\,\mathrm{d}\theta \end{bmatrix} \right.$$
$$\left. + \left(\frac{m}{R} + \frac{\Gamma \mathrm{I}_{m+1}(\Gamma R)}{\mathrm{I}_m(\Gamma R)}\right) \begin{bmatrix} \int_{\Theta_B} \cos(m\theta)\cos(w\theta)\,\mathrm{d}\theta & \int_{\Theta_B} \sin(m\theta)\cos(w\theta)\,\mathrm{d}\theta \\ \int_{\Theta_B} \cos(m\theta)\sin(w\theta)\,\mathrm{d}\theta & \int_{\Theta_B} \sin(m\theta)\sin(w\theta)\,\mathrm{d}\theta \end{bmatrix} \right] \begin{Bmatrix} C_m^c \mathrm{I}_m(\Gamma R) \\ C_m^s \mathrm{I}_m(\Gamma R) \end{Bmatrix}$$
$$+ \frac{in}{k} \left\{ \begin{Bmatrix} \int_{\Theta_O} \left(\frac{\partial v_s}{\partial r} + \frac{\gamma v_s}{iBZ}\right)\Big|_{r=R} \cos(w\theta)\,\mathrm{d}\theta \\ \int_{\Theta_O} \left(\frac{\partial v_s}{\partial r} + \frac{\gamma v_s}{iBZ}\right)\Big|_{r=R} \sin(w\theta)\,\mathrm{d}\theta \end{Bmatrix} + \begin{Bmatrix} \int_{\Theta_B} \frac{\partial v_s}{\partial r}\Big|_{r=R} \cos(w\theta)\,\mathrm{d}\theta \\ \int_{\Theta_B} \frac{\partial v_s}{\partial r}\Big|_{r=R} \sin(w\theta)\,\mathrm{d}\theta \end{Bmatrix} \right\} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{5}$$

with the weighing function index $w = 0, 1 \ldots N$. The Bessel term $\mathrm{I}_m(\Gamma R)$ has been moved from the system matrix to the vector of unknowns for better numerical behavior. In this way a larger number of constants, N, can be calculated before the matrix becomes ill-conditioned. The row and column corresponding to C_0^s have to be removed from the system, because this constant is meaningless and it's therefore not used in eq. (3). The variables Θ_B and Θ_O are the intervals of the angle coordinate θ where the barriers and openings are located.

Two different boundary conditions were used to obtain the above linear system. For the barriers a zero radial velocity is used; or $\frac{\partial p}{\partial r} = 0$, a Neumann boundary condition. For the openings a uniform impedance is used; or $p = Z\hat{v}^r \rightarrow \frac{\gamma p}{iBZ} + \frac{\partial p}{\partial r} = 0$, a mixed boundary condition. The variable Z is the dimensionless impedance. If $Z = 0$, the mixed boundary condition can be rewritten as the Dirichlet boundary condition $p = 0$.

The impedances at the openings of the layers are determined from lumped acoustical models. The impedances of these lumped models $\bar{Z}_{lumped}$ are defined as pressure divided by volume flow and have the dimension [N m^{-5}], while the impedances at the openings $\bar{Z}$ have the dimension [N m^{-4}]. The conversion between these two types of impedances is done by multiplication with the length of the openings:

$$\bar{Z} = \bar{L}_{\Theta_O} \bar{Z}_{lumped}, \tag{6}$$

with $\bar{L}_{\Theta_O}$ the total length of the openings of the layer. The back volume in the loudspeaker is modeled as a lumped volume and its impedance is applied to the openings of the lower layer. The ear volume is connected to the membrane with a tube. These are modeled as a lumped Helmholtz resonator and its impedance is applied to the opening of the upper layer.

The system of eq. (5) is solvable for the constants C_m^c and C_m^s. The system has similarities to a Fourier series and the resulting solution can be improved by applying the *Lanczos sigma factors* to the calculated constants.

2.3. The coupling terms and the complete model

The membrane is coupled to the two layers in two ways. Firstly, the squeeze velocity of the membrane appears in the source term of the air layer model. This velocity needs to be known at every location on the membrane. Expressed in polar coordinates and the DOFs of the membrane, the dimensionless squeeze velocity for the *lower air layer* is:

$$v_{s,l} = i \left(u_z + \alpha_1 r \sin\theta + \alpha_2 r \cos\theta \right). \tag{7}$$

This result must be multiplied by -1 to obtain the squeeze velocity for the *upper layer*, because its positive direction is defined outward with respect to the layer: $v_{s,l} = -v_{s,u}$

Secondly, the pressure in the air layers can produce forces and moments on the membrane (F and M in eq. (1)). These can be calculated by integrating the pressure in the layers over the membrane surface Ω. Using eq. (3) and (7), this results in:

$$F_l = \iint_\Omega p_l \,\mathrm{d}\Omega = \frac{2\pi R_l}{\Gamma_l} C_{0,l}^c \mathrm{I}_1(\Gamma_l R_l) - \frac{n_l \pi R_l^2}{k_l} u_z, \tag{8a}$$

$$M_{1l} = \iint_\Omega p_l r \sin(\theta) \,\mathrm{d}\Omega = \frac{\pi R_l^2}{\Gamma_l} C_{1,l}^s \mathrm{I}_2(\Gamma_l R_l) - \frac{n_l \pi R_l^4}{4k_l} \alpha_1, \tag{8b}$$

$$M_{2l} = \iint_\Omega p_l r \cos(\theta) \,\mathrm{d}\Omega = \frac{\pi R_l^2}{\Gamma_l} C_{1,l}^c \mathrm{I}_2(\Gamma_l R_l) - \frac{n_l \pi R_l^4}{4k_l} \alpha_2. \tag{8c}$$

These equations are valid for the *lower layer*. Note that only one constant is needed to calculate either the force or one of the moments: C_0^c, C_1^s or C_1^c. However, the value of these constants slightly depends on the number of calculated variables N. The forces and moments for the *upper layer* can be derived similarly. For this layer, a minus signs appears in front of the terms with the constants C, and all layer specific variables change; for example, n_l becomes n_u.

Equations (5) and (1) can be combined to form the complete system. This system has the following structure:

$$\begin{bmatrix} [\mathrm{LRF}]_l & [0] & [S(v_s)]_l \\ [0] & [\mathrm{LRF}]_u & [S(v_s)]_u \\ [F_p]_l & [F_p]_u & [\mathrm{Mech}] \end{bmatrix} \begin{Bmatrix} \{C_m \mathrm{I}_m(\Gamma R)\}_l \\ \{C_m \mathrm{I}_m(\Gamma R)\}_u \\ \{\mathrm{DOF}\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{F_{ex}\} \end{Bmatrix} \tag{9}$$

The only non-zero entry in the system vector on the right-hand side is F_{ex} at the position corresponding to the u_z DOF. The sub-matrices $\left[S(v_s)\right]$ contain the system vectors of eq. (5) split into the contributions of each DOF. Notice that the forces and moments from the air layers as described by eq. (8) contribute to the sub-matrices $\left[F_p\right]$ as well as the sub-matrix $\left[\mathrm{Mech}\right]$.

3. RESULTS

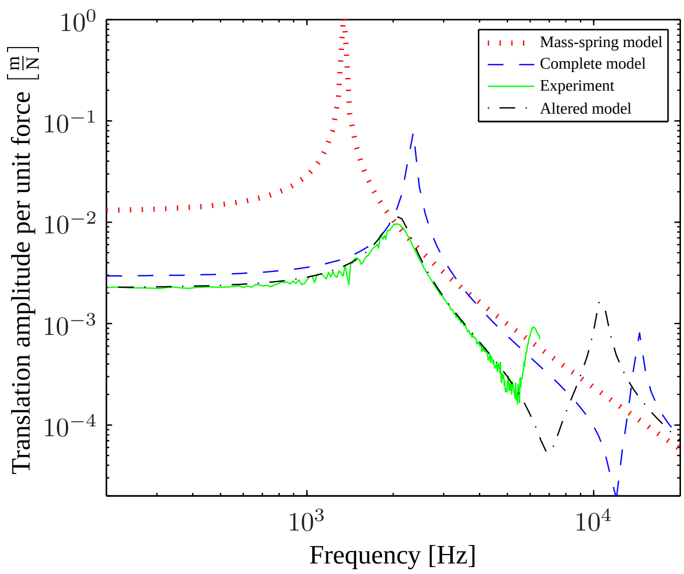

Figure 3. Frequency response of the models and the experiment.

The values of the constants used in this paper are shown in Table 1. Furthermore, the lower layer has eight openings that occupy 32 % of the circumference. The upper air layer has one opening that occupies 30 % of the circumference. The results for several models and an experiment are shown in Fig. 3. In the considered models, the membrane rotations are very small compared to the membrane translation. Therefore, only the results for the the translation DOF $\bar{u}_z$ are shown.

If the impedance on all openings is set to zero, the air layers hardly influence the response of the mechanical system. Under these circumstances, the resulting response cannot be distinguished from the response of the 'mass-spring model' in Fig. 3.

The response of the 'complete model' in Fig. 3 is the model with impedance conditions at the openings. The impedance at the openings of the lower layer represents a lumped acoustical volume of 1e-8 m^3. The impedance at the openings of the upper layer represents a Helmholtz resonator with a volume of 2e-6 m^3, a tube length of 1e-2 m, and a tube radius of 0.5e-3 m. The resulting uniform impedances for the openings of the layers are:

$$\bar{Z}_u = 46.6 i\omega + 1.71\mathrm{e}4 + \frac{1.62\mathrm{e}8}{i\omega}, \tag{10a}$$

$$\bar{Z}_l = \frac{3.40\mathrm{e}10}{i\omega}. \tag{10b}$$

The back volume adds stiffness to the system which decreases the low frequency response and increases the resonance frequency. The Helmholtz resonator adds the resonance and anti-resonance above 10e4 Hz.

The results are compared to experiments that were done using a laser vibrometer. The displacement of the membrane was measured at nine points. The average displacement of these points is shown in Fig. 3. Clearly, there is a difference between the experiments and the complete model. However, considering the rough approximations of lumped volumes and tubes, the results are quite good.

A model that better fits the measurements, can be made by changing the lumped values. Damping, which was neglected in the mechanical model of the suspension, and mass need to be added. The parameters of the tube are changed heavily to obtain the ‘altered model’, which is also shown in Fig. 3. This model’s response resembles the experiment up to a frequency of 5000 Hz. Unfortunately it is not possible to get the second resonance peak to fit the measurements. This indicates that the lumped approximation of a volume and a Helmholtz resonator is not accurate enough.

Besides the differences between the model and the experiment discussed above, the measurement data shows a rotation that is much larger than in the model. Even if the volumes are disconnected from the loudspeaker, the large rotations are present in the measurements. The suspension, that has been modeled as a translation spring and two rotations springs, seems to have a much more complicated behavior than assumed.

4. CONCLUSIONS

A hearing aid loudspeaker membrane is modeled. The mechanics of the membrane is modeled with a lumped mass spring model. For the air layers on both sides of the membrane, the distributed LRF model is used. The other parts of the loudspeaker are modeled with straightforward lumped acoustical models. Despite these rough approximations, the results of the complete model are quite close to the experiments. For a better model, the sub-models of the suspension of the membrane and of the acoustical parts other than the air layers, must be refined.

ACKNOWLEDGEMENTS

Sonion is gratefully acknowledged by the authors for financing this project.

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