ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

A NUMERICAL STUDY OF SOUND-STRUCTURE INTERACTION IN A DUCT STRUCTURE BY THE ACOUSTICAL WAVE PROPAGATOR METHOD

Shuzhi Peng, Li Cheng

Department of Mechanical Engineering, the Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
mmszpeng@polyu.edu.hk

Abstract

The pseudospectral time-domain method has long been used to describe the acoustical wave propagation. However, due to the limitation and difficulties of the FFT, the dispersion error is hard to be avoided and the computational accuracy greatly decreases after the waves arrive at the non-periodic boundary. To resolve this problem, the Lagrange-Chebyshev interpolation polynomials are used to replace the previous FFT. In addition, a mapping method is introduced to overcome the additional time-step restriction. In this paper, several issues are addressed to explore its numerical performances: the numerical accuracy, computational efficiency and stability of this proposed method.

1. INTRODUCTION

Partial differential equations (PDEs) describe a wide array of physical processes such as molecular dynamics, fluid flow and sound propagation. Over the last decade, a great deal of effort has been devoted to the development of numerical methods for solving the PDEs including time-dependent Schrödinger equation and wave equations [1]. Because the time-domain investigation provides important insight into the understanding of governing physical phenomena, various numerical schemes have been developed in parallel in many fields with little across referencing. Balakrishnan *et al*. [2] presented a comprehensive discussion on various expansion schemes. It is worth noting that Kosloff and Tal-Ezer [3] did a pioneer work on the pseudospectral method. Therefore, it is not surprising that great effort has been devoted to finding the optimum numerical method with an efficient, accurate and stable numerical procedure to solve the time-dependent PDEs. Recently, Peng and Pan [4] developed an explicit acoustical wave propagator (AWP) method to describe the time-domain evolution of acoustical waves. However, in practical engineering applications, structures with complex boundary conditions must be treated properly. Therefore, the existing problem is that, the previous AWP method including the Fourier transform scheme, can hardly deal with the non-periodic problems such as asymmetrical boundary conditions.

More recently, as a further development of the AWP method, Peng and Huang [5] introduced the Lagrange-Chebyshev interpolation polynomials to replace the previous FFT scheme. However, despite the improvement on the spatial derivatives, the additional restriction on the time step is still left to be solved. The present paper aims to eliminate this restriction by remapping the Chebyshev-Gauss-Lobatto points. By choosing an optimal parameter γ in the mapped Chebyshev method, a larger time step with higher computational efficiency and stability can be achieved.

2. A DUCT STRUCTURE MODEL AND THE AWP

The theoretical model consists of a one-dimensional duct structure with two different boundaries: a) periodic (rigid walls at both ends); and b) non-periodic (the left-hand side is a rigid wall, the right-hand side is a pressure-release wall).

Acoustical wave motion in the duct is described by the following PDEs:

$$\frac{\partial p}{\partial t} = -\rho_0 c_0^2 \frac{\partial V}{\partial x}, \qquad \frac{\partial V}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}, \tag{1}$$

where p is the sound pressure; and V is the particle velocity along the x-direction in the duct.

To derive the acoustical wave propagator in the duct, we select a state vector ϕ_D consisting of sound pressure p and particle velocity V. From Eq. (1), we have the following system state equation:

$$\frac{\partial}{\partial t}\underbrace{\begin{bmatrix} p \\ V \end{bmatrix}}_{\phi_D} = -\hat{H}_D \begin{bmatrix} p \\ V \end{bmatrix}, \quad \hat{H}_D = \begin{bmatrix} 0 & \rho_0 c_0^2 \dfrac{\partial}{\partial x} \\ \dfrac{1}{\rho_0}\dfrac{\partial}{\partial x} & 0 \end{bmatrix}. \tag{2}$$

Integrating Eq. (2) with respect to time yields $\phi_D(x,t) = e^{-(t-t_0)\hat{H}_D}\phi_D(x,t_0)$, where $e^{-(t-t_0)\hat{H}_D}$ is defined as the acoustical wave propagator (AWP). When the initial values $\phi_D(x,t_0)$ are known, there are two key steps to obtain $\phi_D(x,t)$: a) calculation of the spatial derivatives in $\hat{H}_D$; and b) implementation of the exponential expansion $e^{-(t-t_0)\hat{H}_D}$ by an efficient and accurate method.

3. CALCULATION OF THE SPATIAL DERIVATIVES AND IMPLEMENTATION OF THE AWP

For well-behaved problems (symmetrical structure with periodic boundary conditions), the Fourier transform is very useful to evaluate the spatial derivatives. However, for problems where the natural boundary conditions are non-periodic, the Fourier transform scheme will introduce additional numerical dispersion and the computational accuracy rapidly deteriorates. Therefore, the Lagrange-Chebyshev interpolation polynomials are used to overcome this numerical problem.

3.1 Lagrange-Chebyshev interpolation polynomials scheme for spatial derivatives

For simplification, only the former $\partial p(x,t)/\partial x$ is derived as follows. Normally, the Chebyshev pseudospectral method is based on polynomial interpolation in the canonical interval [-1,1]. However, it can be defined on any finite internal $[x_0, x_N]$ for a general case by means of a linear transform of variable χ which maps [-1,1]. In this scheme, $\partial/\partial\chi$ is represented by a matrix $d_\chi = [d_{i,k}]$ with its elements $d_{i,k}$

$$\partial\tilde{p}(\chi,t)/\partial\chi = \sum_{k=0}^{N} d_{i,k}\,\tilde{p}_k(\chi,t), \tag{3}$$

which can also be expressed as $[\partial\tilde{p}(\chi,t)/\partial\chi] = d_\chi[\tilde{p}(\chi,t)]$. When $N+1$ discrete points in χ-axis are given, $\partial\tilde{p}(x,t)/\partial x$ can be obtained by multiplying $\partial\tilde{p}(\chi,t)/\partial\chi$ with the constant $2/(x_N - x_0)$.

3.2 Non-periodic/periodic boundary conditions

A mathematical model with non-periodic boundary conditions is introduced to describe how the boundary conditions are considered in the spatial derivatives. Here, two different boundary conditions: a rigid wall with $\partial\tilde{p}(x,t)/\partial x = 0$ is imposed on the left-hand side; and a pressure-release wall with $\tilde{p}(x,t) = 0$ is used on the right-hand side. The procedure is described as follows: a) all values at the initial condition $\phi_D(x,t_0)$ are known, so $\partial\tilde{p}(x,t_0)/\partial x$ can be calculated; b) the values at the new time step $t_0 + dt$ can be obtained by using $\phi_D(x,t_0+dt) = e^{-dt\hat{H}}\phi_D(x,t_0)$ for all inner points; then $\partial\tilde{p}(x,t_0+dt)/\partial x$ should be re-calculated from $\partial\tilde{p}(\chi,t_0+dt)/\partial\chi$; and c) the boundary conditions are applied to get $\phi_D(x,t_0+dt)$ on all boundaries. For example, the sound pressure $\tilde{p}(x_0,t_0+dt)$ on the left-hand side is calculated by

$$\begin{aligned}
&\tilde{p}(x_0,t_0+dt) = -\left(d_{0,1}\tilde{p}(x_1,t_0+dt)+\cdots+d_{0,N-1}\tilde{p}(x_{N-1},t_0+dt)+d_{0,N}\underbrace{\tilde{p}(x_N,t_0+dt)}_{=0}\right)\Big/ d_{0,0}\,,\\
&\partial\tilde{p}(x_N,t_0+dt)/\partial x = 2\left(d_{N,0}\tilde{p}(x_0,t_0+dt)+\cdots+d_{N,N-1}\tilde{p}(x_{N-1},t_0+dt)+d_{N,N}\underbrace{\tilde{p}(x_N,t_0+dt)}_{=0}\right)\Big/(x_N - x_0).
\end{aligned} \tag{4}$$

Similarly, for periodic boundary conditions $\partial\tilde{p}(x,t)/\partial x = 0$, the sound pressures $\tilde{p}(x_0,t_0+dt)$ and $\tilde{p}(x_N,t_0+dt)$ are calculated by

$$\begin{aligned}
\tilde{p}(x_0,t_0+dt) &= -\big((d_{0,1}d_{N,N} - d_{0,N}d_{N,1})\tilde{p}(x_1,t_0+dt)+\cdots+(d_{0,N-1}d_{N,N} - d_{0,N}d_{N,N-1})\tilde{p}(x_{N-1},t_0+dt)\big)/(d_{0,0}d_{N,N} - d_{0,N}d_{N,0}),\\
\tilde{p}(x_N,t_0+dt) &= \big((d_{0,1}d_{N,0} - d_{0,0}d_{N,1})\tilde{p}(x_1,t_0+dt)+\cdots+(d_{0,N-1}d_{N,0} - d_{0,0}d_{N,N-1})\tilde{p}(x_{N-1},t_0+dt)\big)/(d_{0,0}d_{N,N} - d_{0,N}d_{N,0}).
\end{aligned} \tag{5}$$

3.3 Chebyshev polynomial expansion schemes with *I*-expansion

The scheme with *I*-expansion is used to implement the exponential propagator $e^{-(t-t_0)\hat{H}_D}$. The solution of Eq. (2) can be rewritten as $\phi_D(x,t) = \left[I_0(R)I + 2I_1(R)\hat{H}_D^{'} + 2\sum_{n=2}^{\infty} I_n(R)T_n\left(\hat{H}_D^{'}\right)\right]\phi_D(x,t_0)$, where I is a unit matrix with the same size as that of $\hat{H}_D^{'}$; and $I_n(R)$ is the *n*th-order modified Bessel function of the first kind. Theoretically speaking, you can choose any large R with enough expansion term $n_{\min}$ to ensure $I_0(R)/I_{n_{\min}}(R)$ reaches the order of 10^{16}. The main difference between the present improvement and the previous AWP method in the Ref. [4] is that $[\partial\tilde{p}(x,t)/\partial x] = 2d_\chi[\tilde{p}(\chi,t)]/(x_N - x_0)$ is included in $\hat{H}_D^{'}$ to replace $F[\partial\tilde{p}(x,t)/\partial x] = ik_x F[\tilde{p}(x,t)]$ included in $\hat{H}_D^{'}$.

3.4 The RK4 method

The RK4 method is expressed by $\phi(x,t+dt) = \phi(x,t)\left[1 - \hat{H}_D dt + \hat{H}_D^2 dt^2/2 - \hat{H}_D^3 dt^3/6 + \hat{H}_D^4 dt^4/24\right]$. The drawback of the RK4 method is that the associated numerical error is usually proportional to the time step. The strategy chosen for the propagating scheme is to expand the evolution operator $AWP = e^{-\hat{H}_d dt}$ based on the Chebyshev polynomials, which has been regarded as the best polynomial approximation. Therefore, a simple function $f(t) = e^{-t}$ is introduced to demonstrate its numerical accuracy. For interpolation approximation, the Chebyshev polynomial expansion $f_{Ch}(t)$ of degree *n* for $f(t)$ over the interval [-1,1] can be written as a sum of $T_j(t)$: $f(t) \approx f_{Ch\_Inter}(t) = \sum_{j=0}^{n} c_j T_j(t)$.

The coefficients c_j are computed with the following formulas

$$c_0 = \frac{1}{n+1}\sum_{k=0}^{n} f(t_k);\; c_1 = \frac{2}{n+1}\sum_{k=0}^{n} f(t_k)t_k;\; c_j = \frac{2}{n+1}\sum_{k=0}^{n} f(t_k)T_j(t_k) = \frac{2}{n+1}\sum_{k=0}^{n} f(t_k)\left(2t_k T_{j-1}(t_k) - T_{j-2}(t_k)\right);\quad j = 2,\cdots,n \tag{6}$$

When $n = 4$, $f_{Ch4\_Inter}(t) = 1.0 - 0.9973173\,t + 0.4995562\,t^2 - 0.1773346\,t^3 + 0.0434341\,t^4$.

For extrapolation expansions, the $f(t) \approx f_{Ch}(t) = \left[I_0(R)T_0(t-t_0) + 2I_1(R)T_1(t-t_0) + \sum_{m=2}^{M} 2I_m(R)T_m(t-t_0)\right] f(t_0)$, where $I_m(R)$ is the *m*th-order Bessel function of the first kind. Different from the previous interpolation expansions, the coefficients $I_m(R)$ and $T_m(t)$ are calculated by known results at present time step.

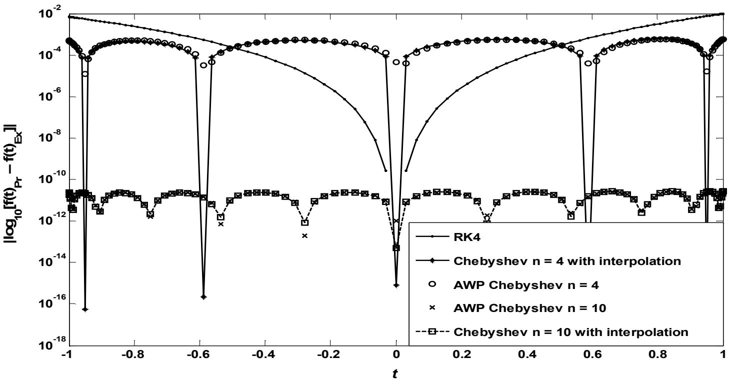

Figure 1. Absolute errors of the RK4, Chebyshev methods compared with the exact solutions for the function $f(t) = e^{-t}$ based on the variable time-step sizes.

There is increasing interest in using a very long time step, even just one step to complete the calculation for some specific problems. The Chebyshev polynomial expansion has much better accuracy for a larger time step (Fig. 1). An overwhelming important performance is that, the approximation error nearly keeps the same order from the first iteration to the last iteration. However, for the RK4 method, as the absolute value of *t* increases, the approximation accuracy becomes worse exponentially. It is the reason that only very small time steps can be adopted and the number of steps required for modelling a complete propagation is large. The Chebyshev polynomial expansion overcomes this disadvantage.

3.5 The mapped Chebyshev method

Chebyshev-Gauss-Lobatto points in Lagrange-Chebyshev interpolation polynomials are highly dense near the boundaries with minimal spacing, which leads to the severe stability condition. Therefore, a modified Chebyshev pseudospectral method is used to improve its restrictive stability condition. A transform algorithm is applied to map these Chebyshev-Gauss-Lobatto points χ_i to another set of points $X_i = \arcsin(\gamma\,\chi_i)/\arcsin(\gamma)$, where γ is an optimal parameter, $\gamma \in [0,1]$ but does not include two end values: 0 and 1. As a result, the minimal spacing near the boundaries is stretched with larger minimal spacing.

Similarly, the spatial derivative of this new function can be obtained by $\partial\tilde{p}/\partial X = \aleph d_\chi \tilde{p}$, where the diagonal matrix $\aleph$ has elements $\aleph_{i,i} = \arcsin(\gamma)\sqrt{1-\gamma^2\chi_i^2}/\gamma$. It is noted that the parameter γ has much significant effects on $\aleph_{i,i}$, $\partial\tilde{p}/\partial X$ and subsequently on the prediction result of sound pressure $\tilde{p}$. When $\gamma \to 0$, $\aleph_{i,i} = \arcsin(\gamma)\sqrt{1-\gamma^2\chi_i^2}/\gamma \to 1$, for the minimal spacing $\Delta X_{\min} = 1-\cos(\pi/N)$ as in standard Chebyshev methods $\Delta\chi_{\min}$, which means the mapped method is functionless. When $\gamma \to 1$, $\aleph_{i,i} = \arcsin(\gamma)\sqrt{1-\gamma^2\chi_i^2}/\gamma = \pi\sin(i\pi/N)/2$, for the minimal spacing $\Delta X_{\min} = |\arcsin(\gamma\chi_{N-1})/\arcsin(\gamma)-1| = 2/N$, which is the same order as the uniform spacing (Fourier case) Δx $(O(L_x/N))$. In other words, the restriction on the time step related to the stability condition has been removed.

4. NUMERICAL EXAMPLES AND RESULTS

4.1 Numerical examples and exact analytical solutions for periodic and non-periodic boundary conditions

First of all, to demonstrate this proposed method, a modified Gaussian impulse is selected as the initial wave packet, which is given below together with boundary conditions,

$$p(x,0) = f(x) = 0.04^2 x^2 (x-x_N)^2 e^{-[(x-x_c)^2/(4\sigma^2)]}, \quad \partial p(x,0)/\partial t = g(x) = 0, \quad \partial p(x_0,t)/\partial x = \partial p(x_N,t)/\partial x = 0, \tag{7}$$

where x_c and σ denote the position and Gaussian factor of the initial wave packet, respectively; and the constant (0.04^2), the terms x^2 and $(x-x_N)^2$ are introduced to ensure the maximum initial value with positive unit, the sound pressures and its first-order spatial derivatives with zero at two ends, respectively. To some extent, a proper function modified can improve the computational accuracy.

4.2 Analysis of numerical accuracy and computational efficiency

For the previous AWP method, Fourier transform scheme was adopted to evaluate the spatial derivative such as $\hat{p}_x = F[\partial p/\partial x] = (jk)F[p(x,t)] = (jk)\hat{p}$. Then the inverse Fourier transform is applied to get $\partial p(x,t)/\partial x = F^{-1}\{\hat{p}_x\}$. Theoretically speaking, the fast Fourier transform can only deal with well-behaved periodic functions. When an initial wave packet arrives at the boundary, for the first-order and second-order derivatives, both unbalanced FFT and balanced FFT have poor approximations. In addition, as the order of the spatial derivatives increases, the approximation error becomes worse gradually.

For the above non-periodic boundary condition, the modified Lagrange-Chebyshev interpolation polynomial method is used for calculating the spatial derivatives. Here, only two spatial derivatives $\partial p(x,t)/\partial x$ and $\partial^2 p(x,t)/\partial x^2$ are demonstrated to evaluate this new feature.

The exact solutions of the first-order and second-order derivatives are given by

$$\partial p(x,t)/\partial x = \sum_{m=0}^{\infty}[-c_m\eta_m \sin(\eta_m x)\cos(\zeta_m t)], \quad \partial^2 p(x,t)/\partial x^2 = \sum_{m=0}^{\infty}[-c_m\eta_m^2 \cos(\eta_m x)\cos(\zeta_m t)] \tag{8}$$

Comparison between the previous Lagrange-Chebyshev method and the modified Lagrange-Chebyshev method [(a) $dp/dX = \aleph d_\chi p$ and (b) $d^2p/dX^2 = \aleph^2 d_\chi^{(2)} p - \arcsin^2(\gamma)\chi d_\chi p$] based on the exact expressions for the initial wave packet given in Eq. (7) was carried out to demonstrate the numerical performance of this new method, where $\tilde{d}_X^{(1)}$ is the diagonal matrix with entries $\tilde{d}_{X_{ii}}^{(1)} = \Re''(\chi_i,\gamma)/(\Re'(\chi_i,\gamma))^3 = \arcsin^2(\gamma)\chi_i$. For the present modified Lagrange-Chebyshev interpolation polynomial method, the maximum errors have increased slightly to 1.088×10^{-14} and 3.7788×10^{-12}, respectively. Therefore, this new method not only keeps high accuracy for calculating the spatial derivatives, but also improves computational efficiency greatly.

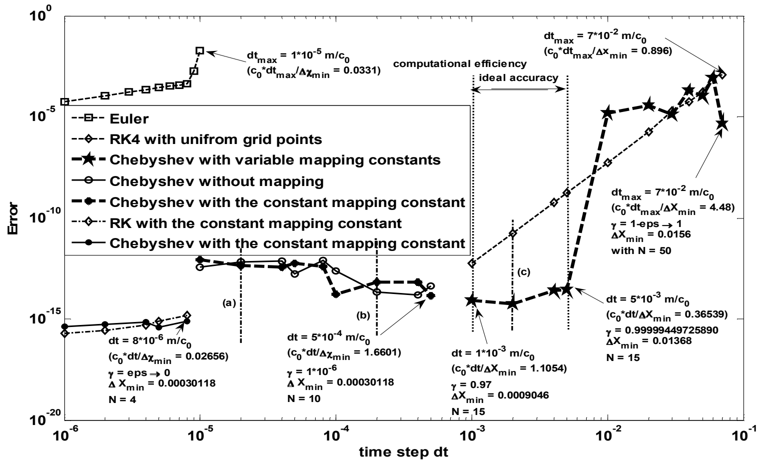

Figure 2. Errors of the Euler, RK4 and AWP methods with/without the mapped Chebyshev method compared the exact solutions in different time steps.

The main parameters used in this computation are given as follows: the speed of sound $c_0 = 344$ m/s, the structure sizes are $x_0 = 0$ m, $x_N = 10$ m, $x_c = 5$ m, $\sigma = 0.5$. One of the main thrusts is to explore the modified present Lagrange-Chebyshev interpolation polynomial method, especially the numerical accuracy, and computational efficiency and stability.

Figure 2 shows the error comparison between the Euler method, the RK4 method, and the Chebyshev method with/without the mapped Chebyshev method. In addition, one motivation of this paper is to investigate the effect of the parameter γ in the mapped Chebyshev method on the prediction results, in particular the numerical accuracy and computational efficiency. The size of the time step used can be roughly divided into three zones: a) very small time step $dt \in [1\times10^{-6} \;\; 1\times10^{-5}]$; b) small time step $dt \in [1\times10^{-5} \;\; 1\times10^{-3}]$; and c) large time step $dt \in [1\times10^{-3} \;\; 7\times10^{-2}]$. Here, it is necessary to mention that, due to non-uniform Gauss-Lobatto points, the traditional error evaluation methods are found with lower accuracy. Therefore, to ensure the accuracy of numerical analysis, the multiple-application trapezoidal rule, an energy integration method with unequal segments ($\int_{x_0}^{x_N} p(x,t)dx$, which represents an integration to the propagation wave function $p(x,t)$ examined in the range of the whole structure considered) is used in the error analysis below. For the Euler method, a very small time step $dt_{Euler} = 1\times10^{-6}\, m/c_0$ is necessary to get good prediction results, but the errors are still large. As the size of the time step increases, the approximation errors increase linearly. When the time step is larger than the critical value of $dt = 8\times10^{-6} m/c_0$ ($c_0 dt/\Delta X_{\min} = 0.02648$), the error increases dramatically. As the time step further increases over the value of $dt = 1\times10^{-5} m/c_0$ ($c_0 dt/\Delta X_{\min} = 0.0331$), the calculation becomes divergent. In the same zone, the mapped Chebyshev method does not have any effect on the calculated results both the RK4 method and Chebyshev method. Similarly, as the size of the time step increases, the approximation error obtained by the RK4 method increases linearly. Compared with the Euler and RK4 methods, the Chebyshev method has much different performance: the error increases between $dt = 1\times10^{-6}$ and $dt = 4\times10^{-6}$, decreases between $dt = 4\times10^{-6}$ and $dt = 5\times10^{-6}$, and starts to increase at $dt = 5\times10^{-6}$. The reason can be caused by the complicated re-combination of these Chebyshev expansion coefficients.

In the second zone, only the Chebyshev methods with/without the mapped Chebyshev method are used to demonstrate the effect of the parameter γ on the calculation errors. Although the two curves have the similar shapes as the solid-dotted line located in the first zone, it is found that the parameter γ with a slight different value of 10^{-6} has significant contribution to the numerical accuracy. According to this clue, it is possible to achieve the maximum benefit of the computational efficiency by choosing an optimal parameter γ and enough expansion term n provided that the calculation keeps within the highly numerical accuracy (the order of error $O(10^{-14})$).

In the third zone, the effect of the uniform grid points and Chebyshev-Gauss-Lobatto points on the numerical accuracy and computational efficiency is investigated in details. At the expense of sacrificing computational accuracy, the maximum time step can be up to $dt_{\max} = 7.0\times10^{-2}\, m/c_0$ ($c_0 dt_{\max}/\Delta x_{\min} = 0.896$) provided that the computation is still convergent. By properly choosing the term n in the Chebyshev expansion and the parameter γ, an optimal result with highly numerical accuracy and computational efficiency with much larger time step used can be obtained, such as $dt = 1.0\times10^{-3}\, m/c_0$ with $\gamma = 0.97$ and $n = 15$; $dt = 5.0\times10^{-3}\, m/c_0$ with $\gamma = 0.99999449725890$ and $n = 15$, as shown in Fig. 2. As the time step further increases, the modified Lagrange-Chebyshev method with the mapped Chebyshev method can still be used in the calculation by increasing both the parameter and expansion terms. However, neither the previous Lagrange-Chebyshev method without the mapped Chebyshev method or the high-order RK method cannot be used in the calculation. It is the reason that their results cannot be shown in Fig. 2.

From the above three zones, three time steps ($dt = 2.0\times10^{-5} m/c_0, dt = 2.0\times10^{-4} m/c_0, dt = 2.0\times10^{-3} m/c_0$) are selected to investigate the effect of the expansion term on the prediction results. First of all, the number of expansion term in the Chebyshev method is examined by the coefficients of the first kind of Bessel function. According to the dynamic range of the modern computer 10^{-16}, the minimum expansion terms are needed to ensure the truncation errors do not contribute to the final result and the sum of the polynomials converges to the order of $I_{n_{\min}}(R)$. For the above three zones, the convergence properties of $I_n(R)$ for three given R (0.0049 for $n = 4$, 0.0493 for $n = 10$, 0.4929 for $n = 15$) are, respectively, illustrated in Fig. 3 (upper panel). Besides the most concern of the possible maximum time step related to computational efficiency, another main concern of this paper is to investigate the effect of the parameter γ in the mapped Chebyshev method on the predication error. Here, two larger time steps ($dt = 2.0\times10^{-4}\, m/c_0$, $dt = 2.0\times10^{-3}\, m/c_0$) were selected to demonstrate this effect, as shown in Fig. 3 (middle panel). For the former $dt = 2.0\times10^{-4}\, m/c_0$, there are several "minimum" values at $\gamma = 1\times10^{-6}$, $\gamma = 1\times10^{-3}$, $\gamma = 4\times10^{-1}$, $\gamma = 6\times10^{-1}$, and $\gamma \to 1$. From $\gamma = 1\times10^{-15}$ to $\gamma = 1\times10^{-9}$, the error keeps the constant ($\varepsilon = 6.6597\times10^{-14}$). The best result ($\varepsilon = 2.2815\times10^{-14}$) can be achieved by taking $\gamma = 1\times10^{-6}$. In addition, during the whole range from $\gamma = 1\times10^{-15}$ to $\gamma \to 1$, the predication results are found to be highly accurate (the order of error below $O(10^{-12})$). Different from $dt = 2\times10^{-4}$, for the latter $dt = 2\times10^{-3}$, there is only one minimum value at $\gamma = 0.9825$ in the range from $\gamma = 0.98$ to $\gamma \to 1$. In particular, the prediction error increases drastically with the increase of the parameter γ. However, the prediction error keeps high accuracy of $O(10^{-12})$restricted within a narrow range from $\gamma = 0.98$ to $\gamma \to 1$. Furthermore, when the time step dt and the parameter γ are fixed, the effect of the expansion term on the prediction errors is shown in Fig. 3 (lower panel). A general knowledge is that: the less the number of expansion terms and the larger the time step, the worse the prediction error ($O(10^{-2})$). As the expansion term n increases, the prediction error step-by-step improved.

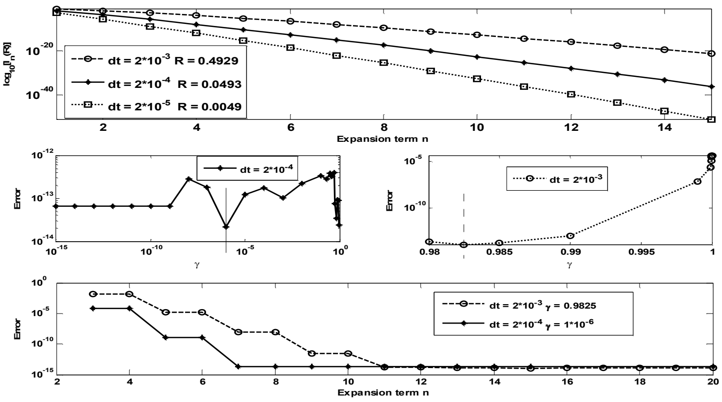

Figure 3. Effects of the expansion term n and the parameter γ on the prediction errors.

For a very smaller time step, the mapped Chebyshev method does not have any effect on the calculated results. For a larger time step, this method dominates the numerical accuracy, computational efficiency and stability completely, which is the main backbone of this paper. The above analysis demonstrates that the proposed method can deal with the non-periodic boundary conditions well. More importantly, this method has not only good numerical accuracy, but also computational efficiency and stability for predicting the time-domain acoustical wave propagation and scattering, especially for a long-term calculation.

5. CONCLUSIONS

Through a detailed study on the numerical accuracy, computational efficiency and stability, the acoustical wave propagator (AWP) technique is further developed to describe the time-domain evolution of acoustical waves. Due to the limitation and difficulties of FFT, the previous Chebyshev-Fourier scheme has been fully replaced by the combined scheme: a) A Chebyshev polynomial expansion scheme to implement the operation of the AWP; and b) the modified Lagrange-Chebyshev interpolation polynomials scheme with the mapped Chebyshev method to evaluate the spatial derivatives in the system operator. One function is introduced to demonstrate that the Chebyshev polynomial expansion has much better accuracy for a larger time step. In addition, the effects of different expansion terms, time steps and the parameter γ in the mapped Chebyshev method on the predicted results are investigated by a numerical example.

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