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DEVELOPMENT OF A SEMI-EMPIRICAL MODEL OF MECHANICAL HYSTERESIS FOR SANDWICH COMPOSITE MATERIALS, AND SYSTEM IDENTIFICATION USING HEURISTIC OPTIMIZATION METHODS

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Abstract

Generally speaking, of the fundamental dynamic mechanical properties - mass, damping, and stiffness, damping is usually the most difficult to quantify. This is perhaps particularly true for composite materials which tend to have substantially higher damping than comparable isotropic materials and therefore having an accurate representation is correspondingly more important. Accordingly, some heuristic optimization techniques for the identification of the dynamic characteristics of honeycomb-core sandwich composite materials have been suggested, such as Particle Swarm Optimization (PSO) and Genetic Algorithms (GA). Experimental measurements of the dynamic responses (in the form of hysteresis loops) of simply-supported composite beam samples have been carried out, and a simplified semi-empirical mathematical model has been developed for such a system, tailored from individual experimental observations of the dynamic behavior of the samples when they are excited at their mid-points by sinusoidal displacement waves. The hysteresis loops that were obtained are for several frequencies and excitation amplitudes around the first mode of vibration. The analytical model contains four unknown system parameters, which must be identified by both optimization techniques utilized. The performance of these optimization methods are compared with computer-generated and experimental hysteresis loops. In addition, the effect of noise contamination in the signals has been studied in order to assess the search accuracy of the optimization algorithms under such conditions.

1. INTRODUCTION

Composite materials are being increasingly used as an alternative to conventional materials for highly demanding structural applications, such as the construction of marine, automobile, and spacecraft structures [1]. Composites are mainly used for this kind of applications because they have the desired properties of high specific strength, specific stiffness and tailorable design, along with a low mass. Damping is one important parameter related to the study of dynamic behavior of composite structures in general. There are numerous types of composites currently available, but the present study deals with the use of a specific kind of composite construction called "sandwich composite", which is a structure consisting of two thin faces bonded to a thick lightweight core [2,3].

Several damping measurement techniques have been developed, and the applicability of each of these methods is normally dependent on the frequency range of interest, and whether or not the damping in a resonant frequency is to be measured. Among the several approaches available for damping measurement, the half power method, logarithmic decrement, and hysteresis loops analysis are generally the most popular.

A mechanical hysteresis loop consists of a phase plot of the force vs. the displacement at a specific point in a structure. For dynamical systems that include damping, the displacement/force relationship is typically out of phase, generating an elliptically-shaped hysteresis loop, and the area enclosed by the hysteresis loop is proportional to the damping existent in the system.

The present paper examines the hysteresis loops technique for the damping analysis of a beam of a sandwich composite material consisting of a paper honeycomb core filled with foam and external layers of interlaced strips of carbon fibers, as shown in Fig. 1. A tailored semi-empirical mathematical model of hysteresis has been developed, and heuristic optimization routines, such as Particle Swarm Optimization (PSO) and Genetic Algorithms (GA) have been created for the system identification process.

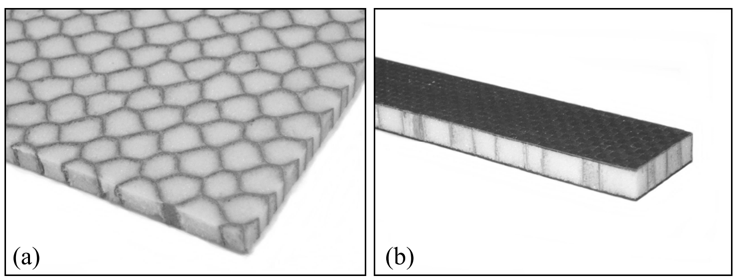

Figure 1. Sandwich composite; (a) detail of the core, (b) composite beam.

2. EXPERIMENTAL SETUP

A schematic of the test apparatus used to obtain experimental hysteresis loops is shown in Fig. 2.

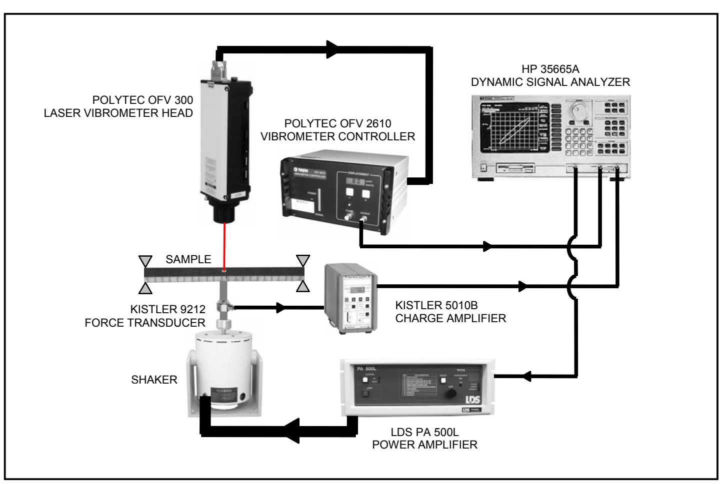

Figure 2. Experimental setup.

The experimental tests consist of a beam sample of the composite material (as in Fig. 1b) having simply-supported conditions, attained by crossing tensioned steel wires at half its thickness near to each end. The beam is excited by a pure sinusoidal displacement wave with a dynamic shaker. This applied displacement is measured at the mid-point of the beam with a laser displacement vibrometer and the resulting force is measured by a force transducer positioned in-line with the shaker contact with the test sample. These signals are collected using a dynamic signal analyzer.

3. MATHEMATICAL MODEL DEVELOPMENT

A baseline model consisting of a linear 1-DOF mechanical system with an equivalent viscous damping has been used as a starting point, with additional tailoring of it to the behavior that has been experimentally observed, as shown in Eq. (1). It should be pointed out that, since the experimental setup consists of a point excitation at the beam center where the sample is simply-supported at both ends, the Euler's beam equation have been converted into this equivalent 1-DOF equation.

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \frac{F(t)}{M_{eq}} \tag{1}$$

As a first step in identifying appropriate modifications to the above model, several hysteresis loops have been obtained experimentally for a material sample, at different excitation frequencies and displacement amplitudes. Two unusual phenomena have been noted after the preliminary identification of the system parameters for individual loops: both, the resonance frequency (ω_n) and the damping ratio (ξ) are linear functions of the peak excitation amplitude X_0, having negative slopes. The damping ratio at fixed excitation amplitudes, however, remained relatively constant for frequencies around the first resonance. The probable cause of the amplitude dependence of the resonance frequency is nonlinear effects arising from the flexing of the wire supports at both ends of the beam, producing a decrease in the effective stiffness of the system. From the observed experimental behavior, functional expressions for the damping ratio (ξ) and the resonance frequency ($\omega_n=2\pi f_n$) are developed. For both cases, linear functions in X_0 will be used (Eq. (2) and (3), respectively). As a result, the modified equation of motion can be represented as shown in Eq. (4), with a closed form steady-state solution shown in Eq. (5), for a sinusoidal displacement excitation $x(t)=X_0\cdot\sin(\omega t)$.

$$\omega_n(X_0) = \omega_0 + S_\omega \cdot X_0 \tag{2}$$

$$\xi(X_0) = \xi_0 + S_\xi \cdot X_0 \tag{3}$$

$$\ddot{x} + 2\xi(X_0)\cdot\omega_n(X_0)\cdot\dot{x} + \left(\omega_n(X_0)\right)^2\cdot x = \frac{F(t)}{M_{eq}} \tag{4}$$

$$\begin{aligned} F(t) = M_{eq}\cdot X_0\cdot\sqrt{\left(\omega_n(X_0)^2-\omega^2\right)^2+\left(2\xi(X_0)\cdot\omega_n(X_0)\cdot\omega\right)^2} \\ \cdot\sin\left(\omega t + \tan^{-1}\left(\frac{2\xi(X_0)\cdot\omega_n(X_0)\cdot\omega}{\omega_n(X_0)^2-\omega^2}\right)\right) \end{aligned} \tag{5}$$

4. HEURISTIC OPTIMIZATION METHODS

Examination of Eq. (2) to (5) shows that the model depends on four parameters, namely ξ_0, S_ξ, ω_0, and S_ω. The characterization of the value of each parameter for specific hysteresis loops (curve-fitting) is a very challenging task due to the nature of the problem. Therefore, heuristic optimization methods were used, where a wide area of the cost surface may be simultaneously scanned, thus finding regions of interest to be analyzed in much more detail, and then progressively approaching to the best solutions.

There exist several popular heuristic optimization methods in the literature, such as Particle Swarm Optimization (PSO), and Genetic Algorithms (GA), which have been used in the present work for extracting parametric solutions from the mathematical model when curve-fitted to synthetic (artificially created) and experimental hysteresis data. These methods are then compared, to assess their search efficiency in this particular problem.

4.1 Particle Swarm Optimization (PSO)

The Particle Swarm Optimization (PSO) is an optimization technique for continuous nonlinear functions (in general), introduced by James Kennedy and Russ Everhart in 1995 [4]. It was originally created by simulation of a simplified social model, where a set of individuals (particles) interact in order to find an optimal solution. In PSO, the particle correction equation is given by Eq. (6),

$$\begin{aligned} &\text{a) } v_{id} = K \cdot \left(v_{id} + \varphi_1 \cdot rand(0,1) \cdot \left(p_{id} - x_{id} \right) + \varphi_2 \cdot rand(0,1) \cdot \left(p_{gd} - x_{id} \right) \right) \\ &\text{b) } x_{id}' = x_{id} + v_{id} \end{aligned} \tag{6}$$

where i is the particle index, d is the dimension index (parameter), g stands for "global", v is the velocity (correction), φ_1 and φ_2 determine the relative influence of the "cognition" and "social" components of the velocity, p is the location of the previous best position, x is the current particle position, and *rand(0,1)* is a uniform random number between 0 and 1. The so-called "constriction factor" (K) was introduced by Maurice Clerc [5], and it improves the control of the velocities, given by Eq. (7), where $\varphi=\varphi_1+\varphi_2$, and $\varphi>4$.

$$K = \frac{2}{\left| 2 - \varphi - \sqrt{\varphi^2 - 4\varphi} \right|} \tag{7}$$

The main advantage of PSO is that it is simple to implement, as compared to other evolutionary computation optimization techniques and generally has an outstanding performance. PSO can also have different neighborhood topologies, namely ring and global topologies. A particle's neighborhood is defined as a set of particles that influence an individual particle movement through the search space. It defines what is considered as P_g (global best position) for a specific particle being evaluated. The larger the neighborhood size, the more particles included in a particular particle's neighborhood [6]. Finally, there are two ways of determining the global best (P_g) position. The first is to only update P_g after every iteration (synchronous update), and the second method is to determine the P_g after each particle has been updated (asynchronous update).

4.2 Genetic Algorithms (GA)

The Genetic Algorithms (GA) technique is also a method for continuous nonlinear functions (in general), created by John Holland [7] during the 1960's and 1970's, and it was further improved by David Goldberg [8] during the 1980's. Although its advantages are similar as those of PSO, the idea behind this search strategy is somewhat different. GA is inspired on the theory of the evolution of species, in which the strongest organisms are expected to survive, propagating their genes to the following generations. There are different versions of GA, where the two main categories are the binary-coded and real-coded representations. In any case, GAs always consist of the steps described by Fig. 3.

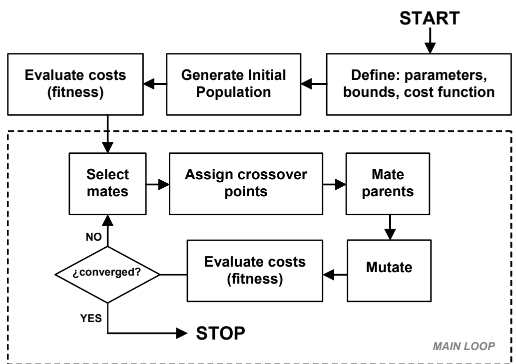

Figure 3. Experimental setup.

5. IMPLEMENTATION

A trial set of 120 computer-generated hysteresis loops (*Synth*) has been generated (at 40 frequencies, and 3 excitation amplitudes each), in order to test the search performance of both PSO and GA for a known global solution. Also, the influence of noise present in the signal has been studied, to see how this affects the optimization results. Finally, another set of 93 hysteresis loops (*Exp*) is obtained from experiments (at 31 frequencies, and 3 excitation amplitudes each) with the test apparatus described earlier, but of course the actual solution is not known in this case. It is expected that, in general, the search algorithms will find parametric solutions relatively close to each other for different runs of the computer codes for the different excitation conditions. The optimization parameters in the PSO routine created are identified as follows: asynchronous PSO, global topology, φ_1=φ_2=2.05, swarm size=500 (population), and iterations=18. As for GA, several versions of it have been tested, but the best results were obtained for: real-coded GA, rank-based parent selection, (μ+μ+L) selection strategy, population size=500, iterations=18, mutation rate=5% (number of mutations in the offspring), and Gaussian mutation amount=30% (amount allowed to move on mutation).

Some additional features were also implemented for both algorithms, which greatly improved their search success. First, a large initial population (at iteration 0, of size=6000) is initially created, in order to find good candidates for the upcoming iterations. Also, an additional partial population is added to the main population, when the best partial solution is found between iterations, consisting of several different combinations of mutated versions of that best solution. The number of members of this additional population depends on the "add-on level" (*L*) set in the algorithm, which is the maximum number of parameters to be mutated in the best solution to create additional candidate solutions. For individuals with *N* parameters, the number of new individuals is given by Eq. (8).

$$AddPopNumber = \sum_{i=1}^{L} \binom{N}{i} = \sum_{i=1}^{L} \frac{N!}{i!(N-i)!} \tag{8}$$

In the extreme case, where L=N (as used in the present study), the result may be simplified as: *AddPopNumber*=2^N-1 (i.e. since N=4 $\Rightarrow$ *AddPopNumber*=15). Finally, the method of error calculation between the trial and model was performed by using the standard Ordinary Least Squares (OLS) error function, comparing the difference between the force data for every displacement sample value.

6. OPTIMIZATION RESULTS

Since GA and PSO work in a random fashion, different runs of the optimization routines produce different (but usually close) results. Four runs for each case were performed in order to check the consistency of the solutions. The search spaces used are shown in Table 1.

Table 1. Search space for the *Synth* and *Exp* hysteresis sets, respectively.

Parameter	***Synth***		***Exp***	
	Min	**Max**	**Min**	**Max**
ξ_0	0	0.1	0	0.05
S_ξ	-10	10	-100	100
ω_0	100	500	400	700
S_ω	-10000	10000	-10000	10000

6.1. Synth Data Set

Initially, optimization tests were performed for the *Synth* set of 120 computer-generated hysteresis loops with different levels of Gaussian random noise (signal-to-noise ratios). The results of the parameter averages ($\bar{x}$) and standard deviations (σ) obtained for PSO are shown in Table 2, whereas the results for GA are shown in Table 3.

Table 2. Optimization results for the *Synth* data set, using PSO.

Parameter	Actual	PSO (noise=0%)		PSO (noise=10%)		PSO (noise=20%)		PSO (noise=30%)	
		$\bar{x}$	σ	$\bar{x}$	σ	$\bar{x}$	σ	$\bar{x}$	σ
ξ_0	**0.0042**	0.004555	8.27E-05	0.004326	0.00049	0.003894	0.000134	0.004259	0.000492
S_ξ	**-0.4**	-0.69209	0.099097	-0.49716	0.358097	-0.17052	0.110643	-0.53369	0.399173
ω_0	**189.12**	189.1119	0.039992	189.0525	0.054989	189.0977	0.036953	189.2253	0.116378
S_ω	**-1256.6**	-1257.47	49.3357	-1203.26	54.32264	-1231.47	24.57532	-1329.27	85.26707

Table 3. Optimization results for the *Synth* data set, using GA.

Parameter	Actual	GA (noise=0%)		GA (noise=10%)		GA (noise=20%)		GA (noise=30%)	
		$\bar{x}$	σ	$\bar{x}$	σ	$\bar{x}$	σ	$\bar{x}$	σ
ξ_0	**0.0042**	0.00399	0.00062	0.004037	0.000227	0.004227	0.000179	0.003931	0.000611
S_ξ	**-0.4**	-0.24299	0.42289	-0.26836	0.214907	-0.42726	0.161295	-0.12807	0.452737
ω_0	**189.12**	189.1605	0.051628	189.1474	0.061878	189.0892	0.106813	189.0812	0.035996
S_ω	**-1256.6**	-1294.08	26.00358	-1271.83	56.29433	-1216.66	88.90028	-1216.04	17.52276

Example comparison plots of some of the trial hysteresis loops, along with the best-fit solutions obtained from the average optimal parameters, are shown in Fig. 4 and 5, for 0% and 20% noise, respectively. It should be noted that similar shapes have been obtained for all runs with both optimization techniques.

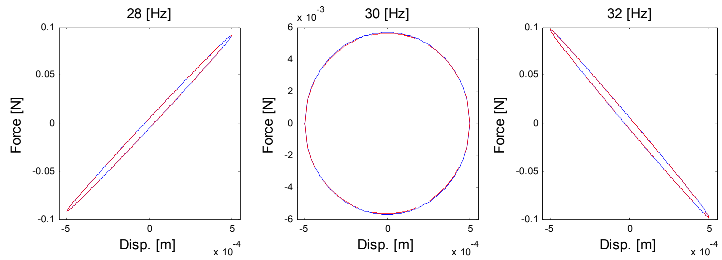

Figure 4. Comparison plots of fitting for some trial hysteresis loops of the *Synth* set; 0% noise; Excitation: 0.5 mm; —— trial, - - - - model.

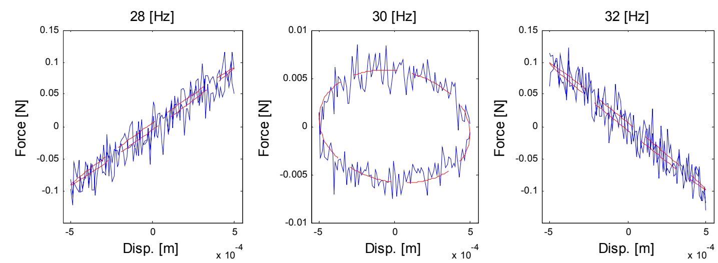

Figure 5. Comparison plots of fitting for some trial hysteresis loops of the *Synth* set; 20% noise; Excitation: 0.5 mm; —— trial, - - - - model.

6.2. Exp Data Set

Parameter identification tests were also performed for the *Exp* set of 93 experimentally-obtained hysteresis loops for a sandwich composite sample around the first resonance frequency. The results obtained in this case are shown in Table 4.

Table 4. Optimization results for the *Exp* data set, using both, PSO and GA.

Parameter	PSO		GA	
	$\bar{x}$	σ	$\bar{x}$	σ
ξ_0	0.009884	7.79E-05	0.009835	0.000125
S_ξ	-25.8709	0.540977	-25.4707	1.01142
ω_0	546.3013	0.099383	546.3183	0.071626
S_ω	-6219.49	850.6676	-6299.07	535.4282

In a similar way as in Section 6.1, an example comparison plot of some of the experimental hysteresis loops used here with their solutions are shown in Fig. 6. Again, similar shapes have been obtained for all runs with both optimization techniques.

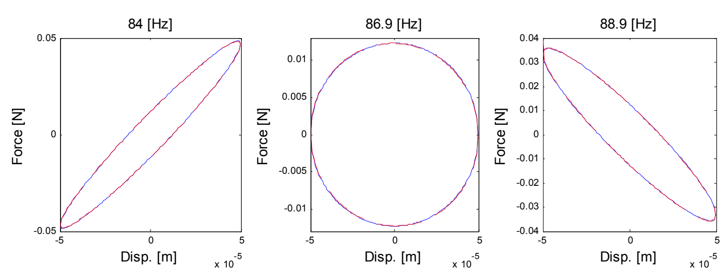

Figure 6. Comparison plots of fitting for some trial hysteresis loops of the *Exp* set; Excitation: 0.05 mm; —— trial, - - - - model.

7. CONCLUSIONS

A study of the identification of model parameters for a sandwich composite material has been presented. The experimental testing apparatus and some representative hysteresis loops that were obtained from it have also been presented. A simplified mathematical model for the representation of the dynamic response of a honeycomb-core sandwich composite material beam sample under a displacement excitation at its mid-point has been developed and tailored based upon experimental observations of the system behavior.

It can be seen from the simulation results obtained from both PSO and GA that the parametric solutions that were found were usually quite close (among different runs) to the actual values of the parameters when the global optimum is known. In addition, it has been observed that in spite of the addition of high levels of noise to the simulated signals, the solutions in these cases still remain close to the global optimum. However, the noise for many cases in the simulation study was deliberately set at very high levels, and the solutions obtained under these conditions are still quite consistent and acceptable for most applications.

The experimentally identified parameters are also quite consistent and lend confidence to both the simplified model and to the identification algorithms. From the experimental results obtained with the optimization techniques, it can be observed that nominal values of the damping ratio are around $\xi \approx 0.0073$, when the results are evaluated into Eq. (3). This is relatively high, even for composite materials, and it is due mainly to relative motion at the various interfaces within the composite and its interaction with the foam filling in the honeycomb core structure. This makes this type of material a good choice when low weight and good vibration energy dissipation performance are of importance for the construction of a structure, such as what is needed in aircrafts.

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