



ROBUST POLE PLACEMENT FOR ACTIVE VIBRATION CONTROL OF SEISMIC-EXCITED BUILDINGS

Haiping Du and Nong Zhang

Mechatronics and Intelligent Systems, Faculty of Engineering, University of Technology, Sydney PO Box 123, Broadway, NSW 2052, Australia <u>hdu@eng.uts.edu.au</u>

Abstract

In this paper, the robust controller design problem for active vibration control of seismicexcited building structures by means of pole placement method is studied. In particular, the building structures are assumed to have parameter uncertainties, and the design of controller will assure the pole locations of the closed-loop system are inside an indicated subregion on the left-hand side of complex plane in spite of the parameter uncertainties. For the building models, the parameter uncertainties dealt with belong to the polytope type uncertainties and are assumed to be the variations of the structural masses, stiffnesses, and damping coefficients. The quadratic stability test using the fixed quadratic Lyapunov function is studied and the sufficient conditions for the existence of a robust stabilising state feedback controller are presented as linear matrix inequalities (LMIs). The performance of the presented approach is demonstrated by numerical simulations on the vibration control of building structure subject to seismic excitation. It is confirmed that the designed controller can effectively attenuate the structural vibration when the parameter uncertainties exist.

1. INTRODUCTION

Active vibration control (AVC) of engineering structures, such as large flexible space structures, tall and slender buildings, towers, and long-span bridges to reduce the excessive vibration received considerable attention in the recent several decades. Various control strategies, such as LQR control, sliding mode control, adaptive control, neural network control, and fuzzy logic control etc., have been proposed and developed to attenuate the effects of structural vibration. However, due to modelling errors, variations of material properties, and changing load environments, the system description for the structural systems inevitably contains uncertainties in different natures and levels [1]. The uncertainty is one of the most critical aspects to a vibration control system since it can affect both the performance and stability of the control system.

This paper is interested in the robust controller design problem for the uncertain structural systems. The parameter uncertainties dealt with are of the polytopic type. The control objective is to assure that the pole locations of the closed-loop system are inside an indicated subregion on the left-hand side of complex plane in spite of the parameter uncertainties. Sufficient conditions for designing such a controller are given in terms of linear matrix inequalities (LMIs). To validate the effectiveness of the approach, the designed controller is applied to reduce the vibration of a seismic-excited building structure. Simulation results show that in spite of the existence of parameter uncertainties, the designed controller can achieve good vibration attenuation performance and keep the system robust stability.

2. STRUCTURAL MOTION EQUATION

Consider an n degree-of-freedom (DOF) linear structure with external disturbance, the equation of motion can be written as

$$\overline{M}\ddot{\mathbf{x}}(t) + \overline{C}\dot{\mathbf{x}}(t) + \overline{K}\mathbf{x}(t) = H\mathbf{u}(t) + Ew(t)$$
(1)

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the floor displacement vector; $\mathbf{u}(t) \in \mathbb{R}^r$ is the control force vector; $H \in \mathbb{R}^{n \times r}$ gives the location of the *r* control forces; w(t) is the ground excitation disturbance; $E \in \mathbb{R}^n$ is an vector denoting the influence of disturbance; \overline{M} , \overline{C} , and \overline{K} are the mass, damping, and stiffness matrices of the structure, respectively.

Using the state variable $\mathbf{q}(t) = [\mathbf{x}^T(t) \ \mathbf{\dot{x}}^T(t)]^T$, the system in (1) can be expressed in state space form

$$\dot{\mathbf{q}}(t) = A\mathbf{q}(t) + B_1 w(t) + B_2 \mathbf{u}(t)$$
⁽²⁾

where

$$A = \begin{bmatrix} 0 & I \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{C} \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ \overline{M}^{-1}E \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \overline{M}^{-1}H \end{bmatrix}.$$

3. EXPRESSION OF UNCERTAIN STRUCTURE

When the time-varying parameters are considered in the model (2), the structure model is becoming a parameter-varying model and this parameter-varying model is expressed as

$$\dot{\mathbf{q}}(t) = A(\xi)\mathbf{q}(t) + B_1(\xi)w(t) + B_2(\xi)\mathbf{u}(t)$$
(4)

where the state space matrices $A(\xi)$, $B_1(\xi)$, and $B_2(\xi)$ are continuous function of ξ which is time-varying parameter vector. Assume matrices $A(\xi)$, $B_1(\xi)$, and $B_2(\xi)$ are constrained to the polytope *P* given by

$$P = \left\{ (A, B_1, B_2)(\xi) : (A, B_1, B_2)(\xi) = \sum_{i=1}^{N} \rho_i(\xi)(A, B_1, B_2)_i, \sum_{i=1}^{N} \rho_i(\xi) = 1, \rho_i(\xi) \ge 0, i = 1, \dots, N \right\}$$
(5)

It is clear that the knowledge of the value of ρ_i defines a precisely known system inside the polytope *P* described by the convex combination of its *N* vertices. Although ρ_i does not necessarily represent the actual time-varying parameter ξ of the dynamical system, there exists a linear relationship between ξ and ρ_i that can be easily determined from the physical model whenever ξ affects affinely the linear system.

4. VIBRATION CONTROL OF UNCERTAIN STRUCTURES

4.1 Pole Placement Requirement

It has been demonstrated that the optimal control design methods actually force the closedloop poles to be located in the complex plane where the damping ratios and the decay ratios of the system are required to obtain the satisfactory response performance [2]. From this point of view, the pole placement technique provides a more direct design procedure to achieve a similar objective as that of the optimal control methods. Therefore, to obtain the satisfied dynamic response, we need to place the closed-loop poles in the region $S(\alpha, r, \theta)$ as defined in Figure 1. Since the step response of a second-order system with poles $\lambda = -\varsigma \omega_n \pm j\omega_d$ is fully characterized in terms of the undamped natural frequency $\omega_n = |\lambda|$, the damping ratio ς , and the damped natural frequency $\omega_d = \omega_n \sqrt{1-\varsigma^2}$, by confining the closed-loop poles in the region $S(\alpha, r, \theta)$, we can ensure a minimum decay rate α , a minimum damping ratio ζ (= $\cos \theta$), and a minimum damped natural frequency ω_d (= $r\sin \theta$) for the closed-loop system. These, in turn, bound the maximum overshoot, the frequency of oscillation, the decay time, the rise time, and the settling time for the closed-loop system transient response.



Figure 1. The system pole placement region $S(\alpha, r, \theta)$.

4.2 Formulation for the Controller Synthesis

The aim of this paper is to investigate the existence of a state feedback control law $\mathbf{u}(t) = K\mathbf{q}(t)$ such that the poles of the closed-loop system

$$\dot{\mathbf{q}}(t) = \left[A(\xi) + B_2(\xi) K \right] \mathbf{q}(t) + B_1(\xi) w(t)$$
(6)

are located in the region $S(\alpha, r, \theta)$, where κ is a constant state feedback gain.

The quadratic stability of the closed-loop system (6) with the pole location specification $S(\alpha, r, \theta)$ is equivalent to the existence of a symmetric positive definite matrix x such that the following linear matrix inequalities

$$A(\xi)X + XA^{T}(\xi) + B_{2}(\xi)Y + Y^{T}B_{2}^{T}(\xi) + 2\alpha X < 0$$
⁽⁷⁾

$$\begin{bmatrix} -rX & A(\xi)X + B_2(\xi)Y \\ * & -rX \end{bmatrix} < 0$$
(8)

$$\begin{bmatrix} \sin\theta \left[A(\xi)X + B_2(\xi)Y + X(\xi)^T + Y^T B_2(\xi)^T \right] & \cos\theta \left[A(\xi)X + B_2(\xi)Y - XA(\xi)^T - Y^T B_2(\xi)^T \right] \\ * & \sin\theta \left[A(\xi)X + B_2(\xi)Y + XA(\xi)^T + Y^T B_2(\xi)^T \right] \end{bmatrix} < 0$$

$$\tag{9}$$

are feasible [3]. Where Y = KX, and the notation * is used to represent a block matrix which is readily inferred by symmetry.

Since the uncertain closed-loop system (6) belongs to the polytope *P* described by its vertices, the sufficient conditions assuring the desired pole locations are given by the existence of a common matrix $X = X^T > 0$ satisfying (7)-(9) at the vertices of *P*. In fact, using the conditions $\sum_{i=1}^{N} \rho_i(\xi) = 1$ and $\rho_i(\xi) \ge 0$, the following LMIs can be easily derived from (7)-(9):

$$A_{i}X + XA_{i}^{T} + B_{2_{i}}Y + Y^{T}B_{2_{i}}^{T} + 2\alpha X < 0$$
(10)

$$\begin{bmatrix} -rX & A_i X + B_2 Y \\ * & -rX \end{bmatrix} < 0$$
(11)

$$\begin{bmatrix} \sin\theta \left[A_i X + B_{2_i} Y + A_i X + (B_{2_i} Y)^T \right] & \cos\theta \left[A_i X + B_{2_i} Y - X A_i^T - (B_{2_i} Y)^T \right] \\ * & \sin\theta \left[A_i X + B_{2_i} Y + X A_i^T + (B_{2_i} Y)^T \right] \end{bmatrix} < 0$$
(12)

 $i = 1, 2, \dots, N$.

Suppose (X^* , Y^*) is a feasible solution of the above LMIs, then the state feedback gain is obtained as $K = Y^*(X^*)^{-1}$.

5. APPLICATION TO SEISMIC-EXCITED BUILDINGS

In this example, the three-storey shear-beam building model considered in [4] is studied. The active bracing system (ABS) is installed at the first floor to control the vibration of the structure. The structural parameters are $m_i = 1000$ kg, $c_i = 1.407$ kN s/m and $k_i = 980$ kN/m, where i = 1, 2, 3 respectively.

The equation of motion of the three-storey shear-beam building model is obtained similar to equation (2), in which

$$A = \begin{bmatrix} 0 & I \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{C} \end{bmatrix}$$

where

$$\overline{M}^{-1}\overline{K} = \begin{bmatrix} k_1 / m_1 & -k_2 / m_1 & 0 \\ -k_1 / m_1 & k_2 / m_1 + k_2 / m_2 & -k_3 / m_2 \\ 0 & -k_2 / m_2 & k_3 / m_2 + k_3 / m_3 \end{bmatrix}$$

and $\overline{M}^{-1}\overline{C}$ can be obtained by replacing k_i by c_i where i = 1, 2, 3. Also,

$$B_1 = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix}^T$$

and

 $B_2 = \begin{bmatrix} 0 & 0 & 0 & 1/m_1 & -1/m_1 & 0 \end{bmatrix}^T$

In this paper, the parameter uncertainties of the structure are considered as the variations of masses, stiffnesses, and damping coefficients of all floors, and assume that the uncertainties of masses, stiffnesses and damping coefficients are 30% of their nominal values, i.e. the mass can be varied between 700 and 1300 kg, the stiffiness can be varied between 0.7×980 and 1.3×980 kN/m, and the damping coefficient can be varied between 0.7 × 1.407 and 1.3 × 1.407 kN s/m. Since the mass, stiffness, and damping coefficient are identical for every floor in this example, we can define the uncertainty parameters ξ_1 as ki/mi, ξ_2 as the ci/mi, and ξ_3 as 1/mi, then the vertices ρ_i (i = 1, 2, 3, 4, 5, 6, 7, 8) of the polynomial set for matrices A and B_2 can be defined.

The design objective is to reduce the structural vibration excited by seismic disturbance by constraining the closed-loop poles in the region $S(4, 400, \pi/2)$ regardless of the existence of the parameter uncertainties. Using the approach presented in Section 4.2, we can obtain such a state feedback controller.

In order to show the time domain performance of the system, the El Centro 1940 earthquake excitation of which peak acceleration is scaled to 0.12g is applied to the system (6). For the nominal system, the responses of the open-loop (passive) system (u(t)=0) and the closed-loop (active) system are compared in Figure 1, where only the interstorey drift of the first floor is shown for clarity.



Figure 2. Interstorey drift of the first floor for the nominal system. Dot line is for the passive system, solid line is for the active system.

For detailed comparison, the maximum open- and closed-loop interstorey drifts, $x_{i \max}$, i = 1, 2, 3, floor absolute accelerations, $\ddot{x}_{i \max}$, i = 1, 2, 3, and control force u_{\max} are summarized in Table 1. It can be seen from Figure 1 and Table 1 that better responses are obtained for the closed-loop case with the designed controller.

Quantities	Passive	Active	
$u_{\rm max}$ (kN)	0	5.06	
$x_{1 \max}$ (cm)	1.26	0.41	
$x_{2 \max}$ (cm)	0.99	0.23	
$x_{3 \max}$ (cm)	0.59	0.13	
$\ddot{x}_{1 \max} (m/s^2)$	3.40	2.65	
$\ddot{x}_{2 \max} (m/s^2)$	4.36	1.00	
$\ddot{x}_{3 \max}$ (m/s ²)	5.81	1.29	

Table 1. Peak response quantities for nominal system under El Centro earthquake excitation

To show the robustness of the closed-loop system when parameter uncertainties exist, the statistical peak response quantities of the interstorey drifts and absolute accelerations of the uncertain system with 500 randomly generated masses, stiffnesses, and damping coefficients are listed in Table 2, where the maximum, minimum, and average values for both the passive and the active systems are given. It can be seen from this table that in spite of the existence of uncertain parameters, which induces large variations in the peak responses of the passive system, the active system with the designed controller can still keep the peak responses lower. It confirms that the robust stability and robust performance of the closed-loop system with the designed controller are achieved.

 Table 2. Statistical peak response quantities for system with randomly generated system parameters under El Centro earthquake excitation

Quantities	Passive		Active			
	Maximum	Minimum	Average	Maximum	Minimum	Average
$u_{\rm max}$ (kN)	0	0	0	6.58	3.58	5.19
$x_{1 \max}$ (cm)	2.74	0.63	1.40	0.72	0.23	0.43
$x_{2 \max}$ (cm)	2.10	0.47	1.10	0.32	0.13	0.23
$x_{3 \max}$ (cm)	1.15	0.27	0.64	0.20	0.07	0.13
$\ddot{x}_{1 \max}$ (m/s ²)	4.89	2.57	3.46	3.22	1.80	2.62
$\ddot{x}_{2 \max} (m/s^2)$	6.99	3.38	4.74	1.13	0.88	1.00
$\ddot{x}_{3 \max}$ (m/s ²)	8.49	4.33	5.93	1.31	1.09	1.27

For showing this clearly, the time-domain responses for system with randomly generated m_i =895 kg, c_i =1.4623 kN s/m, and k_i = 1215.9 kN/m are plotted in Figure 3 for both the passive and the active systems, where only the interstorey drift of the first floor is shown for clarity. It can be seen from this figure that the active system response is much smaller than the passive system even when the system parameters are different from the nominal system.

Figure 4 shows the closed-loop poles for the 500 randomly generated uncertain systems,

which are far away from their nominal specifications. It can be seen that closed-loop poles are all located within the specified region.



Figure 3. Interstorey drift of the first floor for the uncertain system. Dot line is for the passive system, solid line is for the active system.



Figure 4. Closed-loop system poles with randomly generated system parameters. The poles of the closed-loop system vertices are represented by diamond mark.

6. CONCLUSIONS

This paper presents a pole placement approach for designing a robust controller to attenuate the vibration of civil engineering structures under seismic excitation. Based on the quadratic stability condition, the required state feedback control gain matrix can be determined by solving finite number of LMIs. The parameter uncertainties dealt with in this paper belong to the polytope type and the designed controller gain is constant. Therefore, the real-time measurements for the time-varying parameters, such as mass, stiffness, and damping coefficient, are not necessary. Simulation example shows that the controller designed using the presented approach can effectively perform the attenuation objective even when the system has larger parameter uncertainties.

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