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SHIMMY EVALUATION USING THEORETICAL MULTIBODY MODEL AND EXPERIMENTAL TECHNIQUES

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Abstract

Shimmy is the vibration in the steering rotational direction during steady-state running and gives annoyance to passengers. Passenger's comfort is a critical characteristic in vehicle. In this study, both a theoretical linear model and a numerical model are used to predict and optimize the frequencies of shimmy related modes and the level of vibration for a vehicle. A theoretical multibody linear model of 48-DOF was used for quasi-static analysis and modal analysis. The theoretical model was verified by comparing the results of modal analysis with those of a modal experiment for front suspension system using CADA-X program. The quasi-static analysis results of the theoretical model were compared with those of an ADAMS model. A full vehicle model using ADAMS was also verified with the results of the modal experiment and chassis dynamometer experiments for shimmy reproduction. From experiments and simulations, it was found that the wheel longitudinal vibration mode was the most dominant source of shimmy vibration. In addition, 13 design parameters of the front suspension system including compliance and geometric factors for reducing shimmy level were selected based on the experimental results. By performing 27 orthogonal simulations using Taguchi methodology, an optimal combination of design parameters was constructed. Through modal analysis of the theoretical model with the optimized design parameters, it was found that the wheel longitudinal vibration mode changed into two local vibration modes. Finally, a simulation of the numerical model verified that the suggested design parameters resulted in shimmy vibration reduction.

1. INTRODUCTION

The source of generation of rotational moment around kingpin axis, which leads to lateral vibration of tie-rod and subsequently rotational vibration of steering wheel, is identified by non-uniformity, unbalance, eccentricity, or run-out of wheel-tire assembly. Shimmy has a significant influence on ride comfort and maneuvering stability due to its resonant characteristics. Once shimmy occurs, it does not vanish until vehicle speed is significantly increased or decreased. Since it is virtually impossible to eliminate the source of shimmy mentioned above, it is necessary to study the optimization of suspension system, which is the transmitting path of vibration from wheel-tire assembly to steering wheel. Kimura [1] classified low speed shimmy and high speed shimmy according to the occurring velocity. The former is a self-excited oscillation of suspension system and steering wheel, and the latter is caused by dynamic inequality of the system and disturbance. Dödlbacher [2] investigated shimmy phenomenon by using 19-DOF model with engine and links, and defined a percentage as contribution rate for the effecting element to shimmy, for instance, geometry of suspension system 30 %, fore-aft stiffness of suspension system 25 %, fore-aft stiffness of suspension member 25 %, torsional stiffness of steering system 10 % and moment of inertia at steering wheel 10 %. Although a number of researches on vehicle shimmy vibration have been carried out, most of them take either mathematical or complex numerical simulation model into consideration. In this study, theoretical vehicle model is constructed by applying linear analysis method with small displacement assumptions, and quasi-static and modal analyses are conducted using this model. Numerical simulation model is also constructed and predicted using multibody dynamics software ADAMS. Besides, the path of vibration transmission is investigated by observing dynamic behavior of front suspension system through shimmy reproduction experiment. We find out shimmy related modes through modal testing. Finally, a numerical simulation is carried out to verify that by applying suggested design parameters of Taguchi methodology results in alleviating shimmy vibration.

2. THEORY AND MODELING

2.1 Linear elasto-kinematic governing equation

There have been a wide variety of researches about kinematic and dynamic analyses on multibody systems consisting of rigid bodies [3]. Although it is relatively easy to determine the position of the purely kinematic system, it is difficult to determine the position of the rigid body system that is connected by elastic elements and constrained by kinematic constraints. The conventional method of finding static equilibrium of multibody system is to solve nonlinear constraint Equation (1) and force equilibrium Equation (2) [4, 5].

$$\Phi(\boldsymbol{q}) = 0 \tag{1}$$

$$\Phi_{\boldsymbol{q}}^{\boldsymbol{T}} \lambda + \boldsymbol{f}^{\boldsymbol{A}} = 0 \tag{2}$$

Where $\boldsymbol{q}$ represents general coordinate vectors of a rigid body system in Equation (1). λ, Φ_q, and $\Phi_q^T \lambda$ represent Lagrange's multiplier, Jacobian and constraint force in Equation (2) respectively. f^A is external forces applied on a rigid body system. These are represented in the form of nonlinear Equation (3), and are generally solved by iteration methods, such as Newton-Rhapson method, in order to obtain $\boldsymbol{q}$ and λ. However, it is difficult to estimate the initial value of λ and to obtain exact Jacobian matrix, which results in degrading the convergence of the solution.

$$\begin{bmatrix} \Phi_{\boldsymbol{q}} & 0 \\ (\Phi_{\boldsymbol{q}}^{\boldsymbol{T}}(\lambda) + \boldsymbol{f}^{\boldsymbol{A}})_{\boldsymbol{q}} & \Phi_{\boldsymbol{q}}^{\boldsymbol{T}} \end{bmatrix} \begin{Bmatrix} \Delta \boldsymbol{q} \\ \Delta \lambda \end{Bmatrix} = -\begin{Bmatrix} \Phi \\ \Phi_{\boldsymbol{q}}^{\boldsymbol{T}}(\lambda) + \boldsymbol{f}^{\boldsymbol{A}} \end{Bmatrix} \tag{3}$$

However, with small displacement assumptions, difference of Equation (1), $\Delta\Phi = 0$, can be satisfied while the system undergoes external forces. Therefore, an elasto-kinematic governing equation with Jacobian matrix of the kinematic constraint equation is as follows.

$$\begin{bmatrix} \boldsymbol{K} & \Phi_{\boldsymbol{q}}^{\boldsymbol{T}} \\ \Phi_{\boldsymbol{q}} & 0 \end{bmatrix} \begin{Bmatrix} \Delta \boldsymbol{q} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f} \\ 0 \end{Bmatrix} \tag{4}$$

2.2 Quasi-static equilibrium analysis with piece-wise linear assumption

If we assume a small displacement, Equation (4) can be satisfied in deformed positions resulted from continuous external forces. The piece-wise linear assumption makes it possible for the above mentioned elasto-kinematic governing equation to be applied. With this assumption, we can induce the following series equation, which determines the coordinates of the next step by obtaining the displacement increment $\Delta \boldsymbol{q}^{\boldsymbol{n}-1}$ from the coordinates of the previous step $\boldsymbol{q}^{\boldsymbol{n}-1}$.

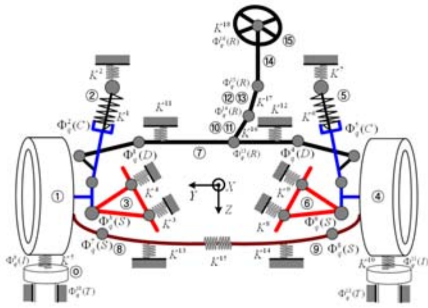
Figure 1. Theoretical model for quasi-static analysis.

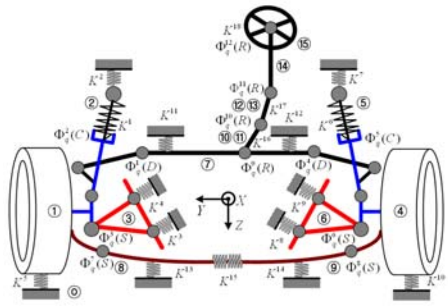
Figure 2. Theoretical model for modal analysis.

$$\begin{bmatrix} \boldsymbol{K}(\boldsymbol{q}^{\boldsymbol{n}}) & \Phi_{\boldsymbol{q}}^{\boldsymbol{T}}(\boldsymbol{q}^{\boldsymbol{n}})^{\boldsymbol{T}} \\ \Phi_{\boldsymbol{q}}(\boldsymbol{q}^{\boldsymbol{n}}) & 0 \end{bmatrix} \begin{Bmatrix} \Delta \boldsymbol{q} \\ \lambda \end{Bmatrix}^{\boldsymbol{n}} = \begin{Bmatrix} \Delta \boldsymbol{f} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \Phi_{\boldsymbol{q}}(\boldsymbol{q}^{\boldsymbol{n}-1})^{\boldsymbol{T}} \lambda^{\boldsymbol{n}-1} \\ 0 \end{Bmatrix} \tag{5}$$

According to small rotation of local coordinate systems, coordinate transformation matrix also has to be updated by Equation (6), and any vector represented by global coordinate system can be updated by Equation (7). With these series equations, quasi-static behavior of multibody system can be analyzed easily by any linear methods such as Gauss elimination method.

$$\boldsymbol{A}^{\boldsymbol{n}} = \begin{bmatrix} 1 & -\Delta\pi_{\boldsymbol{z}} & \Delta\pi_{\boldsymbol{y}} \\ \Delta\pi_{\boldsymbol{z}} & 1 & -\Delta\pi_{\boldsymbol{x}} \\ -\Delta\pi_{\boldsymbol{y}} & \Delta\pi_{\boldsymbol{x}} & 1 \end{bmatrix}^{\boldsymbol{n}-1} \boldsymbol{A}^{\boldsymbol{n}-1} \tag{6}$$

$$\boldsymbol{v}^{\boldsymbol{n}} = \boldsymbol{v}^{\boldsymbol{n}-1} - \tilde{\boldsymbol{v}}^{\boldsymbol{n}-1} \Delta\pi^{\boldsymbol{n}-1} \tag{7}$$

Suspension parameters have been obtained by applying Equations (5), (6) and (7) to a Mcpherson suspension. A theoretical linear model of a 48-DOF shown in Figure 1 has been constructed in order to simulate Suspension Parameter Measurement Device (SPMD) test. Vertical force has been applied on the center of the jack in order for the wheel to move in vertical direction. For model verification, results of SPMD analysis have been compared with those of ADAMS simulation.

2.3 Theoretical modal analysis

Dynamic characteristics of vehicle suspension system can be effectively investigated through modal analysis. In order to find out dominant modes related to shimmy and changes in those modes according to design modifications, theoretical modal analysis for front suspension system has been conducted. Based on the theory suggested above, 48-DOF mathematical model has been constructed by modeling Mcpherson suspension system as 9 rigid bodies connected by elastic elements and joints shown in Figure 2.

From linear static equations, dynamic equation can be induced. By differentiating Equation (3) with respect to time, velocity and acceleration constraint equations for rigid body system can be obtained with small motion assumption. Taking constraint forces into consideration, the following dynamic equation can be obtained.

$$\boldsymbol{M}\Delta\ddot{\boldsymbol{q}} + \boldsymbol{C}\Delta\dot{\boldsymbol{q}} + \boldsymbol{K}\Delta\boldsymbol{q} + \Phi_{\boldsymbol{q}}^{\boldsymbol{T}}\lambda = \boldsymbol{f}(\boldsymbol{t}) \tag{8}$$

If we divide general coordinate vector $\Delta\boldsymbol{q}$ and Jacobian matrix $\Phi_{\boldsymbol{q}}$ into $\Delta\boldsymbol{q} = \left[\Delta\boldsymbol{u}^{\boldsymbol{T}}, \quad \Delta\boldsymbol{v}^{\boldsymbol{T}}\right]^{\boldsymbol{T}}$ and $\Phi_{\boldsymbol{q}} = \left[\Phi_{\boldsymbol{u}}, \quad \Phi_{\boldsymbol{v}}\right]$ respectively, constraint equation can be written as follows.

$$\Phi_{\boldsymbol{u}}\Delta\boldsymbol{u} + \Phi_{\boldsymbol{v}}\Delta\boldsymbol{v} = 0 \tag{9}$$

Since submatrix $\Phi_{\boldsymbol{u}}$ of Jacobian matrix $\Phi_{\boldsymbol{q}}$ is not singular, Equation (9) can be arranged and written as follows.

$$\Delta\boldsymbol{u} = -\Phi_{\boldsymbol{u}}^{-1}\Phi_{\boldsymbol{v}}\Delta\boldsymbol{v} \tag{10}$$

That is, equation of motion of $\boldsymbol{n}$ general coordinate system can be transformed into that of $\boldsymbol{k}$ independent coordinate system $(\boldsymbol{n} \geq \boldsymbol{k})$. Transformation equation and transformation matrix are shown below.

$$\Delta\boldsymbol{q} = \boldsymbol{D}\Delta\boldsymbol{v} \tag{11}$$

$$\boldsymbol{D} = \begin{bmatrix} -\Phi_{\boldsymbol{u}}^{-1}\Phi_{\boldsymbol{v}} \\ \boldsymbol{I} \end{bmatrix} \tag{12}$$

If we carry out coordinate transformation by substituting Equation (11) into Equation (7), and then conduct forward multiplication of $\boldsymbol{D}^{\boldsymbol{T}}$, following equation of motion of $\boldsymbol{k}$ independent coordinates can be obtained.

$$\boldsymbol{M}_{\boldsymbol{D}}\Delta\ddot{\boldsymbol{v}} + \boldsymbol{C}_{\boldsymbol{D}}\Delta\dot{\boldsymbol{v}} + \boldsymbol{K}_{\boldsymbol{D}}\Delta\boldsymbol{v} = \boldsymbol{f}_{\boldsymbol{D}} \tag{13}$$

where $\boldsymbol{M}_{\boldsymbol{D}} = \boldsymbol{D}^{\boldsymbol{T}}\boldsymbol{M}\boldsymbol{D}$, $\boldsymbol{C}_{\boldsymbol{D}} = \boldsymbol{D}^{\boldsymbol{T}}\boldsymbol{C}\boldsymbol{D}$, $\boldsymbol{K}_{\boldsymbol{D}} = \boldsymbol{D}^{\boldsymbol{T}}\boldsymbol{K}\boldsymbol{D}$ and $\boldsymbol{f}_{\boldsymbol{D}} = \boldsymbol{D}^{\boldsymbol{T}}\boldsymbol{f}$.

If we remove damping term and external force for eigenvalue analysis, simple dynamic equation can be obtained as follows.

$$\boldsymbol{M}_{\boldsymbol{D}}\Delta\ddot{\boldsymbol{v}} + \boldsymbol{K}_{\boldsymbol{D}}\Delta\boldsymbol{v} = 0 \tag{14}$$

The solution satisfying the following eigenvalue problem exists in this system,

$$[\boldsymbol{K}_{\boldsymbol{D}}][\Psi] = [\boldsymbol{M}_{\boldsymbol{D}}][\Psi]\lfloor\omega^2\rfloor \tag{15}$$

where $\lfloor\omega^2\rfloor$ and $[\Psi]$ are eigenvalue matix and eigenmode matrix, respectively. In case that there are $\boldsymbol{k}$ number of independent coordinates, $\boldsymbol{k}$ number of eigenvalues and eigenmodes can be obtained by coordinate transformation through Equation (11).

3. NUMERICAL ANALSYSIS

3.1 SPMD and Shimmy analysis

ADAMS model has been constructed using geometry, inertia, stiffness, and damping data of front suspension system, and SPMD simulation under the same condition as quasi-static analysis has been carried out. The result of SPMD analysis was found to agree well with those by ADAMS simulation result. Shimmy simulation has been conducted with the ADAMS simulation model which was verified to have kinematic and dynamic characteristics of the subject vehicle [8, 9]. Simulation has been carried out under the driving condition of constant speed of 120 km/h on random road profile without steering input. Road profile of ISO D level was used in order for random excitation to be applied to wheels.

4. EXPERIMENTS

4.1 Shimmy reproduction experiment

Experiment has been conducted in order to observe dynamic behavior of each component of suspension system using chassis dynamometer while shimmy occurs. Vehicle speed was increased from 40 km/h to 140 km/h with constant acceleration for the investigation of shimmy trend with respect to vehicle speed. Also, additional unbalanced masses of 0 g, 20 g, 40 g, 60 g, and 80 g were attached on the wheel in each experiment for looking into the effect of wheel unbalance on shimmy.

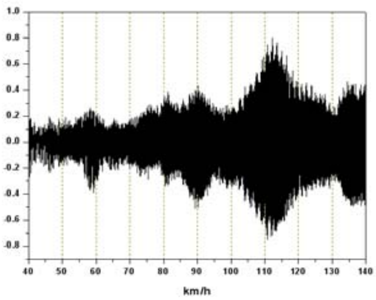

Figure 3. Steering wheel rotational acceleration.

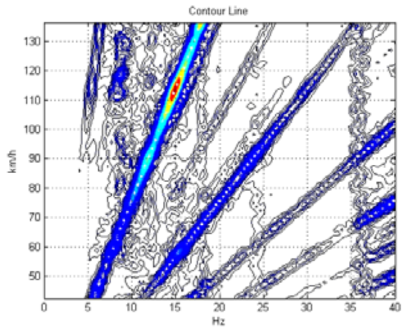

Figure 4. Contour plot of steering wheel.

Signals were collected through 24 channels in order to analyze dynamic behavior of each part of front suspension system. Sampling frequency was 160 Hz and frequency resolution was 0.0195 Hz. The rotational acceleration is extracted by calculating difference of two vertical direction signals measured on both the sides of steering wheel. Figure 3 shows time response of rotational vibration on the steering wheel when vehicle speed is increased from 40 km/h to 140 km/h. Shimmy begins to appear when vehicle speed reaches 100 km/h and salient rotational vibration is observed at around 110 km/h. Shimmy decreases at the velocity of the maximum response, which shows that shimmy is due to the resonance of specific modes of suspension system. Figure 4 represent shimmy response in frequency domain simultaneously with horizontal axis at 15 Hz and vertical axis at 115 km/h. The fact that the response of steering wheel coincides with the angular velocity of wheel means that the vibration on the steering wheel is due to the rotational excitation of wheel with unbalanced mass. With this excitation, steering wheel shows 15 Hz shimmy response, which means that the suspension system has a resonant mode in this frequency.

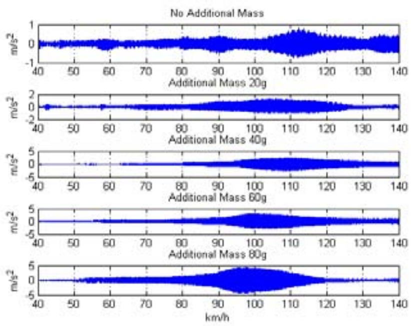

Figure 5. Effect of unbalanced mass.

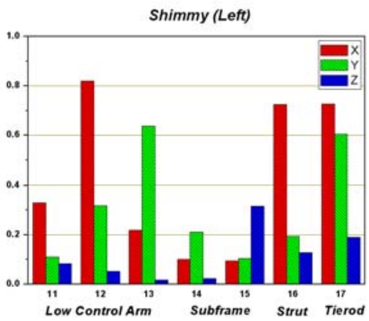

Figure 6. The responses on suspension system.

Tendency of shimmy to the amount of unbalanced mass on the wheel is represented in Figure 5. As unbalanced mass attached to the wheel increases, shimmy remarkably appears more in lower speed, which is due to nonlinear characteristics [6, 7] of suspension system, that is, resonant frequency becomes lower as the magnitude of excitation force $\boldsymbol{F} = \boldsymbol{m_e r}\omega^2$ increases due to increasing unbalanced mass. The magnitudes of response at each measuring point are compared in order to investigate the path of vibration transmission from wheel to steering wheel as shown in Figure 6. As shimmy phenomenon becomes more evident due to increasing unbalanced mass, centrifugal force originated from wheel unbalance excites wheel-knuckle assembly in longitudinal direction. Hence, lower arm vibrates in rotational direction on the whole with A-bush being rotational center. Consequently, tie-rod moves in lateral direction resulting in subsequent rotational vibration of steering wheel, or shimmy.

4.2 Modal experiment

Shimmy reproduction experiment indicates that shimmy is due to a resonance of specific modes of suspension system. In order to find out dominant modes related to shimmy, modal experiment [10, 11] has been conducted for front suspension system. Front suspension is excited on the wheels by electromagnetic exciter with random function up to 256 Hz which is generated by CADA-X. Acceleration signals have been measured on the same points as shimmy reproduction experiment. Natural frequencies and natural modes of the system have been extracted from signals using CADA-X (Figure 7).

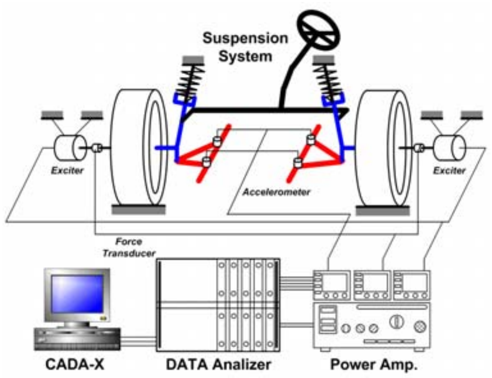

Figure 7. Experiment setup.

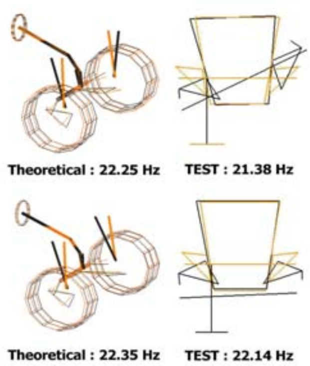

Figure 8. Wheel longitudinal vibration modes.

Table 1. Comparison of mode results of theoretical analysis and experiment [Hz].

Mode shape		Theoretical	Experiment
Wheel Hop Mode	In phase	13.13	13.99
	Out of phase	13.45	
Wheel Longitudinal Movement Mode	In phase	22.25	22.14
	Out of phase	22.35	21.38
Wheel Camber Motion Mode	In phase	28.22	27.55
	Out of phase	29.44	26.58

According to the result of modal experiment, wheel longitudinal vibration mode accords with the dynamic behavior of suspension system in shimmy reproduction experiment. To sum up, when frequency of excitation caused by rotation of unbalanced mass of wheel increases as vehicle speed becomes faster, resonance of wheel longitudinal vibration mode occurs at the time excitation frequency coincides with the natural frequency of the mode, which gives rise to lateral vibration of tie-rod that is connected to steering wheel. Natural frequencies and mode shapes obtained from modal experiment and theoretical modal analysis are compared in Table 1. Results of experiment and analysis coincide well in low frequency bend, which verifies the validity of the theoretical model. Especially, as shown in Figure 8, there is a very good agreement in the result of experiment and analysis for the wheel longitudinal vibration mode which is considered to be the most dominant source of shimmy vibration.

5. DESIGN OPTIMIZATION

5.1 Sensitivity analysis and design optimization using Taguchi methodology

Taguchi methodology [12] has been applied to suspension system in order to find out design parameters sensitive to shimmy and to determine optimal values of those parameters. 13 design parameters which are related to the wheel longitudinal vibration have been selected, and numerical simulation has been conducted in order to minimize the amplitude of vibration in frequency band of 13~18 Hz. Since it is smaller-the-better type problem, signal-to-noise ratio is as follows.

$$\eta = -10\log_{10}\left[\frac{1}{n}\sum_{i=1}^{n} y_i^2\right] \tag{16}$$

Using orthogonal array of 13 factors including geometry and compliance of suspension system and 3 levels for each factor, 27 orthogonal simulations are conducted in order to minimize interactions among design factors, and the effect of each factor is obtained through Analysis of Variance (ANOVA). Among 13 factors, ***C, J, L, M*** of which F-value is over 4 are set to be design parameters and the rest to be error terms. The following optimal combination of design factors has been obtained through ANOVA.

$$A_2, B_3, C_3, D_2, E_1, F_1, G_2, H_1, I_2, J_2, K_2, L_2, M_1 \tag{17}$$

Error variation (σ_e^2) of factor effects is ± 0.2590, and this satisfies 95 % confidence interval. S/N ratio of design factors C_3, J_2, L_2, M_1 and improvement are predicted as follows.

$$\begin{aligned} \eta_{opt} &= m + (m_{C3} - m) + (m_{J2} - m) + (m_{L2} - m) + (m_{M1} - m) \\ &= 19.533\,(dB) \\ \Delta\eta &= \eta_{opt} - \eta_{initial} = 2.2955\,(dB) \end{aligned} \tag{18}$$

In order to predict changes in dynamic behavior of suspension system after design optimization, modal analysis has been conducted by applying the optimal combination of design parameters to 48-DOF mathematical model. As a result, it is observed that there are some changes in mode shapes while natural frequencies remain almost the same. Particularly, there is a significant change in wheel longitudinal vibration mode which participates most in shimmy. Wheel longitudinal vibration mode, which moved as a whole, splits into two local modes with independent movement of right wheel and left wheel respectively (Figure 9). This shows that shimmy can be controlled by suppressing global motion of suspension system through appropriate design modification.

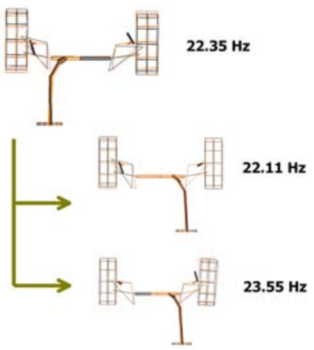

Figure 9. Mode separation using optimized values.

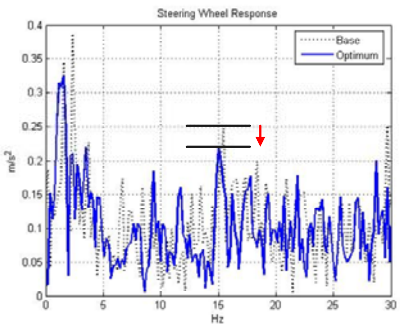

Figure 10. Simulation results.

Optimal combination of design parameters obtained above is applied to ADAMS model to predict the level of shimmy reduction after design optimization. Rotational acceleration on the steering wheel is measured under 120 km/h constant speed driving condition. As a result, shimmy response of 15 Hz has been decreased by more than 10 % as shown in Figure 10.

6. CONCLUSIONS

In this study, 48-DOF theoretical linear model of front suspension system was constructed and quasi-static analysis was carried out. The result was compared with that of numerical simulation conducted with multibody dynamics analysis software ADAMS. Also, modal analysis was carried out with theoretical model and the results were compared with those of modal experiment in order to validate the model. Wheel longitudinal vibration mode was found to be the most dominant mode in shimmy reproduction experiment. Furthermore, it is observed that wheel longitudinal vibration mode splits into two local modes with independent movement of right wheel and left wheel after optimization of design parameters using Taguchi methodology. This resulted in reducing shimmy response of 15 Hz and its level was predicted to decrease by more than 10 % in numerical simulation. Theoretical and numerical models for shimmy analysis were developed and they are expected to be used in estimation of shimmy level in the course of developing suspension system. Moreover, optimal combination of design parameters is suggested to improve shimmy phenomenon in the subject vehicle.

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