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NON-PERIODIC LOCALLY RESONANT SONIC MATERIALS

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Abstract

Locally resonant sonic materials (LRSMs) are a new class of meta-materials that can block sound waves well beyond the limit of the conventional mass density law. The LRSMs reported so far are made of local resonators arranged periodically in a matrix, although the period is usually much smaller (< 1/20) than the wavelength in the matrix material. At resonance, the effective wavelength becomes much smaller, and the interference from the individual resonators could become significant. Here we report further study of the properties of LRSMs. We found that in some deliberately structured LRSMs the main acoustic properties is determined only by the resonators, independent of their spatial arrangement. In particular, the acoustic properties of a single resonator resemble that of an assembly of the resonators. The lateral coupling of resonators, when deliberately introduced, generates multi-mode resonances, leading to multiple stop bands in transmissions. The dependence of the resonant frequency f on the mass of the weight M in the single-weight resonator follows the simple mass-and-string relation, namely $f \propto 1/\sqrt{M}$. The large phase shift associated with the transmission peak that accompanies the stop band provides an approach for designing zoneplates to focus low frequency sound waves, in much the same way as optical zoneplates. Simple estimations show that effective focusing could be achieved for a plate of the diameter of one wavelength and phase shift difference of less than 110 degrees across different zones.

1. INTRODUCTION

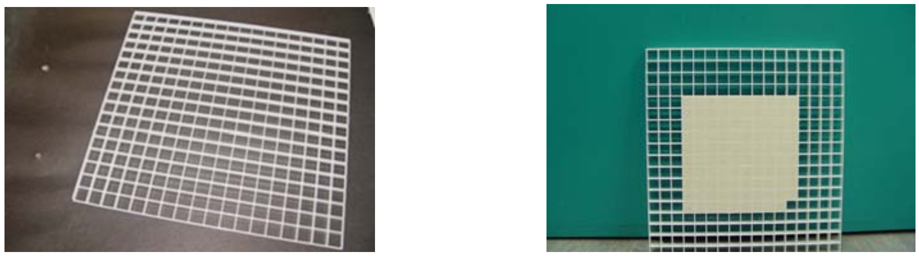
Phononic crystals (PCs) are composite materials with periodic variation of elastic and acoustic properties [1]. They are in the general category of meta-materials, which are artificial materials that possess extraordinary properties which are absent in the individual components. Typical PCs are made of identical scatterers of one material forming a periodic lattice that is embedded in another that fills the rest of the space. Sonic waves propagating in PCs will experience repeated scattering at periodic sites, and the combined interference of the scattered waves can lead to waves that propagate in extraordinary ways. Theoretical studies [1 – 4] have shown that within certain ranges of frequencies no wave can propagate (forbidden gaps), some along particular propagation directions relative to the PC orientation (partial gaps) while others in all directions (full gaps). Like in the case of photonic crystals [5], strong scattering is required to produce a gap wide enough so that a full gap can be obtained in all directions.

The locally resonant sonic materials (LRSMs) were first designed as PCs with strong scatterers, since at resonance the scattering cross section and the contrast of effective acoustic properties (effective mass and/or effective modulus) of the scatterers over the surrounding medium are expected to reach maximum. They are a new class of meta-materials that can block sound waves well beyond the limit of the conventional mass density law [6]. The LRSMs reported so far are made of local resonators arranged periodically in a matrix [6 – 8], although the period is usually much smaller (< 1/20) than the wavelength in the matrix material. They form a flat band in the band structure diagrams [9 – 12]. The predicted characteristic features of LRSMs are a dip followed by a peak in the transmission spectrum, while the phase spectrum changes by nearly 180 degrees at the corresponding dip and peak frequencies. At resonance, the effective wavelength becomes much smaller, and the interference from the individual resonators could become significant. There are considerable theoretical works [10 - 12], but there are only limited experimental works [6 – 8].

Here we report the experimental study of the properties of a particular type of LRSMs, in which the local resonators are virtually isolated from one another. If one resonator is let to oscillate, the others will remain virtually inactive. The resonators will oscillate collectively only when a broad acoustic wave is incident onto them. We found that the periodicity plays no critical role in determining the main characteristics of such LRSMs. In particular, the acoustic properties of LRSMs are well preserved even when the waves are passing through a single resonator. The mass dependence of the resonance frequency is also investigated. These properties offer the opportunity to arrange the resonators strategically to achieve particular purposes. For example, by stacking several panels together a broad band sound blocking panel could be formed [8]. By arranging resonators properly on a flat panel a zoneplate to focus low frequency sound becomes possible, in much the same way as optical zoneplates to focus optical waves.

2. EXPERIMENTS

The basic structure of the samples is the same as reported in Ref. 8. A regular unit cell grid is made of a square array of 1.5 cm × 1.5 cm × 2 cm (height) cells, with a lattice constant of 1.6 cm. Each cell is surrounded by four 1.5 cm × 2 cm × 0.1 cm hard plastic walls. A picture of an empty grid array is shown in Fig. 1. A larger square unit cell with the dimensions of 3.1 cm × 3.1 cm × 2 cm (height) is made of four regular unit cells by removing part of the grids. The cells are in regular array simply because of the availability of the raw materials. A steel ball is usually placed at the center of the cell and the rest of the cell space was filled with silicone rubber. Two kinds of rubber were used, one soft and one hard. The soft rubber was Silastic-3133 with elastic and loss modulus of 2×10^6 Pa and 1×10^6 Pa, respectively. The corresponding values of the hard rubber, Wacker M4440 Elastosil, were about 4×10^6 Pa and 1×10^6 Pa. The hard grid effectively block out the interaction between neighbouring cells while maintaining efficient interaction between the incidence acoustic waves and the resonators. There is a large acoustic mismatch between the grids and the rubber, as the sound speeds in the grid material and in the rubber differ by about a factor of 100.



(a) (b)

Figure 1. (a) An array of empty grids for the samples, and (b) a finished sample.

Our measurements are based on modifying the standard method [7, 13]. Impedance tubes are used to generate plane sound waves inside the tube while screening out room noise. The sample slab being measured is placed firmly and tightly between two Brüel & Kjær (B&K) Type-4026 impedance tubes. The front tube contains a B&K loudspeaker at the far end, and two Type-4187 acoustic sensors. A third acoustic sensor with an electronic gain about 100 times of that of the front sensors is placed at the fixture of the back tube. The rest of the back tube after the sensor is filled with anechoic sound absorbing sponge. The frequency of the wave is scanned in a range from 200 Hz to 1500 Hz, while the electric signals are measured by three lock-in amplifiers. More details can be found in Ref. 7. Aluminum boards of 10 mm in thickness with a clear hole of different sizes were used as apertures to select the area of a panel to be measured.

3. RESULTS

3.1 Stacked Samples

The transmission amplitudes of two samples are shown in Fig. 2(a). Both samples (Sample-1 and Sample-2) consist of one 5.6 gram steel ball, 11 mm in diameter, in each soft rubber filled 1.5 cm × 1.5 cm × 2 cm cell. The spectra display typical local resonant behavior with the resonance frequency at 890 Hz (transmission dip), followed by a transmission peak at 1100 Hz. The depth of the dip is over 20 dB in intensity. The red and the green curves are the spectra of Sample-1 and Sample-2 when they were measured individually. Their spectra are almost identical, owing to their nearly identical composition. The black curve in Fig. 2(a) is the transmission amplitude taken from the two samples stacked together. The dip deepens by another 20 dB, which can be regarded as the intrinsic sound attenuation per layer of unit cells. The results also indicate that each ball in the unit cell vibrates as an independent unit, so that its resonance frequency remains unchanged by the presence of other cells. In particular, there is no additional transmission dip which could be made possible by the coupling between two unit cells placed one in front of the other. Instead the resonance is mainly determined by the natural frequency of the unit cell, rather than due to the interference effect of waves scattered by the cells.

Heavier balls were used in the LRSM samples to downshift the resonant frequency. Sample-3 consists of steel balls (16.4 g) 16 mm in diameter in each large 3 cm × 3 cm × 2 cm cells. Its resonant frequency, as seen in Fig. 2(b), is at 490 Hz. Sample-4 consists of 16 mm lead balls (23.0 g) and its resonant frequency is at 360 Hz. Sample-5 consists of 19 mm steel balls (28.4 g) and its resonant frequency is at 270 Hz. The three samples were then stacked together to form effective broad band low frequency sound shield. The measured transmission amplitude is shown in Fig. 2(b) as the black curve. It is seen that the transmission dips of the stacked samples are at almost the same frequencies as the individual ones when they were measured alone. This again shows that the resonant frequency is mainly determined by the natural frequency of the unit cell, rather than due to the interference effect of waves scattered by the cells.

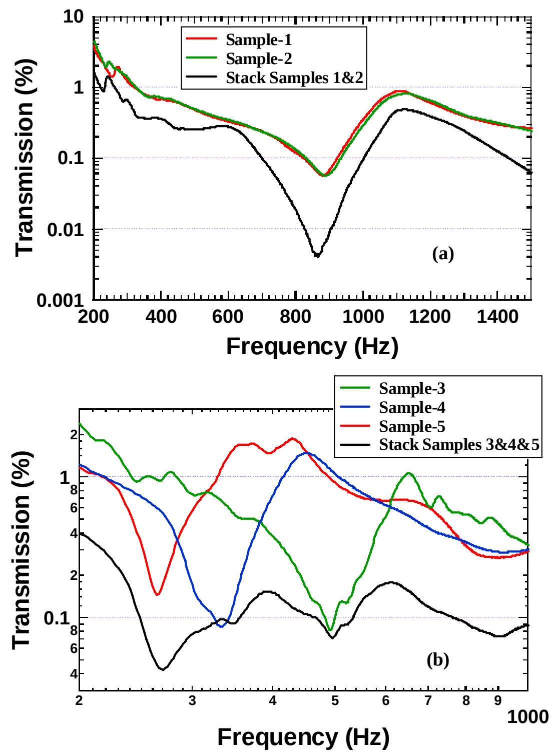


Figure 2. The transmission spectra of LRSM panels.

There is no apparent coupling between adjacent cells one stacked on top of the other, even though they are in contact and there are no rigid grids in between. In other words, the resonators do not couple longitudinally, i. e., in the direction along the wave propagation, due to the strong localization effect.

3.2 Single Cell Spectra

To further verify the local nature of the resonators, we measured the transmission spectra through different area sizes of Sample-1. Within the aperture of 9.0 cm in diameter there were about 27 regular unit cells, while within the 3.0 cm diameter aperture there were 4 cells. The 1.5 cm diameter aperture limited the transmission through a single cell. Figure 3(a) shows the transmission spectra using different apertures. The solid curves are the transmission amplitudes, and the dashed curves are the phase spectra. The transmission dips occurred at the same frequency, but the overall amplitude of the single cell was much reduced due to the reduced apertures. The depths of the dips are comparable among the three spectra, and the phase spectra are about the same. In particular, the characteristic drop in the phase by about 90 degrees is present in all the spectra taken at different aperture sizes. Due to the damping of the rubber the phase shift was smaller than the theoretical prediction which did not include the loss modulus [8]. It is clear that the local resonance feature is well preserved even when only a single cell is excited. Another sample, Sample-6, consists of large unit cells with two 10 mm diameter steel balls in each unit cell. The positions of the balls are at the centers of two diagonal regular cells, as shown in the insert of Fig. 3(b). There are two dips in both the large area (9.0 cm aperture) and single cell (3.0 cm aperture) transmission spectra of the two-weight sample, indicate again that the same local resonance feature is preserved even for a single cell. The resonance frequencies of the single cell deviate slightly (~ 7%) from that of the large area, due to the unavoidable variation in positioning the balls in the cells when the sample was being fabricated. As a result, there are small variations in the resonance frequencies from cell to cell.

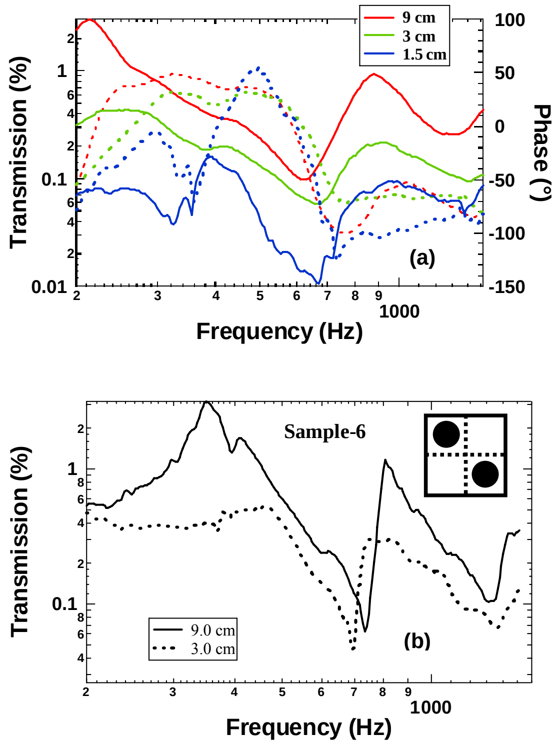


Figure 3. (a) The transmission spectra, including both the amplitude and phase, of LRSM samples through large area and through a single unit cell. The solid curves are the transmission amplitude and the dashed curves are the phase spectra. (b) The transmission amplitudes of LRSM Sample-6 through large area (solid curve) and through a single unit cell (dashed curve).

3.3 Coupling among the Vibrators

The two-weight sample offers some additional interesting features. As there are two weights in a unit cell, there are multiple vibration modes due to the lateral coupling between the weights. One dip is still at about the same frequency as the single weight unit cell, as if the weights were vibrating the same way as before. The other dip is at about 1/2 of the high frequency. The amplitudes (indicated by the depth of the dip) of both vibration modes are reduced, as the oscillation strength is now shared by the two modes. So, as expected, if two weights are linked by the restoring medium (rubber) and are facing the incoming acoustic waves side by side, their vibrations will be coupled. If the cells are further enlarged and contain many weights an elastic band gap could be formed [15].

The stacked samples in section 3.2 represent another coupling geometry. Two resonators from the two stacked samples are in contact with one another. But the incoming wave reaches the front resonator before it reaches the second one at the back. Although their vibrations are expected to be coupled, the strength of the incoming wave is greatly reduced by the front resonator before it reaches the second. Given that the intrinsic sound intensity attenuation per layer of unit cells is over 20 dB at resonance (Fig. 2(a)) and the layer is 2.0 cm thick, the equivalent amplitude attenuation length is less than 0.9 cm (≈ 2.0/ln(10)), as compared to the air borne wavelength of 38 cm at 890 Hz.. The acoustic waves are therefore strongly localized in the longitudinal direction, and the two resonators vibrate as if they were isolated. This is still not fully understood, and requires further theoretical and experimental investigations.

3.4 Mass Dependence

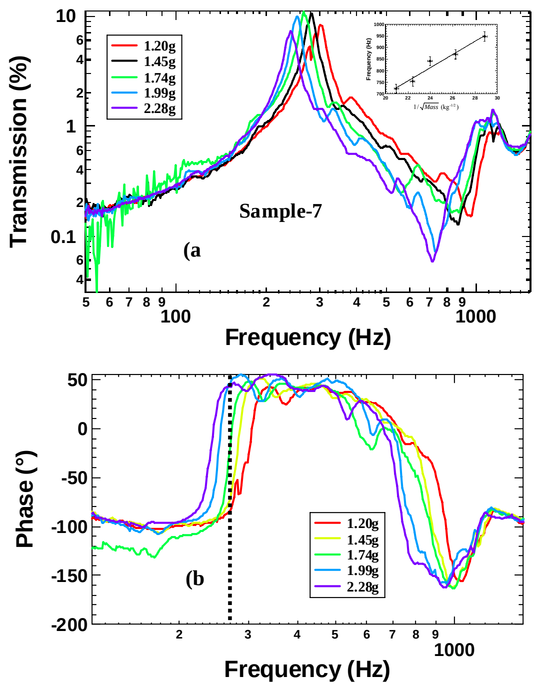


Figure 4. The transmission amplitude (a) and phase (b) of sample-7 with different weight mass.

Next we study the mass dependence of the resonators. It was done with Sample-7 consisting of cylindrical steel weights placed vertically to the panel plane in regular cells. One end surface of the cylinder (7 mm in diameter and 5 mm long) is exposed while the rest is buried in the hard rubber. The effective force constant of the resonator is determined by the geometric dimensions of the weight and the cell, namely the surface area of the cylinder in contact with the rubber and the size of the grid. Additional weights were glued onto the surfaces without changing the lateral geometric dimensions, and the effective force constant remained unchanged. Figures 4(a) and 4(b) show the transmission amplitude and phase for different weights. The transmission dip shifts to lower frequency, as expected. The phase rises by about 150 degrees near the first transmission peak at low frequency around 300 Hz. It then drops by about 150 degrees at the dip frequency near 800 Hz, and finally rises again at the second transmission peak. As the loss modulus is smaller in the hard rubber, the phase shift is larger than in the soft rubber samples and closer to the theoretical prediction. The low frequency peak is due to the vibration of the rubber, because it is present even without the weights. The dip and the higher frequency peak with the corresponding phase shifts are typical features of LRSM's [6 - 8]. In a simple mass-and-spring resonator model, the natural frequency is given by $f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$, where K is the force constant and M is the mass. In the insert of Fig. 4(a) the dip frequency is plotted as a function of $\sqrt{1/M}$. It is seen that the resonant frequency of the local resonators follows the simple mass-and-spring relation. From the slope of the straight line fitting we obtained the effective force constant of the resonator $K = 3.55 \times 10^4$ N/m.

3.5 Zone Plates

Finally, we discuss the possibility of focusing acoustic waves based on the concept of zoneplate. Consider a flat circular panel, and a particular point on its central axis. If all the waves passing through the panel reach the point in phase, then the wave intensity at this point will be greatly enhanced. Taking the radius as one wavelength (λ) of the incoming wave and the distance from the point to the panel center as 1.5λ, the maximum phase difference due to propagation in free space from the center and the edge of the panel is then about 0.3λ (= ($\sqrt{1.5^2+1}-1.5$), or less than 110°. Such phase shift is obtainable near the first transmission peak, as shown in Fig. 4(b). At f = 270 Hz as indicated by the dashed vertical line, the waves through the cells with 1.2 g weight have a phase shift about -150 degrees as compared to the one through the cells with 2.28 g weight, while the transmission amplitudes are comparable. For weights between 1.2 g and 2.28 g the phase shift can be tuned to any value in between. This suggests a way to construct a zoneplate where individual zones having the same weights would give rise to designated phase shift for a wave of given frequency.

4. SUMMARY

LRSMs can exhibit stop frequency bands within which acoustic waves are greatly attenuated. We report a particular type of LRSMs in which the stop band is determined only by the resonant frequency of the local resonators, and independent of the spatial arrangement of the resonators. It is due to the lateral confinement determined by the unit cell structure, and the longitudinal strong localization of the scattered waves by the resonators. The lateral coupling of resonators, when properly designed, generates multi-mode resonances, leading to multiple stop bands. The mass dependence of the resonance frequency follows the simple mass-and-spring relationship. The transmission peak with the associated large phase shift provides a new approach for zoneplates to focus low frequency sound waves, in much the same way as optical zoneplates focusing optical waves. Simple estimations show that effective focusing could be achieved for a plate of the diameter of one wavelength in air and phase shift difference of less than 110 degrees across different zones.

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