

MEASUREMENTS OF VISCOELASTIC PROPERTIES OF CONCRETE STRUCTURE USING BEAM TRANSFER FUNCTION METHOD

Junhong Park¹, Seung Joon Kim¹, Young Jeong², and Jin Yong Jeon²

¹School of Mechanical Engineering, Hanyang University, 17 Haengdang-dong, Seongdong-gu, Seoul, 133-791, Korea
²Department of Architectural Engineering, Hanyang University, 17 Haengdang-dong, Seongdong-gu, Seoul, 133-791, Korea Email: parkj@hanyang.ac.kr

Abstract

Flexural wave propagation characteristics influence the impact noise generation of concrete structures found in apartment floors. The flexural vibration of the concrete structure is affected from its dynamic properties. The purpose of this study is to propose an experimental method of measuring the dynamic characteristics of concrete structures. Using the beam transfer function method, the flexural wave speeds, bending stiffness and their loss factors are measured from which the vibration dissipation capabilities are investigated. Various concrete beam structures were built for measurement. The parameters that have impact on the experimental results were identified. The dynamic stiffness of a concrete structure was determined by its components, and remains constant under the frequency range of interest. The structural loss factor which was similar to each other for various concrete beams was much smaller compared to those of typical polymeric materials. Since the damping in the concrete structure is small, the damping treatments should be a most effective method to reduce vibration and noise generation.

1. INTRODUCTION

The impact noise generated from upstairs is causing many disputes among residents of apartment buildings. To make apartments a safe and quiet residential space, the reduction of noise transferred from neighboring residents through floors is required. The floor impact noise is generated from vibration of concrete slabs and floor coverings in reinforced concrete structures. The concrete slab usually consisted of cement paste, steel bar, coarse and fine aggregates. Its mechanical properties have effects on the structural and acoustical properties. To understand and model the floor impact noise generation mechanisms, the mechanical properties should be measured in the audio frequency ranges.

Various methods for non-destructive evaluation of concrete structures are summarized in

Malhotra and Carino[1]. The resonant frequency method can be used to obtain the dynamic stiffness for flexural vibration[2]. In this method, the relationship between the dynamic stiffness to the resonance frequency of the standardized specimen is utilized. Lin and Sansalone[3,4] calculated this relationship for specimens of thick circular and square bars. The cross-sectional modes lowered the natural frequencies of bars compared to those predicted without considering those modes. Correction factor was suggested for correct evaluation of the elastic modulus. To perform these tests, the specimen of similar size and shape should be used. The method is not applicable when conducting measurements for specimens having different sizes or shapes other than rectangular prisms or circular shapes, or plate structures. Also, the dynamic stiffness is obtained only at a single frequency.

For measuring dynamic stiffness of plate structures, the time of flight method is a widely accepted approach. The ultrasonic wave propagation though the concrete may be used for the time of flight method, but are limited to early-age cement [5]. The ultrasonic wave is difficult to penetrate deeply into concrete structures, and the measured results can be affected from the coarse aggregate since the wave length is comparable to its size. As an alternative approach, the Impact-Echo method has been widely used to measure the dynamic characteristics of concrete [6]. Pessiki and Carino [7] measured the longitudinal wave speeds from the resonance frequency of the concrete cylinder at early ages. The resonance occurred at frequencies where the reflected P-waves travel in phase regardless of the number of reflections. The wave speed is closely related to the strength of the concrete. The method is simple since the specimen was excited at a point and the response was measured at the location close to the point impact. The method was applied to the plate structures by Pessiki and Johnson [8]. The ratio of the wave velocities of plate and rod depends on the Poisson's ratio. Direct P-wave speed measurement was performed using two receivers [9] using the characteristic that the P-wave speed is fastest among the stress waves. This direct P-wave technique was applied to measure the thickness of the slab [10]. For testing large specimens other than thin slab concrete, the measurement of Rayleigh surface wave velocity was proposed by Wu et al [11] by using cross correlation function. This method is limited to the large sample since it requires long distance between source-to-receiver.

Although several different methods for using stress wave propagation have been proposed to measure dynamic stiffness it dose not provide information about the damping characteristics. These studies are limited to measuring elastic properties of concrete. However, the noise generation from a finite structure is affected significantly from the damping properties. The damping properties can show significant frequency dependent variation especially for those of the rubber-modified or lightweight concrete. To investigate the damping properties, the damping ratio are most often measured [12, 13]. More detailed measurements of the damping characteristics are to measure the structural loss factor. The measurements of the grequency dependent variation of the viscoelastic properties requires measurements of the wave propagation characteristics, and was limited to cement paste at very early stages [14].

In this study the propagation characteristics of transverse structural waves on the concrete composite structures are measured from the structural vibration of the sample. For controlled vibration of the concrete beam, it was excited by an impact hammer under free-free boundary conditions. The transfer function between vibrations of the beam at different locations was measured. From the measured transfer function, the wave propagation characteristics and dynamic stiffness were obtained. The frequency dependent variation of the loss factors was measured. Since the natural frequencies for flexural vibration of the concrete structures are much less than those of the longitudinal vibration, the sensitivity of the transfer function method was small enough to obtain dynamic properties accurately at audio frequency.

2. BEAM TRANSFER FUNCTION METHOD

When a sample of bar is excited in the direction normal to its span, the bending and shear deformation dominates the flexural response. This flexural vibration is most significant contributor to the noise generated from concrete slabs. The flexural wave propagation characteristics determine the sound radiation efficiency and the vibration attenuation. For most building floors the fundamental frequency ranges from 10 to several hundred hertz depending on the structural and geometrical properties of the floor. Consequently the flexural vibration and modal response of the structure has significant impacts at audio frequency. To understand the sound radiation characteristics the measurements of flexural stiffness and vibration dissipation characteristics of these complex structures are required at these frequency ranges. The beam transfer function method is a tool to measure the flexural stiffness and vibration dissipation characteristics of the structure. To model the dissipation of vibration energy within a structure, complex stiffness is used for the flexural dynamic properties:

$$\hat{D}(\omega) = D(\omega)[1 + i\eta_D(\omega)] \tag{1}$$

where *D* is the bending stiffness and η_D is the loss factor. The beam transfer function method was proposed to measure these properties of arbitrary beam structures [15]. When the effects of shear deformation and rotary inertia are negligible compared to those of bending deformation, the bending stiffness is measured using the beam transfer function method based on the classical beam theory:

$$D\frac{\partial^4 w}{\partial x^4} + M_b \frac{\partial^2 w}{\partial t^2} = 0.$$
 (2)

where M_b is the bending stiffness and η_D is the loss factor Assuming harmonic motion, i.e., $w(x,t) = \text{Re}\{\hat{w}(x)e^{i\omega t}\}$, the separation of variables to solve equation (2) is performed, and the complex stiffness defined in equation (1) applies to the boundary value problem. The satisfying beam function for the boundary value problem is:

$$\hat{w}(x) = \hat{A}_1 \sin \hat{k}_b x + \hat{A}_2 \cos \hat{k}_b x + \hat{A}_3 e^{\hat{k}_b (x-L)} + \hat{A}_4 e^{-\hat{k}_b x}, \qquad (3)$$

where \hat{k}_b is the wavenumber related to the circular frequency through $\hat{k}_b = (\omega^2 M_b / \hat{D})^{1/4}$. When the free-free beam was excited at its free end, the four boundary conditions are:

$$\frac{\partial^2 \hat{w}(0)}{\partial^2 x} = 0, \ \frac{\partial^3 \hat{w}(0)}{\partial^3 x} = 0, \ \frac{\partial^2 \hat{w}(L)}{\partial^2 x} = 0, \ \hat{D} \frac{\partial^3 \hat{w}(L)}{\partial^3 x} = F.$$
(4a-d)

where F is the force applied at x=L. The following matrix system of equations is obtained from equations (3) and (4) as:

$$\begin{bmatrix} 0 & -1 & e^{-\hat{k}_{b}L} & 1\\ -1 & 0 & e^{-\hat{k}_{b}L} & -1\\ -\sin\hat{k}_{b}l & -\cos\hat{k}_{b}l & 1 & e^{-\hat{k}_{b}L}\\ -\cos\hat{k}_{b}l & \sin\hat{k}_{b}l & 1 & -e^{-\hat{k}_{b}L} \end{bmatrix} \begin{bmatrix} \hat{A}_{1}\\ \hat{A}_{2}\\ \hat{A}_{3}\\ \hat{A}_{4} \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0\\ \frac{F}{k_{b}^{3}\hat{D}} \end{bmatrix}$$
(5)

The above matrix system of equations is solved using symbolic calculations to obtain the predicted transfer function between the beam displacements. Predicted values are compared to the measured values as:

$$\Lambda e^{i\phi} = \frac{\hat{w}(x_1)}{\hat{w}(0)} = \frac{\hat{A}_1 \sin \hat{k}_b x_1 + \hat{A}_2 \cos \hat{k}_b x_1 + \hat{A}_3 e^{\hat{k}_b (x_1 - L)} + \hat{A}_4 e^{-\hat{k}_b x_1}}{1 + \hat{A}_3 e^{-\hat{k}_b L} + \hat{A}_4}.$$
 (6)

where Λ is the amplitude and ϕ is the phases of the measured transfer functions between the displacements. Then, the Newton-Rapson method is applied to solve equation (6) with respect to the complex wavenumber, $\hat{k}_b = k_{br} - ik_{bi}$. After separating the real and imaginary parts, the iterations to solve the above equation are conducted as:

$$\begin{bmatrix} k_{br} \\ k_{bi} \end{bmatrix}_{j+1} = \begin{bmatrix} k_{br} \\ k_{bi} \end{bmatrix}_{j} - \begin{bmatrix} \operatorname{Re} \left\{ \frac{\partial \hat{w}(x_{1})}{\partial k_{br}}, \frac{\partial \hat{w}(x_{1})}{\partial k_{bi}} \right\} \\ \operatorname{Im} \left\{ \frac{\partial \hat{w}(x_{1})}{\partial k_{br}}, \frac{\partial \hat{w}(x_{1})}{\partial k_{bi}} \right\} \end{bmatrix}^{-1} \begin{bmatrix} \operatorname{Re} \left\{ \hat{w}(x_{1}) - \hat{w}(L) A e^{i\phi} \right\} \\ \operatorname{Im} \left\{ \hat{w}(x_{1}) - \hat{w}(L) A e^{i\phi} \right\} \end{bmatrix}$$
(7)

where the subscripts *j* and *j*+1 denote the current and next iterations, respectively. Symbolic computations to solve equation (5) and to calculate the derivatives with respect to wavenumber in equation (7) were performed using commercially available software MATLAB[®]. Using the complex wavenumber obtained through the Newton-Rapson method, the bending stiffness of the beam structure is consequently obtained, Figure 1,

$$\hat{D} = \frac{\omega^2 M_b}{\hat{k}_b^4}.$$
(8)

3. RESULTS AND DISCUSSION

For comparison of dynamic properties of concrete beams, different beams made using different types of cement and coarse and fine aggregates were tested. The mechanical and geometrical properties are given in Table 1. To represent real bare slabs, the steel bar was inserted to Beam 6. Figure 2 shows the measured transfer functions between the displacements during testing of Beam 1-4. Miniature piezoelectric accelerometers (Endevco model 2250-A) were used to measure the vibration response of the beam. Due to its small length the fundamental frequencies are more than 500 Hz, which restricted the frequency ranges that the dynamic stiffness can be obtained. The obtained dynamic stiffness are shown in Figure 3. For the lightweight concrete (Beam 1), the dynamic stiffness was smaller compared to other beams but the loss factor was largest. The loss factor showed continuous increase with the increasing frequency.

To increase the frequency ranges of the dynamic properties the beams of large length was constructed (Beam 5 and 6). Figure 4 shows the measured transfer function. The first resonance was less than 100 Hz. The obtained dynamic stiffness is shown in Figure 5. The bending stiffness was almost constant with frequency. As the frequency increases over 200 Hz, the measured stiffness decreases with increasing frequency. This was resulted due to the shear deformation of the beam that was not taken into account in the classical beam theory. To take into account effects of shear deformation, the Timoshenko beam theory is required [15]. The loss factors ranged from 0.01 to 0.02 depending on the frequency and types of materials used in construction.

6. CONCLUSIONS

The dynamic characteristics of concrete structures were measured at audio frequency. The structural wave propagation characteristics were used in the measurements. To obtain the dynamic properties in the frequency ranges less than 200 Hz, the specimens of small thickness or large length compared to those used for standard strength tests are required. The damping in the concrete slabs were very small (loss factor close to 0.01), and resulted in significant resonant response. This suggests that damping treatments are effective methods to reduce vibration response and sound radiation. The measured properties are useful for sound radiation analysis of concrete structures using numerical methods such as FEM or Rayleigh-Ritz methods.

Beam No.	Cross-section	$M_b(kg/m)$	Dimension of cross-section (m)	Length (m)	
	type				
1	Rectangular	16.6	0.1 (width) x 0.1 (thickness)	0.4	
2	Rectangular	18.2	0.1 (width) x 0.1 (thickness)	0.4	
3	Circular	18	0.1 (diameter)	0.4	
4	Circular	18.4	0.1 (diameter)	0.4	
5	Rectangular	22.5	0.1 (width) x 0.1 (thickness)	2	
6	Rectangular	20.3	0.1 (width) x 0.1 (thickness)	2	
7	Rectangular	3.62	0.04 (width) x 0.03 (thickness)	0.32	

Table 1.	The	geometric and	mechanical	properties	of beams	tested
		0				



Figure 1. Beam transfer function method for measuring dynamic properties of complex structures.



Figure 2. Measured transfer functions (magnitude and phase) for concrete beams of free-free boundary conditions.



Figure 3. The bending stiffness of the concrete beam measured from the transfer function methods in the classical beam.



Figure 4. Measured transfer functions for long (2 m) beams.



Figure 5. The bending stiffness of the concrete beam measured for the long beams.

REFERENCES

- [1] V.M. Malhotra and N.J. Carino, *Handbook on Nondestructive Testing of Concrete* (CRC Press, Boca Raton, 1991).
- [2] ASTM C 215-02, "Standard test method for fundamental transverse, longitudinal, and torsional frequencies of concrete specimens," ASTM International, West Conshohocken, Pa., 2003, 7 pp.
- [3] Y. Lin and M. Sansalone, "Transient responses of thick circular and square bars subjected to transverse elastic impact," *J. Acous. Soc. Am.* **91**, 885-893 (1992).
- [4] Y. Lin and M. Sansalone, "Transient responses of thick rectangular bars subjected to transverse elastic impact," *J. Acous. Soc. Am.* **91**, 2674-2685 (1992).
- [5] Z. Sun, T. Voigt, and S. P. Shah, "Comparison of ultrasonic wave reflection method and maturity method in evaluating early-age compressive strength of mortar," *Cement. Concrete. Comp.* **28**, 307-316 (2006).
- [6] ASTM C 597-02, "Standard test method for pulse velocity through concrete," ASTM International, West Conshohocken, Pa., 2003, 4 pp.
- [7] S.P. Pessiki and N.J. Carino, "Setting time and strength of concrete using the impact-echo method," *ACI Mater. J.* **85**, 389-399 (1988).
- [8] S. Pessiki and M.R. Johnson, "Nondestructive Evaluation of Early-Age Concrete Strength in Plate Structures by the Impact-Echo Method," *ACI Mater. J.* **93**, 260-271 (1996).
- [9] M. Sansalone, J.-M. Lin, and W.B. Streett, "A procedure for determining P-wave speed in concrete for use in impact-echo testing using a P-wave speed measurement technique," *ACI Mater. J.* 94, 531-539 (1997).
- [10] ASTM C 1383-04, "Standard test method for measuring the P-wave speed and the thickness of concrete plates using the impact-echo method," ASTM International, West Conshohocken, Pa., 2004, 11 pp.
- [11] T.-T. Wu, J.-S. Fang, G.-Y. Liu, and M.-K. Kuo, "Determination of elastic constants of a concrete specimen using transient elastic waves," J. Acous. Soc. Am. 98, 2142-2148 (1995).
- [12] J. C. Wang and X. Zeng, "Influence of temperature and pressure on the dynamic properties of rubber-modified asphalt concrete," *J. Mater. Civil. Eng.* **18**, 125-131 (2006).
- [13] J.O.Carneiro, F.J.Q. deMoelo, S.Jalali, V. Teixeira, and M. Tomás, "The use of pseudo-dynamic method in the evaluation of damping characteristics in reinforced concrete beams having variable bending stiffness," *Mech. Res. Commun.* 33, 601-613 (2006).
- [14] Z. Sun, T. Voigt, and S. P. Shah, "Rheometric and ultrasonic investigations of viscoelastic properties of fresh Portland cement pastes," *Cement. Concrete. Res.* **36**, 278-287 (2006).
- [15] J. Park, "Transfer function methods to measure dynamic mechanical properties of complex structures," J. Sound Vib. 288, 57-79 (2005).