ICSV14
Cairns • Australia
9-12 July, 2007

14th International Congress on Sound & Vibration

EFFECT OF AXIAL FORCE, UNBALANCE AND COUPLING MISALIGNMENT ON VIBRATION OF A ROTOR GAS TURBINE

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Abstract

Unbalanced rotors and shaft misalignment are the two main sources of vibration in rotating machinery. These undesired vibrations may destroy critical parts of the machine, such as bearings, gears and couplings. There are lots of investigations about effect of unbalance and coupling misalignment upon the critical speeds and vibration amplitudes in rotary machinery. In this paper, effect of various parameters such as rotational velocity, geometry, variation of temperature and Poisson ratio on the natural frequency of rotating shafts were investigated. The equation of motion was derived from Euler-Bernoulli beam model and strain-stress relations. Results show that the effect of spin softening is more important than axial force. For investigation of unbalance and misalignment response in a multirotor system, FEM is used. The spline coupling was modelled in the form of bending spring. The equation of motion is solved by Newmark scheme. Results show that unbalance and misalignment are important in domain of first and second natural frequency, respectively.

1. INTRODUCTION

Accurate prediction of critical speeds in rotating machinery is of great importance to designers and many attempts have been made to calculate it exactly. External loading can change the lateral natural frequency of a rotating shaft. The effect of externally applied axial force and torque on the lateral vibration of the shafts has been studied by several researchers. For example, Bokian [1] presented changes in the lateral natural frequency of Euler–Bernoulli beams under axial load with various boundary conditions. Inertial forces can also induce axial stresses in shafts and beams. Rotation of a beam about an axis perpendicular to the beam axis has also been studied by researchers, in which centrifugal force directly produces axial stress in the beam. Banerjee [2] used dynamic stiffness matrix for Euler–Bernoulli beam with axial force to analyse the vibration of uniform and tapered rotating beams.
In rotating machinery, unbalance and misalignment of rotors are important reasons that lead to vibration problems. There are lots of investigations about effect of unbalance and coupling misalignment upon the critical speeds and vibration amplitude in rotary machinery. Xu and

Marangoni [3] developed a theoretical model of a complete motor-flexible coupling-rotor system to describe mechanical vibration resulting from misalignment and unbalance. Le and Yu [4] investigated the non-linear coupled lateral torsional vibration model of rotor-bearing-gear coupling system based on the engagement condition of gear coupling.
Dewell and Mitchell [5] considered the lateral vibration frequencies for a misaligned gear coupling. Blooch [6] identified the forces and moments developed by a misaligned gear coupling. Gibbons [7] showed that these forces and moments are developed by different types of misaligned couplings. In their dynamic model the gear coupling was simulated by a very thin beam element. Alfares and et.al [8] investigated the effect of various coupling geometrical parameters on performance and operate of gear coupling.
In this paper, effects of various parameters such as rotational velocity, geometry, variation of temperature and Poisson ratio on the natural frequency of the rotating shafts are investigated. Also using FEM analysis, effects of unbalance and misalignment on the vibration of a milti-rotor system are obtained.

2. NATURAL FREQUENCY OF LATERAL VIBRATION OF A BEAM IN THE PRESENCE OF AXIAL FORCE

In this section, effects of rotation and variation of temperature on the natural frequency of the cylindrical shaft with inner radius a, outer radius b and length L wich rotates with a constant rotational speed Ω are investigated.
The equation of lateral vibration of Euler-Bernoulli beam in the presence of axial force P can be written as:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} - P\frac{\partial^2 w(x,t)}{\partial x^2} + \rho A\ddot{w}(x,t) = 0 \tag{1}$$

where A and I are area and second moment of area of shaft cross-section, respectively.
For a shaft with two bearings at ends, the simply supported boundary condition can be used. The natural frequency given as the first eigenvalue of Eq. (1) is as follows:

$$\omega = \frac{\pi}{\sqrt{\rho A}L}\left[P + \frac{EI\pi^2}{L^2}\right]^{1/2} \tag{2}$$

2.1 Effect of rotation on the natural frequency of a rotating shaft

For calculating the axial force that is produced as a result of shaft rotation, 3-D linear elasticity relations are used. An axisymmetric shaft with a length much larger than its diameter is considered in this study. A plane strain problem is assumed because the bearings suppress axial movement of the shaft. The cylindrical co-ordinate system, in which the Z-axis is coincident with the shaft axis of rotation, is used. Displacement in the directions of r,θ and z are shown by u, v and w, respectively. With applying D'Alembert principle, the equilibrium equation for the shaft in radial direction is as follows:

$$\frac{\partial}{\partial r}\left(r\sigma_r\right) - \sigma_\theta = -\rho\, r^2\Omega^2 \tag{3}$$

where σ_r and σ_θ are the radial and circumferential stresses, ρ is the shaft density and Ω is the constant shaft speed. With applying the stress-strain relations and all assumptions the axial stress in the shaft can be found as follows:

$$\sigma_z = \frac{\upsilon}{4(1-\upsilon)}\left[(3-2\upsilon)(a^2+b^2)-2r^2\right]\rho\Omega^2 \tag{4}$$

The net axial force in the cross-section of shaft, P, is the integral of axial stress on the section of the shaft:

$$P = \int_a^b 2\pi r \sigma_z dr \tag{5}$$

By substituting σ_z from Eq. (4) into Eq. (5), one has

$$P = \upsilon \rho I_p \Omega^2 \tag{6}$$

where I_P is the polar moment of inertia.

Eq. (6) shows that the axial force is proportional to the Poisson ratio and also is proportional to the square of the shaft speed.

Substituting axial force that is produced as a result of shaft rotation, from Eq. (6) into Eq. (2) gives

$$\omega^2 = \frac{\pi^2 E I_d}{\rho A L^2}\left(\frac{\pi^2}{L^2} + \frac{\upsilon \rho I_p \Omega^2}{E I_d}\right) \tag{7}$$

Assuming $I_P / I_d = 2$ which is valid for circular cross-sections, Eq. (7) becomes

$$\frac{\omega^2 - \omega_0^2}{\omega_0^2} = \frac{2\upsilon\rho}{\pi^2 E} L^2 \Omega^2 \tag{8}$$

where ω_0 is the first natural frequency of the non-rotating shaft.

In Figure 1, percent of relative change in the natural frequency caused by shaft rotation is plotted versus shaft speed for a shaft with 1 m length and pin-pin boundary condition. The shaft material is assumed to be steel with: $E = 2\times10^{11} Pa$, $\upsilon = 0.25$ and $\rho = 7800 kg/m^3$.

It can be calculated that increasing the shaft rotation speed up to 20000 r.p.m. will increase $(\omega-\omega_0)/\omega_0 \times 100$ up to nearly 0.45% for a 1-m shaft.

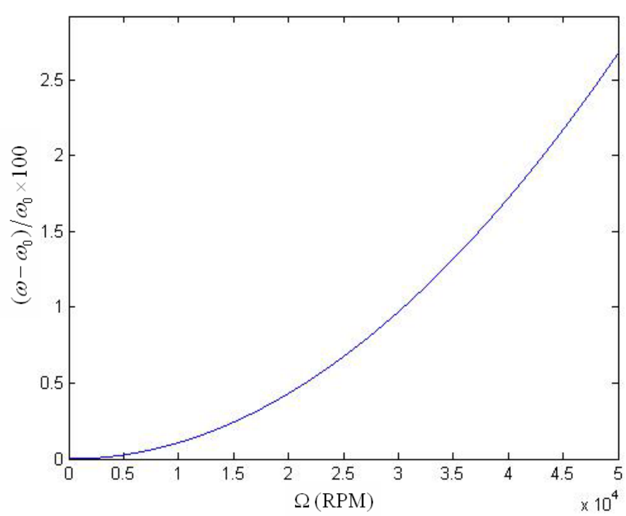

Figure 1. Relative change of natural frequency versus shaft speed with pin-pin boundary condition.

2.2 Effect of variation of temperature on the natural frequency of a beam

Gas turbines are machineries that sustain high variation of temperature. For calculating the axial force that is produced as a result of variation of temperature, assumed the bearings suppress axial movement of the shaft.
Axial stress that induced by variation of temperature amount of ΔT , can be found as follows:

$$\sigma = -E\alpha\Delta T \tag{9}$$

where E, α are Young's modulus of elasticity and coefficient of thermal expansion, respectively.
The axial force in the cross-section of shaft, P , can be found as follows:

$$P = \sigma A = -EA\alpha\Delta T \tag{10}$$

where A is area of cross-section.
Eq. (10) shows that the axial force induced by variation of temperature is proportional to the variation of temperature.
Substituting axial force that is produced as a result of variation of temperature, from Eq. (10) into Eq. (2) gives

$$\omega = \frac{\pi}{\sqrt{\rho A L}}\left[-EA\alpha\Delta T + \frac{EI\pi^2}{L^2}\right]^{1/2}, \ \omega^* = \sqrt{\frac{\rho A}{EI}}L^2\omega = \pi\left[\frac{-\alpha\Delta T}{\lambda^2} + \pi^2\right]^{1/2} \tag{11}$$

where $\lambda = \sqrt{\frac{I}{AL^2}}$ and $\omega^* = \sqrt{\frac{\rho A}{EI}}L^2\omega$ are dimensionless slenderness ratio and natural frequency.
In Figure 2, dimensionless natural frequency, ω^* , versus variation of temperature is plotted according Eq. (11) for a steel shaft, $(\alpha = 14\times10^{-6}/°c)$. Various graphs are plotted for different slender ratio. The results that can be found are as follows:

- Temperature increasing will decrease the natural frequency and vice versa.
- Increasing in temperature has more effect than decreasing in temperature on the natural frequency of the shaft. For example, for a uniform cylindrical shaft with $\lambda = 0.02$ per 100°C increase in temperature, natural frequency about 19% decreases whereas for same decrease in temperature, natural frequency will be increases about 16.8%.
- Effect of variation of temperature on the natural frequency of thinner beams is more than thicker beams.

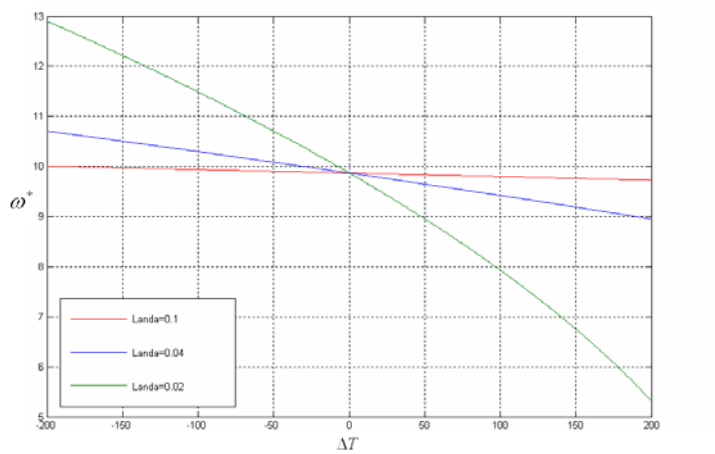

Figure 2. Effect of variation of temperature on natural frequency of a beam, for different slender ratio.

3. EFFECT OF SPIN SOFTENING ON THE NATURAL FREQUENCY OF A SHAFT

The vibration of a spinning body will cause relative circumferential motions, which will change the direction of the centrifugal load which, in turn, will tend to destabilize the structure. As a small deflection analysis cannot directly account for changes in geometry, the effect can be accounted for by an adjustment of the stiffness matrix, called spin softening.
Consider a simple spring-mass system, with the spring oriented radially with respect to the axis of rotation, as shown in Figure 3. Equilibrium of the spring and centrifugal forces on the mass using small deflection logic requires:

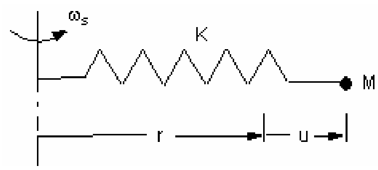

Figure 3.spinning spring-mass system.

Equilibrium of the spring and centrifugal forces on the mass using small deflection logic requires:

$$Ku = \omega_s^2 Mr \tag{12}$$

where u, r and ω_s are radial displacement of the mass from the rest position, radial rest position of the mass with respect to the axis of rotation and angular velocity of rotation, respectively.
To account for large deflection effects Eq. (12) must be expanded to:

$$Ku = \omega_s^2 M(r+u) \tag{13}$$

With extension into three dimensions, the eigenvalue problem can be written in the form:

$$\left| ([K] - \omega_s^2 [M]) - \omega^2 [M] \right| = 0 \tag{14}$$

where ω is the natural frequency of the rotating shaft. Eq. (14) shows that with increase of angular velocity of rotation, the natural frequency of shaft decreases.
It can be calculated that increasing shaft rotation speed up to 20000 r. p. m will decrease $(\omega - \omega_0)/\omega_0 \times 100$ up to nearly 35% for a 1-m shaft. Comparison of Eq. (8) and Eq. (14) shows that the effect0 of spin softening is more important than axial force.

4. EFFECT OF UNBALANCE AND COUPLING MISALIGNMENT ON VIBRATION OF A ROTOR GAS TURBINE

4.1 Equation of motion

The rotor-coupling bearing system is discretized into finite beam elements as shown in Figure 1(a). A typical shaft rotor element is illustrated in Figure 4(b). Each element has two translational and two rotational degrees of freedom for bending mode at each node represented by q1-q8.

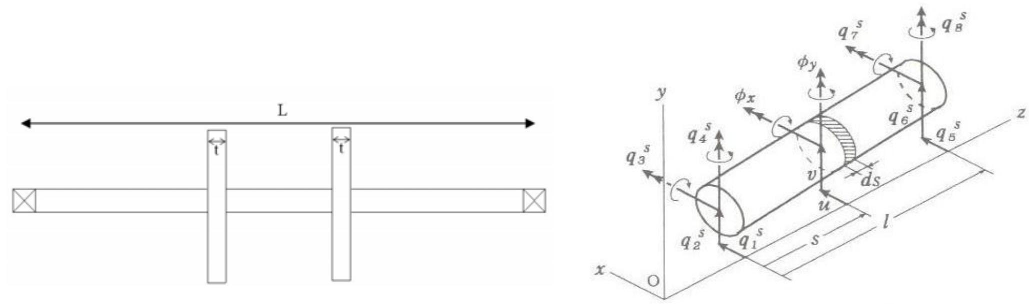

Figure 4. Rotor-coupling-bearing system with typical finite rotor element details.

The equation of motion of the complete rotor system in a fixed co-ordinate system can be written as

$$\{M\}\{\ddot{q}\}+\{C\}\{\dot{q}\}+\{K\}\{q\}=\{F\} \tag{15}$$

where the mass matrix $\{M\}$ includes the rotary and translational mass matrices of the shaft, discs and the spline coupling. The matrix $\{C\}$ includes the gyroscopic moments and the bearing damping. The stiffness matrix $\{K\}$ considers the stiffness of the shaft elements include coupling element and the bearing stiffness. The excitation matrix $\{F\}$ in equation (15) consists of the unbalance and coupling misalignment forces.

4.2 Modeling of unbalance and spline coupling

4.2.1 Unbalance

Vibration caused by mass unbalance is a common problem in rotating machinery. Unbalance occurs if the principal axis of inertia of the rotor is not coincident with its geometric axis. Higher speeds cause much greater centrifugal unbalance forces, and the current trend of rotating equipment toward higher power density clearly leads to higher operational speeds. Centrifugal unbalance forces are as follows:

$$F_x = m_e r\Omega^2 \cos\Omega t \ , \ F_y = m_e r\Omega^2 \sin\Omega t \tag{16}$$

where m_e, r and Ω are unbalance mass, unbalance radius and rotational speed, respectively.

4.2.2 Coupling misalignment

Shaft misalignment is a condition in which the shafts of the driving and the driven machines are not on the same centreline. Misalignment of machinery shafts causes reaction forces and moments to be generated in the coupling, which in turn affect the machines. The reaction forces and moments, which are developed due to angular misalignment, are given in references [6, 7], have been used in the present analysis. The details are presented here briefly as these are introduced in the present work.

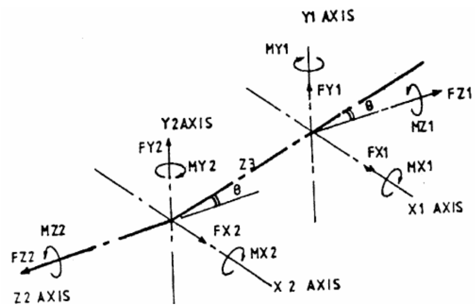

Figure 5. Coupling co-ordinate system of angular misalignment.

Assuming Z1 is the axis of the driving machine, that (+) torque is applied as shown in Figure 4 and the rotation is in the same direction as the applied torque, the reaction forces and moments, which the coupling exerts on the machine's shafts, are as follows:

$$
\begin{aligned}
& MX1 = 0, \quad MY1 = 0, \quad MZ1 = Tq / \cos\theta_3 \\
& MX2 = -K_b\theta_3, \quad MY2 = Tq\sin\theta_3, \quad MZ2 = -Tq \\
& FX1 = (-MY1 - MY2)/Z3, \quad FY1 = (MX1 + MX2)/Z3, \quad FZ1 = K_a\Delta Z + K_a(\Delta Z)^3 / \cos\theta_3 \\
& FX2 = -FX1, \quad FY2 = -FY1, \quad FZ2 = -FZ1,
\end{aligned}
\tag{17}
$$

5. CONCLUSIONS

A typical rotor-coupling-bearing system as shown in Figure 4 considered in the present analysis and the related data are given in Table 1. The analysis carried out by considering unbalance and angular misalignment of the coupling in the rotor system separately, using FEM for flexural vibrations. In addition, effects of spin softening have been applied in calculations. The unbalance mass was located on external edge of the left disc, and also, frequency response diagrams are obtained for this location.

Frequency responses of rotor-coupling-bearing system are shown in Figure 6. It can be seen that in unbalance systems, the first natural frequency is more important than second ones, and then must be attend to the shaft speed don’t lie in the first natural frequency domain. Also in the misaligned systems, against of unbalance systems, the second natural frequency is more important from the first natural frequency, and then must be attend to the shaft speed don’t lie in the second natural frequency domain.

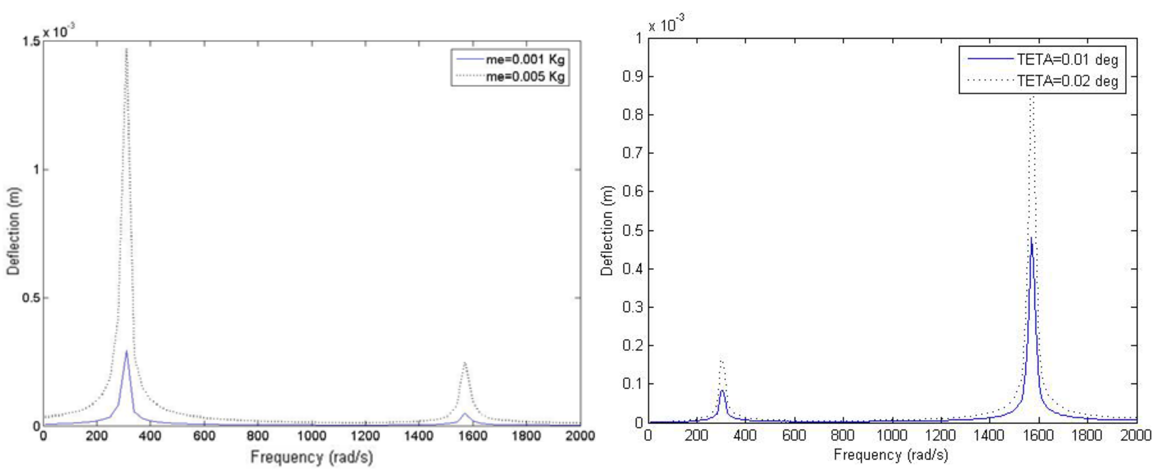

Figure 6. Frequency responses (a) with unbalance (b) with angular misalignment.

Table 1. Rotor-coupling-bearing data.

Torque, *Tq* (N-m)	30
Shaft speed Ω (rad/s)	100
Shaft length, *L* (m)	0.4
Shaft diameter, *D* (m)	0.01
Shaft and discs density and modulus of elasticity, ρ (kg/m^3)	7850
Shaft and discs modulus of elasticity, *E* (Pa)	2×10^{11}
Discs diameter, *d* (m)	0.08
Discs Thickness, *t* (m)	0.01
Bearings stiffness, *k* (N/m)	2.5×10^{5}
Bearings damping, *c* (Ns/m)	100
Coupling length, z_3 (m)	0.01
Bending spring rate per degree per disc pack, K_b (Nm/deg)	30

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