



ADAPTIVE CONTROL OF AN MR MOUNT FOR VIBRATION ATTENUATION

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Abstract

In this paper, an MR mount with flow mode operation is studied for vibration suppression subject to base excitations. Recently, magneto-rheological (MR) fluid has become a popular material for actuator use. There are some good properties associated with MR fluid such as the reversible, controllable, and continuous change of rheological characteristics upon application of magnetic field. However, the dynamic equation of MR mount is highly nonlinear, hence making the controller design an extremely difficult task. This paper aims to develop a semi-active control technique for suppressing vibration of an MR mount subject to its base disturbances. The adaptive control scheme is employed for vibration attenuation. Function approximation technique is used here to represent the unknown system dynamics including the external disturbance in some finite linear combination of the orthogonal basis. The dynamics of MR mount system can thus be proved to be a stable first order filter driven by function approximation errors. Moreover, the adaptive update law can be obtained by using the Lyapunov stability theory. The well-known skyhook control scheme and a controller with constant applied magnetic field are to be compared with the proposed adaptive controller for semi-active vibration control of the MR mount.

INTRODUCTION

For vibration attenuation purpose, many kinds of products have been developed, such as dampers, shock absorbers and suspension system, etc. Severe vibration of a system can cause damage, even leading to safety problem. A lot of passive devices are used in engineering application to absorb or isolate unwanted vibration, but the adaptive ability of passive devices is poor. The semi-active control could provide better performance than the passive control, which has been stated in many fields by many researchers. Compared to the active control, the semi-active control has many advantages, such as low power requirement, higher reliability and simpler structure, etc [1-6].

Recently, smart materials are applied widely and rapidly, such as liquid crystal, piezoelectric ceramics [7], magnetorheological fluids, and electro-rheological fluids [8], etc. MR fluid consists of micron-sized, magnetically polarizable particles dispersed in silicone oil

[9]. MR fluid with viscous damping property can be effectively controlled by the magnetic field (generally controlled by adjusting the current to the electromagnetic coil). Specifically, MR fluid has board operational temperature (-40 to +150 $^{\circ}C$), fast response time (less than milliseconds), low power requirement (2-50 watts), and high yield stress (50–100 *kPa*) [10]. Actuators incorporating these advantages have prominent potentials for vibration suppression, under complex and varied environment.

However, an MR system under operation may contain a time varying disturbance and intrinsic nonlinear properties. Thus, closed loop stability is hard to guarantee by linear controller. In order to improve performance, several control schemes have been discussed in the literatures, such as skyhook, neural network, neuro-fuzzy Control and H_{∞} control, etc. Choi studied the effects of H_{∞} and skyhook control for full vehicle suspensions featuring MR using the method of HILS (hardware-in-the-loop simulation) [11,12]. Kim and Roschke provided a linearization scheme for MR damper behavior using a neural network [13]. Two years later, Schurter and Roschke described a neuro-fuzzy technique to reduce vibration with a MR damper [14]. Yokoyama, Hedrick and Toyama presented a model following sliding mode controller for semi-active suspension systems with MR dampers [15]. Many of these robust control techniques can approximate time varying parameters and attenuate disturbances, yet requiring that all uncertainties be defined in several compact sets.

In this paper, an adaptive sliding controller and function approximation technique are proposed to deal with modeling uncertainty and unknown disturbance [16-18]. It uses a finite linear combination of the orthogonal basis functions to approximate unknown disturbance. Furthermore, not only the convergence of tracking error but also the update law of coefficients can be obtained by applying Lyapunov stability theorem. The paper is organized as follows. Section II gives a brief formulation of MR mount model and its schematic configuration. Section III derives the adaptive sliding control in detail. Moreover, this section uses Lyapunov liked design to obtain the update law of coefficients of the approximation series. Section IV presents the simulation results of the sliding mode controller for semi-active control of an MR mount. In addition, the well-known skyhook control scheme and control with constant current are to be compared with the proposed adaptive controller. Finally in Section V conclusions are briefly made.

PROBLEM FORMULATION

The schematic configuration and its corresponding hydraulic model of a one-dimensional MR mount are shown in Fig. 1. Equation of motion of the system can be derived as [12]:

$$m\ddot{x}(t) + \left\{ b - A\frac{\eta}{h} \right\} (\dot{x}(t) - \dot{y}(t)) + \left\{ k + \frac{A_p^2}{C_1 + C_2} \right\} (x(t) - y(t)) + \frac{C_2 A_p}{C_1 + C_2} \left\{ \frac{2n+1}{n} \frac{2}{Wh^2} \left(\frac{C_2 A_p}{C_1 + C_2} - \frac{A}{2} \right) |\dot{x}(t) - \dot{y}(t)| \right\}^n \frac{2\eta L}{h} = -F_{MR}$$

$$(1)$$

where

$$F_{MR} = \left(\frac{C_2 A_p}{C_1 + C_2} D \frac{2L\tau_{yf}(H)}{h} + A\tau_{ys}(H)\right) \operatorname{sgn}(\dot{x}(t) - \dot{y}(t))$$
(2)



Figure 1. Sketch of an MR mount (left) and its hydraulic model (right)

It is noticed that *m* is the mass, *b* is the damping constant of the rubber, *A* is the flow area, η is the viscosity of the MR fluid, *h* is the gap of the magnetic pole, *k* is the stiffness of the rubber, A_p is the piston area of the upper chamber, C_1 and C_2 are the compliance of the upper and lower chamber, W is the width of the magnetic pole, *L* is the length of the magnetic pole, *n* is the flow behaviour index of Herschel-Bulkey model, x(t) and y(t) represent the displacements at the mass and base, respectively.

We may represent Eqs. (1)-(3) in state space form:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = f(z,t) + Bu \end{cases}$$

$$\tag{4}$$

where $z_1 = x$ and $z_2 = \dot{x}$ are state variables, B and *u* represent the unknown input gain and control input.

$$f(z,t) = \frac{1}{m} \left\{ b - A \frac{\eta}{h} \right\} (\dot{y} - z_2) + \frac{1}{m} \left\{ k + \frac{A_p^2}{C_1 + C_2} \right\} (y - z_1) + \frac{1}{m} \frac{C_2 A_p}{C_1 + C_2} \left\{ \frac{2n+1}{n} \frac{2}{W h^2} \left(\frac{C_2 A_p}{C_1 + C_2} - \frac{A}{2} \right) |\dot{y} - z_2| \right\}^n \frac{2\eta L}{h}$$

$$B = \frac{1}{m} \left(\frac{C_2 A_p}{C_1 + C_2} D \frac{2L}{h} \right)$$
(5)

$$u = \tau_{ys}(H)\operatorname{sgn}(\dot{y} - z_2) \tag{7}$$

Assumption: f(z,t) is an unknown function with unknown variation bound, but it remains continuous and bounded for all admissible z and for all $t \in [t_0, \infty)$.

Remark: Eq. (4) is obtained by assuming that yield stress of the MR fluid in driven flow mode $\tau_{vf}(H)$ is much greater than that in direct shear mode $\tau_{vs}(H)$.

ADAPTIVE SLIDING CONTROLLER DESIGN

In this section, design procedures of adaptive sliding controller for MR mount of Fig. 1 are briefly given. In the beginning, we define sliding surfaces $s = \dot{e} + \lambda e$, where $e = z_1 - z_{1d}$, $\dot{e} = z_2 - z_{2d}$, z_{id} represents the desired value of state, i = 1,2, λ is a parameter to be arbitrarily selected, and the time derivative of *s* can be derived as

$$\dot{s} = (\dot{z}_{2} - \dot{z}_{2d}) + \lambda_{1}\dot{e} = f + Bu - \dot{z}_{2d} + \lambda\dot{e} = (B - \hat{B})u + f + \hat{B}u - \dot{z}_{2d} + \lambda\dot{e}$$
(8)

Eq. (8) can be stabilized by selecting u as

$$u = \frac{1}{\hat{B}} \left[-\hat{f} + \dot{z}_{2d} - \lambda \dot{e} - \eta \frac{s}{\phi} \right]$$
(9)

where \hat{B} and \hat{f} are the estimate values of unknown B and f, respectively. The positive value η is to be determined. ϕ is the width of the sliding boundary layer. Substituting Eq. (9) into Eq. (8), we can obtain

$$\dot{s} = (B - \hat{B})u + (f - \hat{f}) - \eta s / \phi = \widetilde{B}u + \widetilde{f} - \eta s / \phi$$
(10)

where $\tilde{B} = B - \hat{B}$ and $\tilde{f} = f - \hat{f}$. Since f is a time varying uncertainty, the function approximation technique can be applied here to transform the uncertainty into a finite linear combination of the orthogonal basis. Specifically, f and \hat{f} can be represented as

$$f = w^T \varphi + \varepsilon \tag{11}$$

$$\hat{f} = \hat{w}^T \varphi \tag{12}$$

where $w, \hat{w} \in \Re^n$ are weighting vector, $\varphi \in \Re^n$ is the vector of basis function, the positive constant *n* is the number of basis functions used in the approximation, ε is the truncation error. Substituting Eqs. (11) and (12) into Eq. (10) yields

$$\dot{s} = \widetilde{B}u + \widetilde{w}^{\mathrm{T}}\varphi + \varepsilon - \eta s / \phi \tag{13}$$

where $\tilde{w} = w - \hat{w}$. We may select the Lyapunov function candidate as

$$V = \frac{1}{2}s^2 + \frac{1}{2}\widetilde{w}^{\mathrm{T}}Q\widetilde{w} + \frac{1}{2}\rho\widetilde{B}^2 > 0$$
(14)

where $Q \in \Re^{n \times n}$ is a positive definite matrix and ρ is a positive value. Taking time derivative of Eq. (14), we have

$$\dot{V} = s\dot{s} - \widetilde{w}^{T}Q\dot{\dot{w}} - \rho\widetilde{B}\dot{B}$$

$$= s(\widetilde{B}u + \widetilde{w}^{T}\varphi + \varepsilon - \eta s/\phi) - \widetilde{w}^{T}Q\dot{\dot{w}} - \rho\widetilde{B}\dot{B}$$

$$= (\widetilde{B}us - \rho\widetilde{B}\dot{B}) + (\widetilde{w}^{T}\varphi s - \widetilde{w}^{T}Q\dot{\dot{w}}) + (-\eta s/\phi + \varepsilon)s$$
(15)

We may select the update law as

$$\dot{\hat{w}} = Q^{-1}\varphi s \tag{16}$$

$$\hat{B} = \rho^{-1} us \tag{17}$$

To avoid the singularity problem, in Eq. (17) can be modified as

$$\dot{\hat{B}} = \begin{cases} \rho^{-1}su & \text{if } \hat{B} \ge \underline{B} \\ \rho^{-1}su & \text{if } \hat{B} = \underline{B} \\ 0 & \text{if } \hat{B} = \underline{B} \\ \end{array} \quad su \ge 0 \tag{18}$$

where \underline{B} is a known lower bound of B.

By Eq. (16)-(18), we can derive that

$$\dot{V} \le \left(-\eta \frac{|s|}{\phi} + |\varepsilon|\right) |s| \tag{19}$$

If sufficient basis functions are used such that function approximation error $\varepsilon \approx 0$, then Eq. (19) becomes

$$\dot{V} \le \left(-\eta \mid s \mid /\phi\right) \mid s \mid \tag{20}$$

From Eq. (20) we can easily find that the system is uniformly stable and $s, \tilde{B}, \tilde{w} \in L_{\infty}$. By Eq. (13) we can have $\dot{s} \in L_{\infty}$. To acquire the asymptotical stability, we need to prove that $s \in L_2$. It can be proved by Eq. (21)

$$\int_0^\infty s^2 dt = -\frac{\phi}{\eta} \int_0^\infty \dot{V} dt = -\frac{\phi}{\eta} (V_\infty - V_0) < \infty$$
⁽²¹⁾

Since $s \in L_{\infty} \cap L_2$ and $\dot{s} \in L_{\infty}$, by Barbalat's lemma, we can have asymptotical stability and $\tilde{B}, \tilde{w} \in L_{\infty}$; therefore, all estimations remain bounded.

Remark 1: If approximation error cannot be neglected, but there exists a positive constant $\delta > 0$ such that $|\varepsilon| \le \delta$. To cover the effect of this bounded approximation error, Eq. (9) is modified to be

$$u = \frac{1}{\hat{B}} \left[-\hat{f} + \dot{z}_{2d} - \lambda \dot{e} - \eta \frac{s}{\phi} \right] + u_{robust}$$
(22)

Then, we have

$$\dot{V} \le \left(-\eta \frac{s^2}{\phi}\right) + |s|| \varepsilon |+su_{robust}$$
(23)

By select $u_{robust} = -\operatorname{sgn}(s)\delta$, we may also conclude the asymptotical stability of system.

Remark 2: To avoid parameter drift, the σ modification technique [19] can be used to (16) and (18).

$$\dot{\hat{w}} = Q^{-1}\varphi s - \sigma_w \hat{w}$$

$$(24)$$

$$\int e^{-1}su - \sigma_w \hat{R} \qquad if \quad \hat{R} \ge R$$

$$\dot{\hat{B}} = \begin{cases} \rho^{-1}su - \sigma_{B}B & if \quad B \ge \underline{B} \\ \rho^{-1}su - \sigma_{B}\hat{B} & if \quad \hat{B} = \underline{B} \\ 0 & if \quad \hat{B} = \underline{B} \\ \end{cases} \quad su \ge 0$$
(25)

where σ_w and σ_B are small positive constants.

Remark 3: The MR mount control is a semi-active one; therefore control action should follow the actuating condition [7].

$$u = \begin{cases} u & \text{for } z_2(\dot{y} - z_2) > 0\\ 0 & \text{for } z_2(\dot{y} - z_2) \le 0 \end{cases}$$
(26)

SIMULATION RESULTS

The system parameters used in the simulation are shown in Table 1 and n=41 terms of Fourier orthogonal basis are used for function approximation. The sinusoidal disturbances y(t) with frequency between 1Hz~15Hz and amplitude at 1 mm are employed as the base excitation.

Parameter	Specification	Value	Unit
т	load mass	60	kg
b	rubber damping	610	Ns / m
A	flow area	0.0095	m^2
η	MR fluid viscosity	0.8	Ns/m^2
h	gap of magnetic pole	0.01	т
k	rubber stiffness	133240	N/m
A_p	piston area of upper chamber	0.009	m^2
C_1 , C_2	compliance of upper and lower chamber	$\approx 3 \times 10^{-8}$	m^3 / N
W	magnetic pole width	0.45	т
n	flow behaviour index of Herschel-Bulkey model	0.8	

Table 1. System parameters.

The simulation results of vibration control are shown in Fig. 2-4. Fig. 2 shows the frequency response of the system under various control schemes. We can find that the proposed controller has much better performance for vibration attenuation than other controllers. Fig. 3 shows the time response of the corresponding controller nearby the resonant frequency excitation. The proposed controller indeed outperforms other controllers. Fig. 4 compares the approximate values of unknown function \hat{f} with the true values f and depicts the convergence history of the input gain. Though approximation errors do not converge to zero, they are indeed bounded.



Figure 2. Frequency response for 1Hz~15Hz disturbance



Figure 4. Approximation of the unknown function (left) and the convergence history of the input gain (right)

CONCLUSION

This paper proposed an adaptive sliding control with function approximation technique for MR mount. In section III, we have given a brief proof of the stability and update law by Lyapunov stability theory. Although the model parameter and disturbance bounds are not available, we can still obtain good performances from numerical simulation results in section IV. Therefore, the estimates do not converge to actual values, but all remain bounded. With the proposed controller, MR mount can achieve vibration attenuation in a broadband frequency range.

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