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ACTIVE VIBRATION CLAMPING ABSORBER DESIGN

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Abstract

An active Vibration Clamping Absorber (VCA) technique for vibration suppression in flexible structures is proposed and investigated in this paper. The technique uses a Quadratic-Modal-Positive-Position-Feedback strategy to design a simple second-order nonlinear controller that is capable of suppressing structural vibrations at various resonances. The VCA can effectively transfer vibration energy from the main structure to another sacrificial absorber so that large amplitude vibrations in the main structure can be clamped within tolerable limits. The effectiveness of the VCA design is demonstrated through single-mode and multiple-mode control on a flexible cantilever beam system using one sensor/actuator pair. The theoretical analysis and experimental results reveal that the proposed design can be used for real-time control of vibration in large flexible structures.

1. INTRODUCTION

Flexible beam elements constructed with fabrics, composites, polymers, and light metals are increasingly employed in a variety of large structures in aerospace, robotics, marine, and machinery industries. These lighter structures, however, are physically characterised by low structural damping, low stiffness, and low natural frequencies. Consequently, the structures readily experience high-amplitude resonances under external disturbances, such as forces produced by unbalanced rotating machines, reciprocating machines, or shock impacts.

To solve these problems, various control techniques have been proposed, of which modal control is the most widely reported method[1, 2, 3, 4]. One of the advantages of modal control is that it allows each mode of the structure to be controlled independently of the other modes. Because of this characteristic, standard control problems, such as control system sensitivity, observability, and stability problems can be readily addressed.

As a result of recently rapid advancements in smart structure technology interest in modal control has again been revived. Inman's research[2] shows that if modal compensation is used as a control law and designed to roll-off at higher frequencies, spillover is not a problem. However, most of the control methods used in modal control for flexible structures have focused on linear state-space feedback or linear output feedback control strategies using modal displacement or modal velocity as the feedback signal[5, 6, 7]. These methods are very effective for free vibration problems but not for dynamically changing forced vibration problems. Forced vibration applications with changing frequency and amplitude are usually categorised into the class of nonlinear and time-varying systems, therefore, nonlinear control methods are required in the forced vibration control. One traditional passive technique used to solve forced resonance vibration problems is to add an additional mass-spring-damper into the system as a Dynamic Vibration Absorber (DVA) to transfer vibration energy to the sacrificial DVA[8]. For forced vibration problems, nonlinear vibration control techniques can however provide better solutions than linear modal control methods[9, 10]. Therefore, to actively suppress forced vibration in flexible structures which are susceptible to low frequency resonant vibrations, an active Vibration Clamping Absorber (VCA) has been designed. This design combines the DVA and modal control techniques together in a distributed way. VCAs can be built as integrated elements of a structure by using the so-called smart materials, such as piezoelectric materials, magnetostrictive materials, and shape memory alloys. In particular, piezoelectric materials such as *Lead Zirconate Titanate* (PZT) or *Polyvinylidene Fluoride* (PVDF) can be produced as thin films that can be bonded to the surface of large flexible structures using strong adhesive materials. Thus, they can provide spatially distributed information about the structures and are particularly suitable for strain-based sensors and actuators used for active vibration control in large flexible structures. The principle of VCA is to use such smart materials to transfer energy between mechanical structures and electrical sinks. The vibration energy can then be dissipated or absorbed via variable electrical impedances.

2. LINEAR DYNAMIC MODEL FOR FLEXIBLE STRUCTURES

From the principle of modal analysis, it is known that the complete dynamic behaviour of a structure can be discretised as a set of individual modes of vibration, each having a characteristic natural frequency, damping factor, and mode shape. By using these modal parameters to represent the system model, vibration problems at specific resonances can be examined and subsequently solved. Consider the class of flexible systems described by the following generalised wave equation:

$$\mathbf{m}(\mathbf{x})\ddot{\mathbf{w}}(\mathbf{x},\mathbf{t}) + 2\boldsymbol{\zeta}\boldsymbol{\Theta}^{1/2}\dot{\mathbf{w}}(\mathbf{x},\mathbf{t}) + \boldsymbol{\Theta}\mathbf{w}(\mathbf{x},\mathbf{t}) = \mathbf{F}(\mathbf{x},\mathbf{t}), \tag{1}$$

which relates the displacement $\mathbf{w}(\mathbf{x},\mathbf{t})$ of the equilibrium position of a body Ω in M-dimensional space to the applied force distribution $\mathbf{F}(\mathbf{x},\mathbf{t})$. The operator $\boldsymbol{\Theta}$ is a time-invariant symmetric, nonnegative differential operator with a square root $\boldsymbol{\Theta}^{1/2}$, and its domain $\boldsymbol{D}(\Theta)$ is dense in the Hilbert space $H = M^2(\Omega)$. The mass density $\mathbf{m}(\mathbf{x})$ is a positive function of the location $\mathbf{x}$ on the body with a square root $\mathbf{m}(\mathbf{x})^{\mathbf{1/2}}$. Without changing the properties of the above system, Eq. (1) can be normalised by the change of variables, such as using $\mathbf{w}(\mathbf{x},\mathbf{t})/\mathbf{m}(\mathbf{x})^{\mathbf{1/2}}$ to replace $\mathbf{w}(\mathbf{x},\mathbf{t})$. For simplicity and without losing generality, $\mathbf{m}(\mathbf{x}) = \mathbf{I}$ is assumed. From the above condition of operator $\boldsymbol{\Theta}$, it is known that its spectrum contains only separated eigenvalues λ_k with corresponding orthogonal eigenfunctions ϕ_k in $\boldsymbol{D}(\Theta)$, such that $0 \leq \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n, \boldsymbol{\Theta}\phi_k = \lambda_k\phi_k$, and $\boldsymbol{\lambda}_k^{1/2}\phi_k = \omega_k\phi_k$, where ω_k is the k^{th} vibration mode resonance frequency and ϕ_k is the corresponding vibration mode shape of the flexible structure and satisfies the orthogonality condition[11]. According to the nature of Hilbert space, the solutions of Eq. (1) can be expressed as:

$$\mathbf{w}(\mathbf{x},\mathbf{t}) = \sum_{k=1}^{M} v_k(t)\boldsymbol{\phi}_k(x), \tag{2}$$

where $v_k(t)$ is the mode amplitude, and M is the number of modes which should be infinity in theory. However, in practice, it is customary to assume that $\mathbf{w}(\mathbf{x},\mathbf{t})$ can be represented with good fidelity by a truncated mode expression of the form of Eq. (2) where M may be large but finite[12].

Similarly, the distribution of applied point forces at x_i can be discretised and expanded as:

$$\mathbf{F}(x_i,t) = \mathbf{F_e}(x_i,t) + \mathbf{F_c}(x_i,t) = \sum_{k=1}^{M} f_{ek}(t)\phi_k(x_i) + \sum_{k=1}^{M} f_{ck}(t)\phi_k(x_i), \tag{3}$$

where $\mathbf{F_e}$ represents the external excitation forces and $\mathbf{F_c}$ represents the control forces provided by point-force actuators.

In the following analysis, the case of primary resonances is considered and the external force can be defined by a set of M harmonic excitations with amplitude F_k and angular frequency Ω_k close to one of the natural frequencies, i.e., $f_{ek} = F_k \cos(\Omega_k t)$. Substituting Eqs. (2) and (3) into Eq. (1), multiplying through by $\boldsymbol{\phi_k}(x_i)$, integrating over the domain of the structure, and using the orthogonality property of the $\boldsymbol{\phi_k}(x_i)$, it is readily obtained that:

$$\ddot{\boldsymbol{v}}(t) + 2\boldsymbol{\zeta}\boldsymbol{\Delta}^{1/2}\dot{\boldsymbol{v}}(t) + \boldsymbol{\Delta}\boldsymbol{v}(t) = \mathbf{f_e}(\mathbf{t}) + \mathbf{f_c}(\mathbf{t}), \tag{4}$$

where $\boldsymbol{\Delta}^{1/2}$ is a $M \times M$ diagonal matrix with diagonal entries $\omega_1, \omega_2, \cdots, \omega_M$, $v(t) = [v_1(t), \cdots, v_M(t)]^T$, $\mathbf{f_e}(\mathbf{t}) = [f_{e1}(t), \cdots, f_{eM}(t)]^T$, $\mathbf{f_c}(\mathbf{t}) = [f_{c1}(t), \cdots, f_{cM}(t)]^T$ and the damping matrix $\boldsymbol{\zeta} = diag(\zeta_1, \zeta_2, \cdots, \zeta_M)$.

Any one of the modal displacements $v_k(t)$, in non-dimensional form, can be written as:

$$\ddot{v}_k(t) + 2\zeta_k\omega_k\dot{v}_k(t) + \omega_k^2 v_k(t) = F_k \cos(\Omega_k t) + f_{ck}(t), \tag{5}$$

due to the scalar form representation of Eq. (4), where $k = 1, 2, \cdots, M$. The purpose of using the non-dimensional form here is that all the natural frequencies are normalised after introducing a set of dimensionless variables such as non-dimensional modal displacement and non-dimensional time[13].

3. VCA DESIGN

In order to deal with forced vibrations (nonautonomous systems), a Quadratic-Modal-Positive-Position-Feedback (QMPPF) control algorithm has been designed. Based on the QMPPF algorithm, a distributed nonlinear vibration clamping absorber - referred to as VCA for the remainder of this paper, is developed for the structure described by Eq. (5). The purpose of the VCA is to channel the vibration energy to the VCA controller from the structure upon which primary external excitations are imposed. To achieve this purpose, the QMPPF has been designed to provide a control force which is intended to follow the external force, but with opposite phase. This principle is developed from the basic features of the DVA[8]. The design methodology for the active VCA is summarised below.

The structure's k^{th} mode displacement is described by Eq. (5). The QMPPF control law is then designed as:

$$f_{ck}(t) = K_{1k}\omega_k\eta_k^2(t), \tag{6}$$

where K_{1k} is a positive feedback control gain. The term $\eta_k^2(t)$ is the quadratic term of the VCA displacement which can be designed as:

$$\ddot{\eta}_k(t) + 2\xi_k\varpi_k\dot{\eta}_k(t) + \varpi_k^2\eta_k(t) = K_{2k}\varpi_k v_k(t)\eta_k(t), \tag{7}$$

where ϖ_k is a designed angular frequency of the VCA, ξ_k is the VCA controller's damping ratio, and K_{2k} is a control gain. By using the method of multiple scales[14], one can obtain the first-order approximate solutions for Eqs. (5) and (7) as:

$$\begin{aligned} v_k &= a\cos(\Omega_k t + \varphi_1), & (8)\\ \eta_k &= b\cos(\frac{1}{2}\Omega_k t + \varphi_2), & (9) \end{aligned}$$

where a and φ_1 are the amplitude and phase angle of the vector representation of the k^{th} mode of the structure, respectively; b and φ_2 are the corresponding amplitude and phase angle of the vector representation of the VCA's displacement, respectively. Define the two detuning parameters τ and σ as:

$$\begin{aligned} \tau &= \omega_k - 2\varpi_k, & (10)\\ \sigma &= \Omega_k - \omega_k. & (11) \end{aligned}$$

The modulation equations that govern the amplitudes and phases are given by:

$$\left\{ \begin{array}{rcl} \dot{a} &=& -\zeta_k\omega_k a - \frac{K_{1k}}{4}b^2\sin\alpha + f_k\sin\beta, \\ \dot{b} &=& -\xi_k\varpi_k b + \frac{K_{2k}}{4}ab\sin\alpha, \\ a\dot{\beta} &=& \sigma a + \frac{K_{1k}}{4}b^2\cos\alpha + f_k\cos\beta, \\ b(\dot{\alpha}+\dot{\beta}) &=& (\tau+\sigma)b + \frac{K_{2k}}{2}ab\cos\alpha. \end{array} \right. \tag{12}$$

The parameters α, β, and f_k in Eq. (12) are defined as $\alpha = \tau t + \varphi_1 - 2\varphi_2, \beta = \sigma t - \varphi_1$, and $f_k = F_k/4$. These parameters are deliberately designed in the VCA to be tuneable for control purposes. For example, a threshold value F_G for external excitations can be designed so that when the amplitude of external excitations is below this threshold, the VCA will not take any action, i.e., $b = 0$. While the amplitude of external excitations is greater than F_G, the VCA will clamp the structure's response to a limit and transfer the vibration energy to itself. Solving the equilibrium points of Eq. (12), the value of this threshold force F_G can be calculated as:

$$F_G = \frac{4}{K_{2k}}\sqrt{(\sigma^2 + \zeta_k^2\omega_k^2)\left[\frac{1}{4}(\tau+\sigma)^2 + \xi_k^2\varpi_k^2\right]}. \tag{13}$$

This value can be determined experimentally by increasing the excitation force F_G until the structural vibration amplitude is just acceptable. Then the feedback control gain K_{2k} can be designed according to the above equation. In order to keep F_G small so that the corresponding vibration amplitude is small, the design parameter ξ_k of VCA should be small.

The control force generated by the VCA can be obtained by substituting Eq. (9) into Eq. (6) as:

$$f_{ck}(t) = K_{1k}\omega_k(\eta_k)^2 = K_{1k}\omega_k\frac{b^2}{2} - K_{1k}\omega_k\frac{b^2}{2}\cos(\Omega_k t), \tag{14}$$

where the second component of the right side of Eq. (14) represents a control force that can equal the external excitation ($f_k \cos \Omega_k t$) but with opposite phase.
When the VCA is not activated, i.e., $b = 0$,

$$a = \frac{f_k}{\sqrt{\sigma^2 + \zeta_k^2\omega_k^2}}. \tag{15}$$

When the VCA is activated, i.e., $b \neq 0$,

$$a = 4\frac{\sqrt{\xi_k^2\varpi_k^2 + \frac{1}{4}(\tau+\sigma)^2}}{K_{2k}}, \tag{16}$$

$$b = \frac{4}{\sqrt{K_{1k}K_{2k}}}\left\{\left[\frac{\tau+\sigma}{2}\sigma - \zeta_k\omega_k\xi_k\varpi_k\right] \pm \left[\frac{K_{2k}^2 f_k^2}{16} - (\frac{\tau+\sigma}{2}\zeta_k\omega_k + \xi_k\varpi_k\sigma)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \tag{17}$$

It is clear from Eq. (16) that the amplitude of vibration in the structure is independent of the amplitude of the external force f_k. This is because the excitation energy has been transferred to the VCA controller and the structural vibration has been clamped.

4. EXPERIMENTAL STUDIES FOR THE VCA

A simple cantilever beam system was selected and used as a research vehicle to evaluate both the QMPPF control strategy and the VCA controller. In the following studies, primary resonant excitations are considered as they cause serious vibrations in the structure when $\Omega_k = \omega_k$. The bending modal parameters of the beam system were determined experimentally using modal testing method [15] and are shown in Table 1. To verify the theoretical analyses, a physical

Table 1. Modal Characteristics of the Beam System

Characteristics	Mode 1	Mode 2	Mode 3
Natural frequency (Hz)	11.4	68.5	149.8
Modal damping	0.0030	0.00002	≈ 0

system was constructed to test the active structural vibration control system. The test system comprises a $250 \times 13 \times 0.6$ mm mild steel beam with a strain gauge sensor and a piezoceramic actuator patch, mounted on a 100N shaker. The output of the strain gauge is proportional to the modal displacement on the point where the strain gauge is installed. The system was controlled using a dSpace digital controller. A schematic diagram of the physical system is shown in Fig. 1(a). The physical cantilever beam system is shown in Fig. 1(b) where the middle structures are not in contact with the beam but used to support the signal cables.

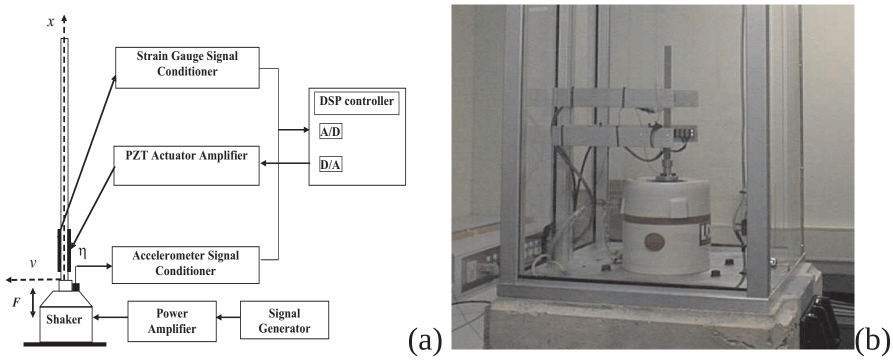

Figure 1. (a) Schematic of the controlled cantilever beam system. (b) The physical cantilever beam system.

When the shaker's frequency is tuned to 11.4Hz (i.e., the first mode frequency) and the acceleration produced by the shaker is $1.2g$, the first mode resonance causes large amplitude vibrations with the beam's tip displacement over 200 mm (peak to peak). After the vibration is fully developed, the VCA controller is switched on at the 20^{th} second time mark. Figs. 2(a) and 2(b) show the experimental results of the structure's first-mode time-response and the enlarged part of (a) around the 24 second time mark, respectively. The experimental results confirm the expected results from the theoretical analysis.

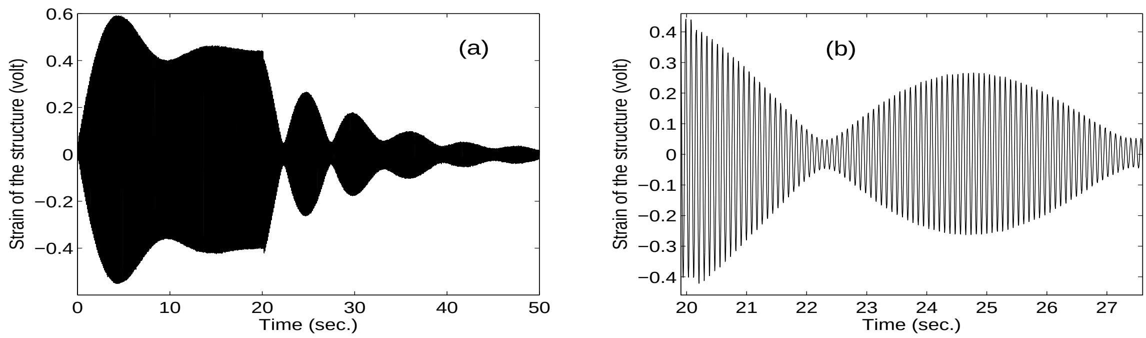

Figure 2. Experimental results of the first-mode time-response under the VCA control: (a) the sensor response v_1 and (b) zoomed part of v_1.

When the shaker's frequency is tuned to 68.5Hz (i.e., the second mode frequency) and the acceleration produced by the shaker is 1.0g, the second mode resonance causes large amplitude vibrations. Figs. 3(a) and 3(b) show the experimental results of the structure's second-mode time-response and the enlarged part of (a) around the 11 second time mark, respectively. It can be seen that the structural vibration has been successfully suppressed even when the external force frequency is at the second resonant frequency.

When the beam is excited under a multiple frequency sinusoidal excitation with frequencies of 11.4Hz and 68.5Hz, and the acceleration produced by the shaker is 3.0g, the combined resonances cause even larger amplitude vibrations. Fig. 4(a) shows the experimental results of the structure's combined mode time-response around the 25 second time mark. It can be seen that the structural vibration has been successfully suppressed even when the external force frequencies are close to the first and second harmonics of the beam. The experimental results further validate the theoretical analysis of multiple mode control. The Power Spectrum Density analysis of multiple mode vibration is shown in Fig. 4(b). The suppression effect achieved by the VCA can be more than 30dB.

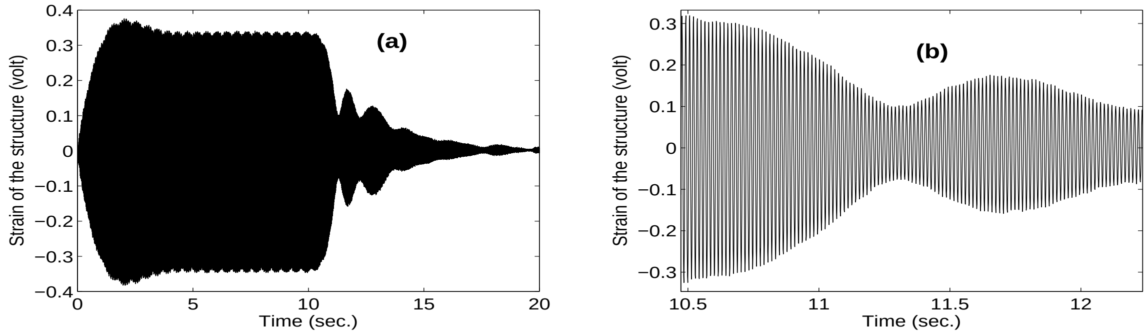

Figure 3. Experimental results of the second-mode steady-state time-response under the VCA control: (a) the sensor response v_2 and (b) zoomed part of v_2.

5. CONCLUSIONS

The effectiveness of the VCA design based on the QMPPF strategy has been validated under single-mode and multiple-mode control on a flexible cantilever beam system with a single sensor and actuator pair. The experimental responses obtained from the physical system have demonstrated that the VCA can be used to control multi-mode resonances in flexible structures.

It should be noted that the method used in the multiple mode control case is based on the assumption that the structural natural frequencies are widely spaced and independent of each other[3]. Under this assumption, the response of the system can be represented by a series of SDOF systems. Therefore, the VCAs can be designed so that each controls a different mode of the system. If there is significant interaction between two modes[13], this assumption may be invalid, in which case other control methods would be needed.

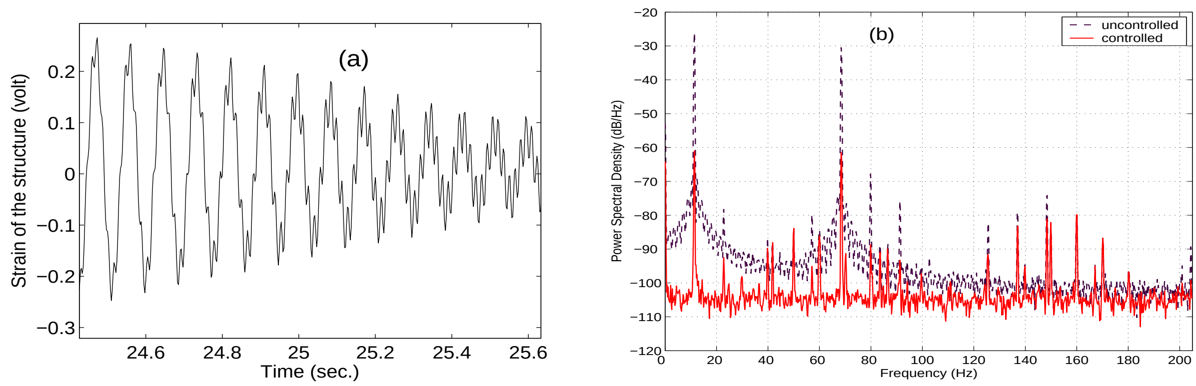

Figure 4. Experimental results of the first- and second-mode time-response under the VCA control: (a) the sensor response and (b)Power Spectral Density with and without the VCA control.

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