



# RESPONSE OF HULL PLATES DUE TO TURBULENT BOUNDARY LAYERS

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**Abstract** A model has been developed for the prediction of velocity levels of ships hull plates which are excited by turbulent boundary layers. The model is based on the theory developed by Corcos. It is found that the velocity of the hull plates strongly depends on the speed of the ship. The acoustic power induced in the hull can be reduced if the thickness of the hull plates is increased. Other parameters like frame distance and height of hull plate are of secondary importance. The effect of turbulent boundary layer excitation is most efficiently reduced by changing the hull shape or by changing the transmission path from the hull plates to the accommodation decks.

## **1. INTRODUCTION**

A turbulent boundary layer-TBL- can be developed in a fluid close to a structure at sufficiently high flow velocities. For a ship travelling through water, the TBL, when developed, excites the hull plates. Part of the acoustic power induced in the hull is transmitted as vibrations in the ship structure to the accommodation spaces. The structures enclosing the accommodations radiates noise into these spaces. The acoustic power radiated due to this effect is proportional to the power induced in the hull structure by the TBL. It is therefore essential to determine the relative importance of the TBL effect as compared to the acoustic power induced in the ship structure by other sources like water jets, engines etc. A model describing the effect of a TBL or rather the acoustic power induced in hull plates as function of ship speed, plate geometry etc could possibly also be used to minimize the effect of the TBL by changing the hull design.

The first parameter that determines the nature of turbulence in a boundary layer is the Reynolds number *R* defined as  $R = UL/v_f$  where *U* is the velocity of the fluid well away from a structure, *L* is the distance of the origin of the boundary layer and  $v_f$  the kinematic viscosity of the fluid. The viscosity for water is  $1.284 \cdot 10^{-6} \text{ m}^2/\text{s}$ . As discussed in Blake [1] turbulence can build up spotwise for  $R \approx 2 \cdot 10^6$ . For  $R \approx 10^7$  the turbulence is fully developed. The thickness of the boundary layer in a fluid having a turbulent flow close to a structure is given by  $\delta$  and is defined as the distance from the wall at which the mean flow

velocity is  $0.99 \cdot U$  where U is the free velocity well away from the wall. Within the boundary layer eddies are formed due to shear effects in the fluid caused by the velocity profile. The principal boundary layer eddies are moving across the wall at the convection velocity  $U_c$ . Both experimental and numerical simulations indicate, according to Howe [2], that  $0.5 \cdot U \leq U_c \leq 0.7 \cdot U$  with only a weak dependence on frequency.

#### 2. THE CORCOS MODEL

The pressure *p* caused by a turbulent boundary layer on a plate, oriented in the x - y – plane, not only depend on time *t* but also on the coordinates *x* and *y*. The auto correlation function of the pressure p(x, y, t) on the plate is defined as the ensemble average of the pressure *p* at the coordinates (x, y, t) and the pressure at the coordinates  $(x + \xi_x, y + \xi_y, t + \tau)$ . For a pressure *p*, stationary in two dimensional space and time, the ensemble average depends only on the separation  $\xi_x$  and  $\xi_y$  in space and  $\tau$  in time. The auto correlation function  $R_{pp}$  is thus defined as

$$R_{pp}(\xi_x,\xi_y,\tau) = E\left[p(x,y,t)p(x+\xi_x,y+\xi_y,t+\tau)\right]$$
(1)

where  $E[\]$  represents the ensemble average. The cross power spectral density between the two signals observed at two positions separated by the vector  $(\xi_x, \xi_y)$  is defined by  $S_{pp}(\xi_x, \xi_y, \omega)$  as

$$S_{pp}(\xi_x,\xi_y,\omega) = \int_{-\infty}^{\infty} d\tau \cdot R_{pp}(\xi_x,\xi_y,\tau) \cdot \exp(-i\omega\tau)$$
(2)

where  $\omega$  is the angular frequency. The cross power spectral density has been investigated by Corcos [3]. The model proposed by Corcos has during the last few decades been widely used for many different types of problems. The model is applicable for  $\omega \delta / U > 1$  in the immediate neighbourhood of the so called convective ridge [2]. For defining the model let a hull plate be oriented in the x-y-plane of a Cartesian coordinate system. The coordinates for the corners of the rectangular plate are (0,0),  $(L_x, L_y)$ ,  $(0, L_y)$  and  $(L_x, 0)$  where  $L_x$  and  $L_y$  are the lengths of the plate. A turbulent flow across the plate in the direction of the x-axis is the seat of local pressure fluctuations. It is assumed that the field is statistically stationary and homogeneous. Corcos assumes that the cross power spectral density, eq. (2), can be expressed as

$$S_{pp}(\xi_x,\xi_y,\omega) = \Phi_{pp}(\omega) \cdot \exp\left[-\gamma_1 \left|\omega\xi_x / U_c\right|\right] \cdot \exp\left[-\gamma_3 \left|\omega\xi_y / U_c\right|\right] \cdot \exp\left[i\omega\xi_x / U_c\right]$$
(3)

The point auto spectrum of the pressure is  $\Phi_{pp}(\omega)$ . The parameters  $\gamma_1$  and  $\gamma_3$  are determined from experimental data.

The power spectral density of the velocity of a plate excited by a pressure fluctuation described by eq. (3) is according to Newland [4] given by

$$S_{\nu\nu}(x, y, \omega) = \frac{1}{(2\pi)^2} \int dk_x dk_y \omega^2 S_{pp}(k_x, k_y, \omega) |G(x, y, k_x, k_y, \omega)|^2$$
(4)

where  $G(x, y, k_x, k_y, \omega)$  is Green's function and  $S_{pp}(k_x, k_y, \omega)$  is the spatial Fourier transform of cross power spectral density  $S_{pp}(\xi_x, \xi_y, \omega)$  as

$$S_{pp}(k_x, k_y, \omega) = \iint d\xi_x d\xi_y \cdot S_{pp}(\xi_x, \xi_y, \omega) \cdot \exp\left[-i(k_x\xi_x + k_y\xi_y)\right]$$
(5)

The definition (5) in combination with eq. (3) yields

$$S_{pp}(k_x, k_y, \omega) = 4 \cdot \frac{\Phi_{pp}(\omega) \cdot \frac{\omega \gamma_1}{U_c} \cdot \frac{\omega \gamma_3}{U_c}}{\left[ \left( \frac{\omega \gamma_1}{U_c} \right)^2 + \left( \frac{\omega}{U_c} - k_x \right)^2 \right] \cdot \left[ \left( \frac{\omega \gamma_3}{U_c} \right)^2 + k_y^2 \right]}$$
(6)

For a simply supported plate, width  $L_x$  and height  $L_y$  the function  $G(x, y, k_x, k_y, \omega)$  is as discussed in [4] given by

$$G = \sum_{m,n} W_{mn} \cdot \varphi_{mn}(x, y) \cdot I_{mn} \quad ; \quad W_{mn} = \frac{4}{L_x L_y \mu [\omega_{mn}^2 (1 + i\eta) - \omega^2]}$$
(7)

In eq. (7)  $\mu$  is the mass per unit area of the plate,  $\omega_{mn}$  the angular frequency corresponding the eigenmode m,n. The loss factor of the plate is  $\eta$ . The eigenmodes of the simply supported plate are  $\varphi_{mn}(x, y) = \sin(m\pi x/L_x) \cdot \sin(n\pi y/L_y)$ . The parameter  $I_{mn}$  in eq.(7) is for a simply supported plate defined as

$$I_{mn} = \frac{(m\pi/L_x)(n\pi/L_y)[1 - \cos(m\pi)\exp(ik_xL_x)][1 - \cos(n\pi)\exp(ik_yL_y)]}{[k_x^2 - (m\pi/L_x)^2]\cdot[k_y^2 - (n\pi/L_y)^2]}$$
(8)

#### 3. FLUID LOADING AND BOUNDARY CONDITIONS

The fluid loading on a hull plate can be considerable. For a plate in flexure with a fluid loading on one side the wavenumber  $\kappa$  is according to for example [6] the solution to  $\kappa^4 = \kappa_o^4 \left(1 + \rho / (\mu \sqrt{\kappa^2 - k^2})\right)$  where  $\kappa_o$  is the wavenumber for the plate in vacuum, *k* the wavenumber in the fluid,  $\rho$  the density of the fluid and  $\mu_o$  the mass per unit area of the unloaded plate. The wavenumber  $\kappa$  for flexural waves is obtained from the expression  $\kappa^4 = \mu \omega^2 / D$  where *D* is the bending stiffness of the plate. For  $\kappa >> k$ , the total mass of fluid loaded plate is approximately  $\mu \approx \mu_o + \rho / \kappa$ .

The loss factor  $\eta$  for a fluid loaded plate is modelled as  $\eta = 0.025 \times f^{-0.275}$  where *f* is the frequency. This result is based on measurements.

#### 4. TURBULENCE MODEL

Corcos' Model, eq. (3), is used to describe the wall pressure fluctuations. In eq. (3)  $\omega$  is angular frequency,  $\xi_{x,y}$  denotes spatial separation in the *x*- and *y*-direction.  $\Phi_{pp}$  is the TBL wall pressure auto-spectrum and taken from measurements [7]. The convection velocity  $U_c$ was approximated from measurement results by  $0.80 \times U_0$ , where  $U_0$  is the free-stream velocity. The stream wise and span wise coherence parameters  $\gamma_1$  and  $\gamma_3$  were initially taken as 0.125 and 0.7 respectively as suggested in [7].

## 5. ENERGY OF PLATE DUE TO RANDOM EXCITATION

Although the eqs. (4) through (10) can be used to calculate the response of the hull plates a simplified approach can be used for estimating the influence on plate velocity of certain material and geometrical parameters of the hull plates. The acoustic power input to a plate depends on the point mobility of the structure at the excitation point as well as the pressure fluctuations across the plate itself. For a TBL excitation of a hull plate the pressure fluctuations on the structure depends on the speed of the vessel and the flow around the hull. The flow depends on the shape of the hull. Once the lines of the ship as well as speed have been determined the excitation forces on the hull plates are also determined. The parameters which possibly could be varied at this stage are frame distance, plate thickness and damping. The power spectral density  $S_{FF}$  is  $S_{\Pi} = S_{FF} \cdot \text{Re } Y$  where Y is the point mobility of the structure. Within a frequency band  $\Delta f$  the time average of the power input  $\overline{\Pi}_{\Delta f}$  is

$$\overline{\Pi}_{\Delta f} = \int S_{\Pi} df = \int S_{FF} \operatorname{Re}(Y) df$$
(9)

For white noise excitation across the plate area  $S_o$  the power input  $\overline{\Pi}_{\Delta f}$  is proportional to

$$\overline{\Pi}_{\Delta f} \propto \int df dS_o \operatorname{Re}(Y) \tag{10}$$

It is readily shown, see for example [8], that for frequencies well above the first natural frequency of the plate the integral (9) is reduced to

$$\overline{\Pi}_{\Delta f} \propto \int df dS_o \operatorname{Re}(Y) \propto S \cdot \operatorname{Re} Y_{\infty} \cdot \Delta f$$
(11)

where  $Y_{\infty}$  is the point mobility of an infinite plate of the same material and thickness as the finite plate. The result (11) is in the high frequency region independent of the boundary conditions of the finite plate. Thus, for a plate excited within a frequency band by white noise the power input to the structure is proportional to the real part of the point mobility of an infinite plate of the same material and the same thickness as the finite plate, or quite simply  $\overline{\Pi}_{\Delta f} \propto \operatorname{Re} Y_{\infty} \propto 1/\sqrt{D\mu}$  where *D* is the bending stiffness of the plate and  $\mu$  its mass per

unit area. For a fluid loaded plate  $\mu \approx \mu_o + \rho / \kappa$ . In general the last mass term dominatesconsequently

$$\overline{\Pi}_{\Delta f} \propto \frac{1}{\sqrt{D\rho/\kappa}} \propto \sqrt{\frac{\kappa}{D}}$$
(12)

The bending stiffness *D* is proportional to the thickness *h* as  $h^3$  and the wavenumber  $\kappa$  as  $h^{-1/2}$  resulting in

$$\overline{\Pi}_{\Lambda f} \propto h^{-7/4} \tag{13}$$

The total energy  $\overline{E}_{\Delta f}$  of the plate within the frequency band is obtained from

$$\overline{\Pi}_{\Delta f} = \omega \eta \overline{E}_{\Delta f} \propto \mu < \overline{\nu}^2 > \tag{14}$$

where  $\langle \overline{v}^2 \rangle$  represents the time and space average of the velocity squared of the plate. The apparent mass of the fluid loaded plate is as before  $\mu \approx \mu_o + \rho/\kappa$ . Thus  $\mu \propto 1/\kappa \propto h^{1/2}$  and

$$\langle \overline{\nu}^2 \rangle \propto h^{-9/4} \tag{15}$$

The input power to the structure and its velocity is reduced if the thickness of the plate is increased. For a doubling of the plate thickness the input power is reduced by 5 dB whereas the velocity level of the plate is reduced by 7 dB. For a 50% increase of the plate thickness the level of the input power is reduced by 3 dB and the velocity level by 4 dB. According to (5) the power input to the plate is independent of the loss factor of the structure. The kinetic energy of the plate and thus also time and space averages of the velocity squared of the plate are proportional to the power input and inversely proportional to the total loss factor of the structure. Thus by increasing the losses of the structure or hull plate the velocity squared of the plate is decreased though the input to the structure is constant and independent of the losses. The energy transmission from the excited hull plates to the adjoining structures is not primarily determined by resonant transmission but rather by forced transmission as discussed in [9]. Consequently, added damping to the hull plates would decrease the velocity level of the plate itself but the energy transmission to the adjoining structures would only be reduced marginally. In order to reduce the velocity levels of a structure, for example a deck, far away from the hull plates which are excited by TBL, damping layers should be applied to the structure or deck itself rather than on the hull plates. The only geometrical parameter which can be changed to significantly alter the power input to or the velocity of a hull plate is the thickness of the hull plate. Boundary conditions, width and length of the plate as well as damping is of secondary importance. This implies that the only practical way to reduce the power input to the hull from TBL excitation is to reduce the pressure fluctuations on the hull by optimizing the hull shape. Due to considerations of the weight and cost of the ship the thickness of the hull plates can in general not be increased. The effect of the TBL excitation is most efficiently reduced by changing the transmission path in the ship structure itself or by adding damping layers to the accommodation decks. The energy transmission to deck structures can be reduced by using spot welds instead of continuous welds between deck

plates and vertical plate structures coupled to the hull. The importance of junctions with respect to energy transmission in ship structures is discussed in [10].

## 6. NUMERICAL RESULTS

The hull plates were modelled as simply supported aluminium plates. The plates were assumed to be flat, isotropic, and without pre-stress. The width, length and thickness of the plate were 0.7m, 1.5m and 4mm respectively. The response of the hull plate can be determined by a numerical surface integration of the pressure excitation over the area of the plate or by calculating the average response at five different points on the surface of the plate. The last approach is very fast and the deviation from the correct result is negligible. Figure 1 compares the auto spectral density of the velocity for the four flow speeds. As expected an increase in speed generally results in an increase in response. The velocity level  $L_{\nu}$  in dB re

 $10^{-9}$  m/s is obtained as  $L_v = 10 \cdot \log(S_{vv}) + 180$  when  $S_{vv}$  is given in SI units. The velocity level of the hull plates is increasing rapidly as function of the flow speed u. In the frequency range 630 to 1250 Hz the velocity level  $L_v$  in dB in 1/3 octave bands is approximately given as  $L_v = 10 \cdot \log u^{10} + C_1$  where  $C_1$  is some constant. Thus by increasing the speed from 25 to 30 knots the velocity level of the plate is increased by 8 dB, from 25 to 35 knots results in 14 dB and from 25 to 40 knots in a 20 dB increase.

Predicted results show that it is only in the low frequency region, below 200 Hz, that the frame distance significantly influences the plate response. For frequencies well above the first plate resonance frequency the plate dimensions are of minor importance with respect to the plate response.

Figure 2 shows the plate response  $S_{vv}$  for the three different plate thicknesses of 4, 6 and 8mm. In the high frequency range a doubling of the plate thickness gives a reduction of the velocity level by approximately 7 dB. A 50% increase results in a reduction of the velocity level by approximately 4 dB. The velocity level  $L_v$  of the hull plate as function of plate thickness *t* is in the high frequency region approximately given by  $L_v = 10 \cdot \log(h^{-9/4}) + C_2$  where  $C_2$  is some constant.

## 7. CONCLUSIONS

The main design parameters which influence the velocity level of hull plates excited by a turbulent boundary layer is the speed of the ship, the shape of the hull and the thickness of the hull plates. A damping layer on the hull plates would decrease the velocity level of the plate but the input power to the plate is not influenced by the damping layer. A damping layer would therefore have a very limited effect on the noise levels in the accommodation spaces on the main deck and above. The frame distance has a very small effect on the noise levels in the mid and high frequency ranges. The shape of the hull is crucial with respect to the acoustic input power to the hull.



Figure 1. Response as function of flow speed. Blue, 25 knots; red, 30 knots; green, 35 knots; yellow, 40 knots.



*Figure 2. Response as function of thickness t at 40 knots. Red, 4 mm; blue, 6 mm; green, 8 mm.* 

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