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INTERNAL RESISTANCE OPTIMIZATION OF A HELMHOLTZ RESONATOR IN NOISE CONTROL OF SMALL ENCLOSURES

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Abstract

This paper examines the influence of the internal resistance of a Helmholtz resonator on the noise control in a small enclosure. The absorptive process mainly occurring within the neck of a Helmholtz resonator provides the resonator with a damping (internal resistance) property, which directly dissipates the input energy in the resonator. The remaining non-dissipated energy is re-radiated back to the enclosure, such forming an effective secondary sound source, and resulting in acoustic interaction with the primary source. If the internal resistance of the resonator is low, the acoustic interaction between the enclosure and the resonator sharply splits the targeted resonance peak of the enclosure into two parts, and the peak response is significantly attenuated within a very narrow frequency band. By appropriately increasing the internal resistance at the resonance of the resonator, the working bandwidth can be enlarged at the expense of sacrificing the control performance due to the decreased amplitude. However, if the resistance is over-increased, the strength of volume velocity out of the resonator aperture becomes too low; compromising the effective acoustic interaction with the enclosure and resulting in insignificant control at the targeted resonance peak. In this paper, a mathematical model describing the acoustic interaction of a resonator and a small enclosure is presented. An analytical solution is obtained on the pressure field inside the enclosure and the radiation of the resonator. Based on the analytical solutions, an energy reduction index describing the strength of acoustic interaction in the enclosure with a resonator is defined. Series of numerical simulations are conducted to illustrate the influence of the internal resistance on the energy reduction and on the dissipated and re-radiated energy. Finally, the optimal internal resistance is obtained. Experimental results using one resonator are also carried out and compared with simulation results.

I. INTRODUCTION

Low frequency noise inside small enclosures always has always been a concern in many engineering problems. Active noise control is a promising approach in low-frequency noise controls. However, its cost of implementation, robustness of the control system and the requirement of a large space hamper its application in small enclosures. Passive techniques such as Helmholtz resonators still show many appealing features in terms of implementation, control quality and cost effectiveness.

The acoustic interaction between an enclosure and with Helmholtz resonators has been extensively investigated in the past. Fahy and Schofield [1] built an advanced model to explore some underlying physics of the resonator. The experimental results on internal resistance [1] showed that there exists an optimal damping value when other parameters were fixed. With a view to broaden the application of resonators, Cummings [2] extended Fahy and Schofield's single resonator and single room mode model to multiple resonator coupled with multi room-mode. In his multi-mode model, resonators were taken as pseudo point sources. In order to solve the singularity problem caused by point source, the sound pressure of that pseudo point source at its own location was calculated by the averaged sound pressure at the surface of a small equivalent pulsating sphere. Li and Cheng [3] recently proposed a new model to analyze the interaction between acoustic resonator array and a room with multiple modes. In that work, the resonator was treated as a point source with volume velocity influenced by enclosure. Instead of building the linear equation at the aperture of resonators, they considered the acoustic equilibrium in the volume of room to avoid the singularity problem previously encountered when the linear equation set was solved. By comparing the theoretical results with experimental results, the model shows very good approximation to real multi-mode setup.

Despite the consistent effort that was made in Helmholtz resonator design, researchers are still looking into approaches to improve the noise control performance using a Helmholtz resonator array, which involves heavy experimental measurements on a trial-and-error basis. This paper presents a systemic design tool to optimize the internal resistance of a Helmholtz resonator. It is divided into four sections. The mathematical model of acoustic interaction between an enclosure and one single resonator, the theory of energy optimization, and investigation of the internal resistance of the resonator are presented in Section II. Section III gives numerical and experimental results to examine the internal resistance of resonators. Finally, conclusions for optimal design of a resonator are presented in Section IV.

II. THEORY

A general model for describing the acoustic interaction between an enclosure and only one Helmholtz resonator is presented, and then, the energy optimization theory is given. Formulas of dissipated energy and radiated energy by an acoustic resonator, which depends on the internal resistance of the resonator, are derived. Throughout the paper, the superscripts and subscripts E, R, and S indicate the variables associated with "Enclosure", "Resonator", and "Source", respectively.

A. Acoustic interaction between an enclosure and a single Helmholtz resonator

The inhomogeneous wave equation governing the pressure field in the enclosure reads:

$$\nabla^2 p(\mathbf{r},t) - \ddot{p}(\mathbf{r},t) = -\rho_0 \dot{q}(\mathbf{r},t)\,, \tag{1}$$

where $p(\mathbf{r}, t)$ is the acoustic pressure, q is the volume velocity source strength density distribution in the volume and surface of the enclosure. Assuming that a set of N harmonic sources with volume velocity source strength density $q^S{}_1\, q_1^S$, q_2^S, …, q_N^S are located at the points $\mathbf{r}_1^S$, $\mathbf{r}_2^S$, …, $\mathbf{r}_N^S$ form the primary sound field in the enclosure, and the single resonator with volume velocity source strength density q^R centered at the point $\mathbf{r}^R$ (center of the resonator aperture) forms the secondary sound field in the enclosure. Notice that the volume velocity directed out of the resonator has the same sign as that of the primary sound source, i.e., the positive sign is directed out off the source into the enclosure. The volume velocity source strength density out of the acoustic resonator is computed from $q^R(t)$= $p(\mathbf{r}^R,t)$/Z, in which Z is defined as the output acoustic impedance at the aperture of the resonator , and $p(\mathbf{r}^R,t)$ is the sound pressure at $\mathbf{r}^R$ provided that the largest dimension of the resonator aperture is much smaller than the sound wavelength of interest, equation (1) becomes

$$\nabla^2 p(\mathbf{r},t) - \frac{1}{c^2}\ddot{p}(\mathbf{r},t) = -\rho_0\left[\frac{\dot{p}(\mathbf{r},t)\delta(\mathbf{r}-\mathbf{r}^R)}{Z} + \sum_{n=1}^{N}\dot{q}_n^S(t)\delta(\mathbf{r}-\mathbf{r}_n^S)\right], \tag{2}$$

where $\delta(\mathbf{r}\text{-}\mathbf{r}^R)$ is a three dimensional Dirac delta function..

Acoustic pressure $p(\mathbf{r},t)$ can be decomposed on the basis of eigenfunctions of the enclosure: $p(\mathbf{r},t)=\Sigma\ \Psi_j(t)\,\varphi_j(\mathbf{r})$, $\Psi_j(t)$ is the j^{th} modal response, and $\varphi_j(\mathbf{r})$ is the j^{th} eigenfunction. Substituting this modal expansion into Eq. (2) and applying orthogonality properties of the eigenfunctions yield a discrete acoustic equation

$$\ddot{\psi}_j(t) - \frac{cz_0}{V^E}\frac{1}{Z}\frac{\varphi_j\left(\mathbf{r}^R\right)}{\Lambda_j}\sum_h\left[\varphi_h\left(\mathbf{r}^R\right)\dot{\psi}_h(t)\right] + \left(\gamma_j^E\right)^2\psi_j(t) = \frac{cz_0}{V^E}\sum_{n=1}^{N}\frac{\tilde{\varphi}_j\left(\mathbf{r}_n^S\right)}{\Lambda_j}\dot{q}_n^S(t)\,,\ (j=1, 2, 3, \ldots, J) \tag{3}$$

where $z_0=\rho_0 c$; z_0 is the characteristic impedance of the fluid, V^E is the volume of the enclosure, Λ_j is the mode normalization factor, given by $\Lambda_j = \int_V\left[\varphi_j(\mathbf{r})\right]^2 dV \big/ V$, $\tilde{\varphi}_j(\mathbf{r}_n^S)$ is the averaged $\varphi_j(\mathbf{r}_n^S)$ over the volume of the nth source (first source), and γ_j^E is the j^{th} eigenvalue of the enclosures, which holds on the homogeneous wave equation. The eigenvalue can be expressed as $\gamma_j^E = \omega_j^E + iC_j^E$, in which the real part is the angular frequency and the imaginary part is an equivalent *ad hoc* damping coefficient.

Assuming all time dependent variables are harmonic, i.e. $\Psi_j(t)=P_h e^{i\omega t}$ and $q_n^S(t) = Q_n^S e^{i\omega t}$, then, in the absence of the resonators, the modal response P_j can be solved from Eq. (3) as

$$\left(P_j\right)_{\text{without resonator}} = \frac{i\omega cz_0}{\omega^2 - \left(\gamma_j^E\right)^2}\frac{1}{\Lambda_j V^E}\sum_{n=1}^{N}\tilde{\varphi}_j\left(\mathbf{r}_n^S\right)Q_n^S\,,\ (j=1, 2, 3, \ldots, J). \tag{4}$$

When a resonator is installed, the modal response of P_j can be solved from Eq. (3) as

$$P_j = \underbrace{\frac{i\omega c z_0 \sum_{n=1}^{N} \tilde{\varphi}_j\left(\mathbf{r}_n^S\right) Q_n^S}{\left[\left(\gamma_j^E\right)^2-\omega^2\right]\Lambda_j V^E}}_{\text{Contribution of the first sound field}} - \underbrace{\frac{\left[\dfrac{1}{\left(\gamma_j^E\right)^2-\omega^2}\dfrac{\varphi_j\left(\mathbf{r}^R\right)}{\Lambda_j}\right]\dfrac{1}{Z}\left(\dfrac{cz_0\omega}{V^E}\right)^2 \sum_h \left[\dfrac{\varphi_h\left(\mathbf{r}^R\right)\sum_{n=1}^{N}\tilde{\varphi}_h\left(\mathbf{r}_n^S\right)Q_n^S}{\left[\left(\gamma_h^E\right)^2-\omega^2\right]\Lambda_h}\right]}{1-i\dfrac{cz_0\omega}{Z}\dfrac{1}{V^E}\sum_h \dfrac{1}{\left(\gamma_j^E\right)^2-\omega^2}\dfrac{\left[\varphi_h\left(\mathbf{r}^R\right)\right]^2}{\Lambda_h}}}_{\text{Contribution of the acoustic resonator}}, \quad (j=1, 2, 3, \ldots, J). \qquad (5)$$

The first term on the right hand side of Eq. (5) is the contribution coming from the primary sound field, and the second term is the effect of inserting an acoustic resonator into the enclosure.

B. Energy optimization

When inserting a designed resonator to target a resonance peak of the enclosure, two new coupled frequencies were produced on the either side of original resonance frequency [1]. It is hard to carry out parameters' optimizations of resonator by selecting just a single frequency as control target since the presence of those two new coupled frequencies. It is necessary to involve major frequency components around the resonance peak [4] for optimizations of resonator. To this end, energy in a frequency band is determined as the objective function for the optimization of resonator location and damping. The theory of the energy optimization is derived as below.

By using $p(\mathbf{r},t)=\sum \Psi_j(t)\,\varphi_j(\mathbf{r})$ and $\Psi_j(t)=P_h e^{i\omega t}$, the energy within a bandwidth $[\omega_1, \omega_2]$ is calculated based on spatially averaged mean-square pressure :

$$E^R\left[\omega_1,\omega_2\right]=\int_{\omega_1}^{\omega_2}\left[\frac{1}{V}\int_V\left(\frac{1}{T}\int_0^T\left[\mathrm{Re}(p)\right]^2 dt\right)dV\right]\mathrm{d}\omega=\frac{1}{2}\Delta\omega\sum_j \mathrm{Re}\left(\Lambda_j\right)\sum_i\left|P_j(\omega_i)\right|^2, \qquad (6)$$

where $\Delta\omega$ is the numerical integration step. The energy reduction is defined as the ratio of energy level $E^R[\omega_1, \omega_2]$ with resonator to that $E^0[\omega_1, \omega_2]$ without resonator.

$$ER\left[\omega_1,\omega_2\right]=-10\log_{10}\frac{E^R\left[\omega_1,\omega_2\right]}{E^0\left[\omega_1,\omega_2\right]}=-10\log_{10}\frac{\sum_j\left[\mathrm{Re}\left(\Lambda_j\right)\sum_i\left|P_j^R(\omega_i)\right|^2\right]}{\sum_j\left[\mathrm{Re}\left(\Lambda_j\right)\sum_i\left|P_j^0(\omega_i)\right|^2\right]} \quad , \qquad (7)$$

where ω_i varies in bandwidth$[\omega_1, \omega_2]$. If local noise control is expected, a small V around this region can be selected to calculate $E^R[\omega_1, \omega_2]$. However, if global noise control is expected, the whole enclosure volume should be selected for the calculation of $E^R[\omega_1, \omega_2]$.

C. Damping optimization for Helmholtz resonators

As mentioned before, inserting a lightly damped resonator into a lightly damped enclosure can split the controlled harmonic peak to two peaks. In order to enlarge the working bandwidth of a resonator, damping material is introduced to abase those harmonic peaks between the two coupled frequencies. However, excessive damping is not effective, as reported in reference [1]. Therefore, there exists an optimal damping to implement optimal control performance of a resonator.

The optimization of internal resistance will be carried out based on the energy reduction described in Eq. (7). When the internal resistance of the inserted resonator varies, the modal response P_j^R of enclosure varies, correspondingly. Two important types of energy, dissipated energy and radiated energy by resonator, are the key parameters to evaluate the control performance of a resonator, which are influenced by resonator resistance. Firstly, the relationship between dissipated energy and resistance is investigated.

The motion of lumped mass in the resonator neck follows the Newton's second law:

$$\rho_0 L^R S^R \ddot{x}(t) + z_0 S^R L^R R \dot{x}(t) + \frac{\rho_0 c^2 (S^R)^2}{V^R} x(t) = -p(\mathbf{r}^R, t) S^R \quad , \tag{8}$$

where $x(t)$ is the particle displacement in the resonator neck, which is assumed positive when it points to the enclosure, and V^R the body volume of resonator, S^R the cross sectional area of the neck, L^R the effective neck length, and R the internal resistance.

Assuming the movement of this lumped mass is a harmonic variable, namely $x(t)=Xe^{i\omega t}$ and considering the decomposition of pressure based on mode shape function shown above, the amplitude of displacement is solved from Eq. (8) as:

$$X = -\frac{1}{\left[\left(\omega^R\right)^2 - \omega^2 + icR\omega\right]} \frac{1}{\rho_0 L^R} \sum_{h=1}^{J} \varphi_h(\mathbf{r}^R) P_h \quad . \tag{9}$$

Because energy is defined in a bandwidth [ω_1, ω_2], parameters used to calculate energy must be mean square values in the same band. Thus, based on Eq. (9), the mean square velocity in this band can be given as:

$$\left[\overline{v^2}\right]_{\omega_1,\omega_2} = \int_{\omega_1}^{\omega_2} \left(\frac{1}{T} \int_0^T \left[\mathrm{Re}(v)\right]^2 dt \right) d\omega = \frac{1}{2\left(\rho_0 L^R\right)^2} \Delta\omega \sum_j \frac{\omega_j^{\,2} \sum_{h=1}^{J} \left|\varphi_h(\mathbf{r}^R) P_h\left(\omega_j\right)\right|^2}{\left[\left(\omega^R\right)^2 - \omega_j^{\,2}\right]^2 + \left(cR\omega_j\right)^2} \quad . \tag{10}$$

By using the definition of mean square velocity in a band in Eq. (10), the averaged dissipated energy of a resonator in the band [ω_1, ω_2] is:

$$E_d^R = S^R R_i \left[\overline{v^2}\right]_{\omega_1,\omega_2} = \frac{S^R}{2\rho_0 L^R} \Delta\omega \sum_j \frac{cR\omega_j^{\,2}}{\left[\left(\omega^R\right)^2 - \omega_j^{\,2}\right]^2 + \left(cR\omega_j\right)^2} \sum_{h=1}^{J} \left|\varphi_h(\mathbf{r}^R) P_h\left(\omega_j\right)\right|^2 \quad , \tag{11}$$

where $R_i = z_0 L^R R$ is the specific acoustic resistance of a resonator.

After discussing the dissipated energy by a resonator, the influence of damping effects on the sound field radiated by a resonator will be investigated as the follows. Since sound pressure in the enclosure is the summation of the primary and secondary sources formed by the inserted resonator, the sound pressure radiated from the resonator can be directly analyzed when the primary field source is turned off. Assuming that the sound source velocity strength of primary source is zero, and the lumped mass in the neck of resonator vibrates with the displacement $x(t)=Xe^{i\omega t}$ [X can be calculated by Eq. (9)], the modal response is calculated by:

$$\left(P_j\right)_{only\_resonator} = i\frac{1}{\left(\gamma_j^E\right)^2-\omega^2}\frac{cz_0\omega}{V^E}\frac{1}{Z}\frac{\varphi_j\left(\mathbf{r}^R\right)}{\Lambda_j}\sum_h \varphi_h\left(\mathbf{r}^R\right)P_h \quad , \tag{12}$$

where P_h is calculated by Eq. (5). Substituting $(P_j)_{only\_resonator}$ into Eq. (6), the space averaged energy radiated only by a resonator can be calculated.

III. NUMERICAL SIMULATIONS AND EXPERIMENTS

Numerical simulations on internal resistance are conducted in the following. The coupling model between the enclosure and a resonator has been previously validated [3]. One room, with dimensions l_x=1 m, l_y=0.7 m and l_z=1.22 m, was used as enclosure for experiments and simulations. All physical parameters used are tabulated in TABLE I. Numerically 216 enclosure modes are used in modal superposition. Natural frequencies with damping are calculated by formula $\gamma_j^E = \omega_j^E + iC_j$, in which the real part ω_j is the angular frequency of the jth enclosure mode without damping and the image part is the jth *ad hoc* damping coefficient $C_j = \omega_j^E / 2Q_j$. Q_j is set to 56 for the rigid mode (000) and 46 for other modes. The eigenfunctions $\varphi_j(\mathbf{r}^R)$ of the enclosure for thermalviscous boundary conditions were presented in reference [2]. One square source with dimensions of 100 mm in the x- and y-directions and zero in z-direction was located at (100, 55, 1) mm to drive the first sound field. One Brüel & Kjær Type 4189 ½" microphone located at (0.84, 0.03, 1.06) m was used to measure SPL in the enclosure.

TABLE I. Physical parameters

Physical parameter	value
Ambient temperature, T (^{o}C)	15
Speed of sound, c (m/s)	340.3
Density of air, ρ_0 (kg/m^3)	1.2
Specific heat ratio of air, γ	1.402
Thermal conductivity of air, κ [W/(m·K)]	0.0263
Specific heat at constant pressure of the air , C_p [J/(kg·K)]	1.01×10^3
Coefficient of shear viscosity, μ (Pa.·s)	1.85×10^{-5}

For optimization of internal resistance, mode (101) was selected as targeted mode. The calculated resonance frequency for this mode is 219.8Hz and the experimental value is 224Hz. A Helmholtz resonator was designed to target this mode and fixed at (0.1, 0.3, 0) m. The internal diameter of its neck is 21mm and the physical neck length is 58mm. The internal diameter and length of its body are 74mm and 67mm, respectively. The optimization frequency band was from 200Hz to 240Hz. The Q-factor of the resonator was varied from 1 to 100, corresponding to a variation range of the resistance R from 1.15 to 115.21 mks Rayls. The energy reduction was calculated by Eq. (7) with different damping values. The dissipated energy was calculated by Eq. (11). The radiated energy was calculated by Eq. (6).

FIG.1(a) shows the variation of energy reduction versus the internal resistance. When the resistance R is equal to 4.11 mks Rayls, the energy reduction in the enclosure reaches the maximum value 2.314 dB. Thus this resistance is the optimal one based on the energy approach. FIG.1(b) shows the variation of radiated energy from the resonator and FIG.1(c) shows the energy dissipated by resonator. It is obvious that the variation tendency in FIG.1(c) is similar to that in FIG.1(a). Therefore, the influence of resonator resistance on the energy reduction in enclosure is dominated by the dissipation ability of resonator. From FIG. 1(a, b, c), it is observed that when the resistance is much small, the radiation from the resonator is efficient and the dissipated energy is very small. Therefore, more energy is returned back to the enclosure from the resonator aperture at this circumstance. With the increase of resistance, more sound energy is dissipated by the resonator and the radiated energy is small. However, both the dissipated energy and radiation energy decreases after the internal resistance of the resonator increases over the optimal value.

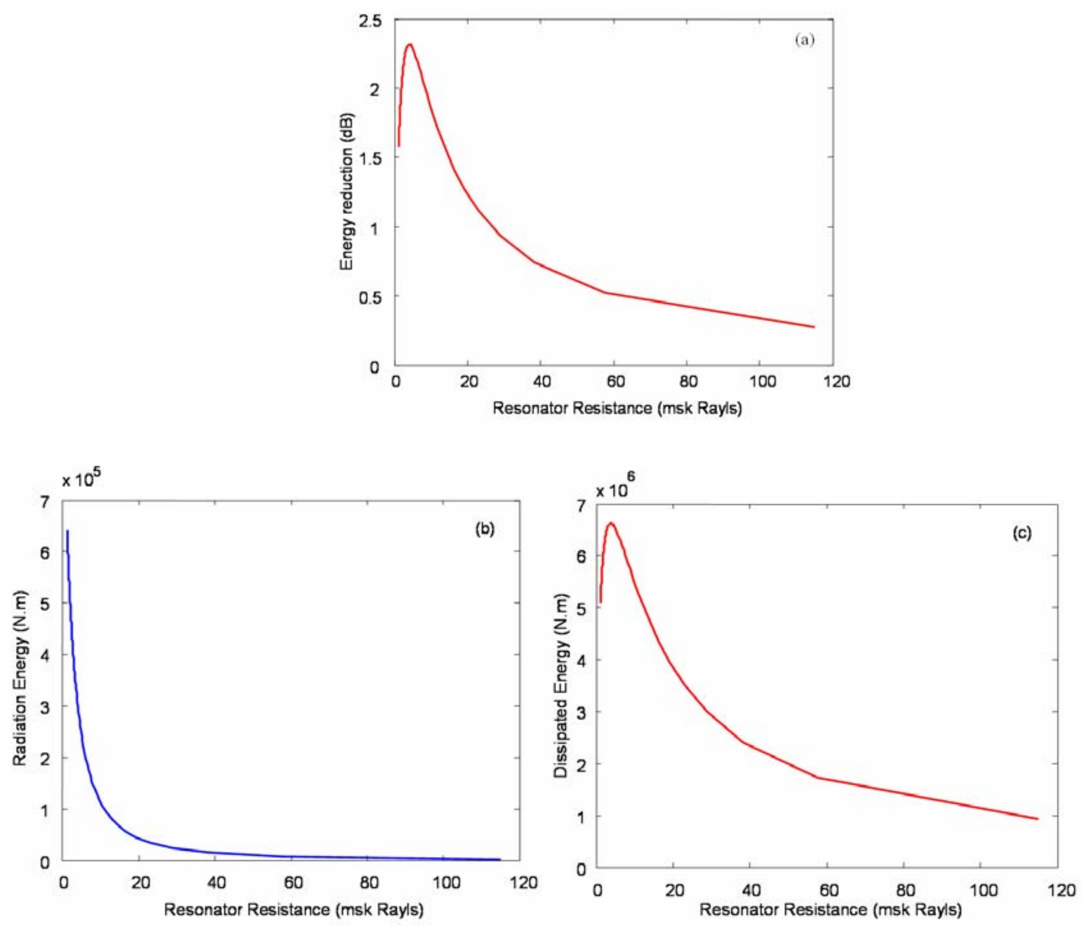

FIG.1. (a) Energy reduction in enclosure measured at (0.84, 0.03, 1.06) m; (b) Radiated energy by resonator; (c) Dissipated energy by resonator.

FIG. 2 shows the experimental results in terms of sound pressure level (SPL) at the microphone position (0.84, 0.03, 1.06) m when resonator resistances are equal to 3.6 and 18.2 mks Rayls. The resonator resistance 3.6 mks Rayls is near the simulated optimal resistance 4.11 mks Rayls, and a large SPL reduction up to 6.9dB at 224Hz was observed. A control over the off-target frequency in a relatively broadband is also obtained from FIG. 2 since the resonator is coupled with all enclosure modes. The internal resistance 18.2 mks Rayls, much larger than the optimal resistance, results in a very low vibrating velocity of the lumped mass inside the resonator neck and subsequently lower energy dissipation inside the enclosure and low radiated energy out of the resonator aperture. Only a 0.66dB SPL reduction at 224Hz was obtained when the internal resistance of the resonator is 18.2 mks Rayls.

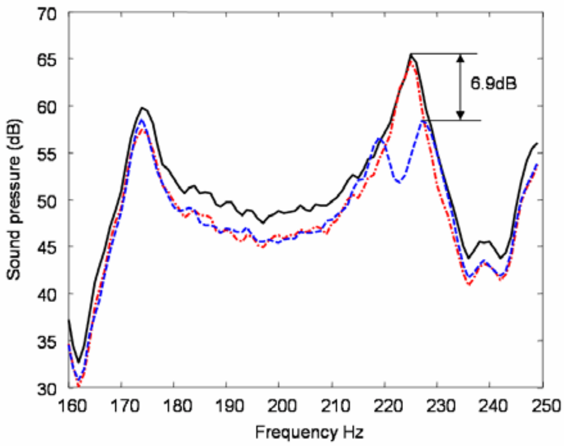

FIG.2. SPL curves at (1.0.84, 0.02, 1.06) m. Experiment: without resonator —; with a damped resonator r = 3.6 mks Rayls — —; with a damped resonator, r = 18.2 mks Rayls.— · —.

IV. CONCLUSIONS

A theoretical method for the optimal design of a Helmholtz resonator internal-resistance is presented based on the maximization of energy reduction. Numerical results show that the dissipated energy dominates the control performance of the resonator (a good cutoff between the peak reduction and the resonator working bandwidth), which greatly depends on both the internal resistance in the resonator neck and the vibrating velocity of the lumped mass in the neck. A large vibrating velocity of the lumped mass (when the internal resistance is small) can provide a large re-radiation volume velocity, which reacts to the primary sound field to sharply reduce the targeted peak by means of acoustic interaction. The measured results show that the optimally designed internal resistance can provide a better control performance for the resonator. Using a Helmholtz resonator with an optimal internal resistance, a SPL reduction up to 6.9dB was obtained around the targeted resonance with 15 Hz bandwidth.

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