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APPLICATIONS OF A FINITE-ELEMENT MODE-MATCHING METHOD TO ACOUSTIC DESIGN OF THE BYPASS DUCT OF TURBOFAN ENGINES

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Abstract

A finite-element mode-matching approach for duct acoustics with flow and circumferentially varying liners is presented. This general prediction method comprises two stages. The first stage is a fully numerical procedure to determine the acoustic modes in ducts of arbitrary cross-section and mean flow profiles. The second stage is a numerical mode-matching method using a modified matching technique in order to deal more accurately with liner discontinuity with flow. By matching modal expansions at the interface between different uniform duct segments, the effect of axial variations of impedance can be modeled with far fewer parameters than would be required for a 3-D numerical transmission analysis. An analytic radiation model can also be integrated with the mode-matching procedure in order to obtain far-field directivity for tones and broadband multi-mode noise. Validation results of the method using analytical results are presented. An application of particular interest to the acoustic design of turbofan engines is the effect of hard patches in lined bypass ducts. These hard patches are introduced to meet other mechanical or aerothermal design requirements but deteriorate the performance of acoustic treatments by reducing the liner area and by introducing significant modal scattering inside the duct. A parametric study of the geometry of a rectangular hard patch in a straight annular duct was carried out using the finite-element mode-matching method. The effect of the hard patch is discussed both in terms of modal scattering inside the duct and in terms of noise radiated to the far field.

1. INTRODUCTION

The prediction of sound propagation in duct with flow is particularly important for the design of low-noise aircraft engines. The optimization of liners for the intake and the bypass ducts is crucial for reducing fan noise from modern high-bypass ratio engines. Several numerical and analytical methods have been developed for the prediction of sound attenuation in lined ducts with flow. For analytical analysis, a method of choice is to use a modal decomposition of the

acoustic field which reduces significantly the complexity of the solution procedure compared to standard numerical approaches. Unfortunately, analytical expressions for the modal basis are only available for simple academic cases (straight ducts with circular or annular cross-sections and with uniform liners). For circular and annular ducts with circumferentially varying liners, it is possible to devise semi-analytical methods by using an azimuthal Fourier decomposition of the impedance [1] or by using multimodal expansions [2].

The present work relies on a hybrid approach where the modes of the duct are determined by a finite element approximation of the eigenvalue problem. This allows for ducts with arbitrary cross-section, liner distribution and flow profiles [3, 4]. The modes are then matched at each liner discontinuity to determine the modal scattering introduce by the discontinuity. When flow is present, a particular issue is the definition of the matching conditions at an interface. In contrast with more standard mode-matching techniques used in the no flow case, modified matching conditions are used in the present work in order to get a better description of the modal scattering.

In this paper, the finite-element mode-matching procedure is described. Then, as an illustration of the method, the influence of hard-patches in a lined bypass-ducts is studied. These hard patches are introduced to meet mechanical or aerothermal design requirements but are detrimental to the performance of acoustic treatments by reducing the liner area and by introducing significant modal scattering inside the duct.

2. THE FINITE-ELEMENT MODE-MATCHING METHOD

The numerical mode-matching method can be applied to a straight duct composed of a series of segments. In each segment, the wall impedance is allowed to vary circumferentially but is constant in the axial direction. The duct carries an axial sheared mean flow. The mean flow axial velocity $w_0(x, y)$, density $\rho_0(x, y)$ and pressure $p_0(x, y)$ are independent of the axial position z but can have arbitrary profiles in the cross-section.

2.1. Modes

The first stage in the analysis is to obtain the modes in each segment of the duct. For time-harmonic problems with an $e^{i\omega t}$ time dependence, the complex amplitude $p(x, y, z)$ of the pressure field inside the duct satisfies Pridmore-Brown's equation

$$\left(i\omega + w_0\frac{\partial}{\partial z}\right)\left[\frac{1}{c_0^2}\left(i\omega + w_0\frac{\partial}{\partial z}\right)^2 p - \Delta p\right] + 2\frac{\partial}{\partial z}\left(\nabla_\perp w_0 \cdot \nabla_\perp p\right) = 0 \ , \tag{1}$$

where $\nabla_\perp$ is the gradient operator in the cross-section of the duct. Solution are thought of the form $p = P(x, y)e^{-ik\lambda z}$ where $k = \omega/c_0$ is the free-field wavenumber. This yields the following eigenvalue problem:

$$(1 - \lambda M_0)\left[\Delta_\perp P + k^2[(1 - M_0\lambda)^2 - \lambda^2]P\right] + 2\lambda\left[\nabla_\perp M_0.\nabla_\perp P\right] = 0 \ , \tag{2}$$

where $M_0 = w_0/c_0$ is the flow Mach number. This equation is supplemented with Myer's boundary condition for wall impedance [5]:

$$\frac{\partial P}{\partial n} = \frac{\rho_0 c_0 k}{iZ} \left(1 - M_0 \lambda\right)^2 P \ , \tag{3}$$

where Z is the liner impedance.

Equations (2) and (3) represent an eigenvalue problem defined on the duct cross-section S with eigenvalue λ and eigenvector P. It is solved numerically using a standard finite element technique. The cross-section is approximated with a mesh of finite elements and the eigenvector $P(x, y)$ is approximated using linear or quadratic shape functions. This yields and algebraic eigenvalue problem that can be readily solved with standard eigenvalue solvers such as LAPACK.

In preparation of the mode-matching method, the modes obtained numerically are discriminated into left-running and right-running modes using the modal power as a criterion to identify the direction of propagation of each mode. The modes are then sorted according to their cut-on ratio.

2.2. Mode matching

For an interface located at $z = 0$ between two segments of the duct, one can write two different modal decompositions of the pressure field:

$$p(x, y, z) = \sum_{\alpha=1}^{N} \left[A^+_{1,\alpha} e^{-ik^+_{1,\alpha} z} P^+_{1,\alpha}(x, y) + A^-_{1,\alpha} e^{-ik^-_{1,\alpha} z} P^-_{1,\alpha}(x, y) \right] \ , \quad \text{for } z < 0 \tag{4}$$

$$p(x, y, z) = \sum_{\alpha=1}^{N} \left[A^+_{2,\alpha} e^{-ik^+_{2,\alpha} z} P^+_{2,\alpha}(x, y) + A^-_{2,\alpha} e^{-ik^-_{2,\alpha} z} P^-_{2,\alpha}(x, y) \right] \ , \quad \text{for } z > 0 \tag{5}$$

where the wavenumbers $k^{\pm}_{n,\alpha}$ and mode shapes $P^{\pm}_{n,\alpha}$ have been obtained numerically for each segment n. One has to obtain the linear relationship between the modal amplitudes $A^{\pm}_{\alpha}$ in one segment in terms of the amplitudes in the other segment. This is done by enforcing conservation of mass and momentum at the interface. This constraint is formulated with a weak variational statement which is also discretized using the finite element method. This yields an algebraic problem of the form

$$\mathbf{X} \begin{pmatrix} \mathbf{A}^+_2 \\ \mathbf{A}^-_1 \end{pmatrix} = \mathbf{Y} \begin{pmatrix} \mathbf{A}^+_1 \\ \mathbf{A}^-_2 \end{pmatrix} \ , \tag{6}$$

where the matrices $\mathbf{X}$ and $\mathbf{Y}$ are calculated from the mode shapes and wavenumbers.

Standard mode-matching strategies are based on the continuity of pressure and axial velocity at the interface between two segments. However, the use of conservation of mass and momentum leads to a more general formulation. With no flow this formulation reduces to the standard formulation imposing continuity of pressure and displacement. But with flow, the modified formulation includes an additional contribution from the boundary of the cross-section which accounts for the flux of momentum through the infinitely thin vortex sheet present above the liner (this vortex sheet is part of Myers' impedance model [5]).

For a duct composed of more than two segments, the linear equations (6) obtained at each

interface can be combined using an iterative method [7]. This yields the scattering matrix of the duct which relates the modal amplitudes at the duct inlet to the modal amplitudes at the exhaust.

2.3. Radiation to the far field

For a given incident sound field at the inlet, the mode-matching method provides the modal amplitudes at the duct exhaust. This information can be used to calculate the sound radiated to the far field. An analytical model for the sound radiation from a semi-infinite jet pipe is used to describe the sound radiation from the duct exhaust, including the refraction effect of the jet mixing layer [6]. This idealized model of noise radiation from bypass ducts is an extension of the Munt solution. The duct is annular with a semi-infinite outer wall and an infinite center body. A cylindrical vortex sheet separates the jet from the ambient flow. For each outgoing mode at the exhaust this model provides the sound directivity in the far field. It is then possible to combine these directivity patterns for a particular modal decomposition.

2.4. Validation

To illustrate the accuracy of the eigenvalue solver, it is tested against analytical solutions for the modes in an annular duct with uniform lining. The parameters of the duct are: hub-tip ratio 0.7, Mach number 0.4, specific impedance $Z = 2 + i$ and Helmholtz number 14.8 (based on the outer radius). Figure 1 shows the numerical error on the wavenumber $k\lambda$ for different mesh resolutions. As the mesh resolution is increased the numerical error decreases over the whole range of modes. It is interesting to note that the error is also increasing with the cut-on ratio of the modes because the shapes of the higher order modes are more oscillatory and are less accurately resolved by the finite element mesh.

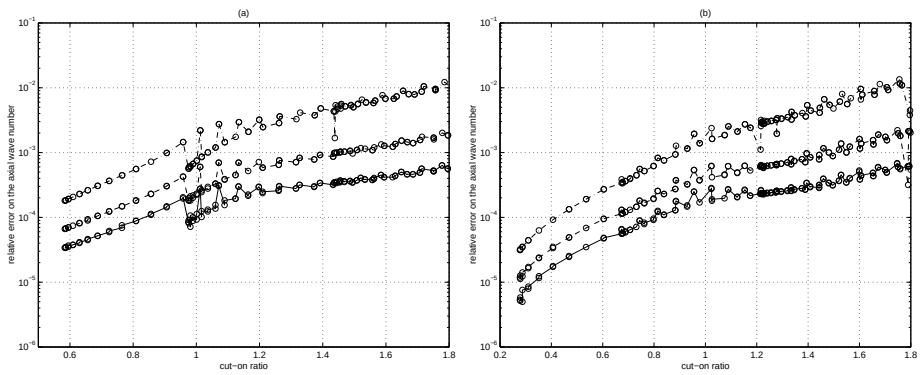

Figure 1. Relative error on the wavenumbers with unstructured meshes for (a) the positive modes and (b) the negative modes. Mesh resolution is 13.6 (dot-dashed line), 22 (dashed line), 31.5 (solid line) points per wavelength.

3. APPLICATION TO LINER HARD PATCHES

The mode-matching method is now used to study the influence of hard patches on the performance of acoustic treatments for turbofan bypass ducts. Hard patches are small areas of a lined surface where the acoustic treatment has been removed. Hard patches can be due to the presence of other components of the engine that prevent the use of liner in some areas of the bypass

duct. Maintenance also requires to remove small part of the liners which are then replaced by repair patches. Hard patches are detrimental to the performance of liners because they reduce the area of treated surfaces and they also introduce significant modal scattering by redistributing the acoustic energy onto low-order modes which are less attenuated.

The configuration considered in this study is an annular duct (with hub-tip ratio 0.78) with a uniform mean flow (with Mach number 0.447). The Helmholtz number is 18.6 (based on the outer radius). The duct is composed of four segments. The first and last segments have hard walls. The third segment is fully lined with a specific impedance $Z = 2 - i$. The second segment is similar except for a hard patch on the outer wall. The configuration of the liner on the duct walls is illustrated in figure 2. The area of the patch is kept constant (it represents 5% of the lined area of the outer wall). The aspect ratio of the patch is varied and is characterized by its azimuthal width given as a fraction of the outer wall perimeter (so 100% corresponds to an axisymmetric configuration).

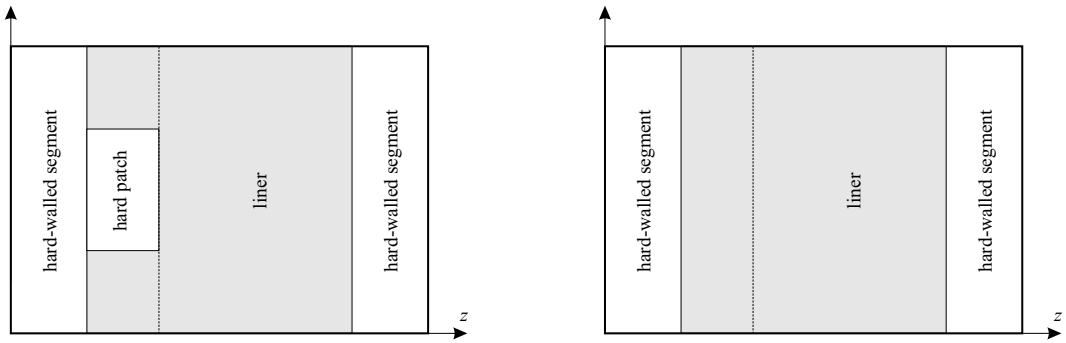

Figure 2. Schematics of the liner distribution on the outer duct wall (left) and the inner duct wall (right).

3.1. In-duct propagation

The modal scattering introduced by the hard patch is shown in figure 3 where the modal power for each mode at the duct exhaust is plotted for different patch widths. The incident acoustic field is the mode (10,1) with a unit acoustic power. Compared to the non axisymmetric configurations, it is clear that the case where the patch width is 100% presents very little modal scattering. With the non-axisymmetric patches however the energy of the incident acoustic mode is significantly redistributed onto other modes, especially low order modes which are less attenuated by the liner.

3.2. Noise radiated to the far field

The description of the acoustic field at the duct inlet is useful, but it is crucial to consider also the noise radiated to the far field. The radiation efficiency can vary significantly between modes. It is therefore important to assess the effect of hard patches in terms of the sound radiated to the far field. For the present test case we consider an ambient flow with Mach number 0.219.

Figure 5 shows the insertion loss of the far-field acoustic intensity for different aspect ratios of the patch. For these results the incident acoustic field is composed of all the cut-on modes with an equal energy distribution and no correlation. In line with the analysis of the in-duct propagation, it is observed again that the insertion loss due to an axisymmetric patch is relatively small compared to non-axisymmetric patches. The introduction of non-axisymmetric

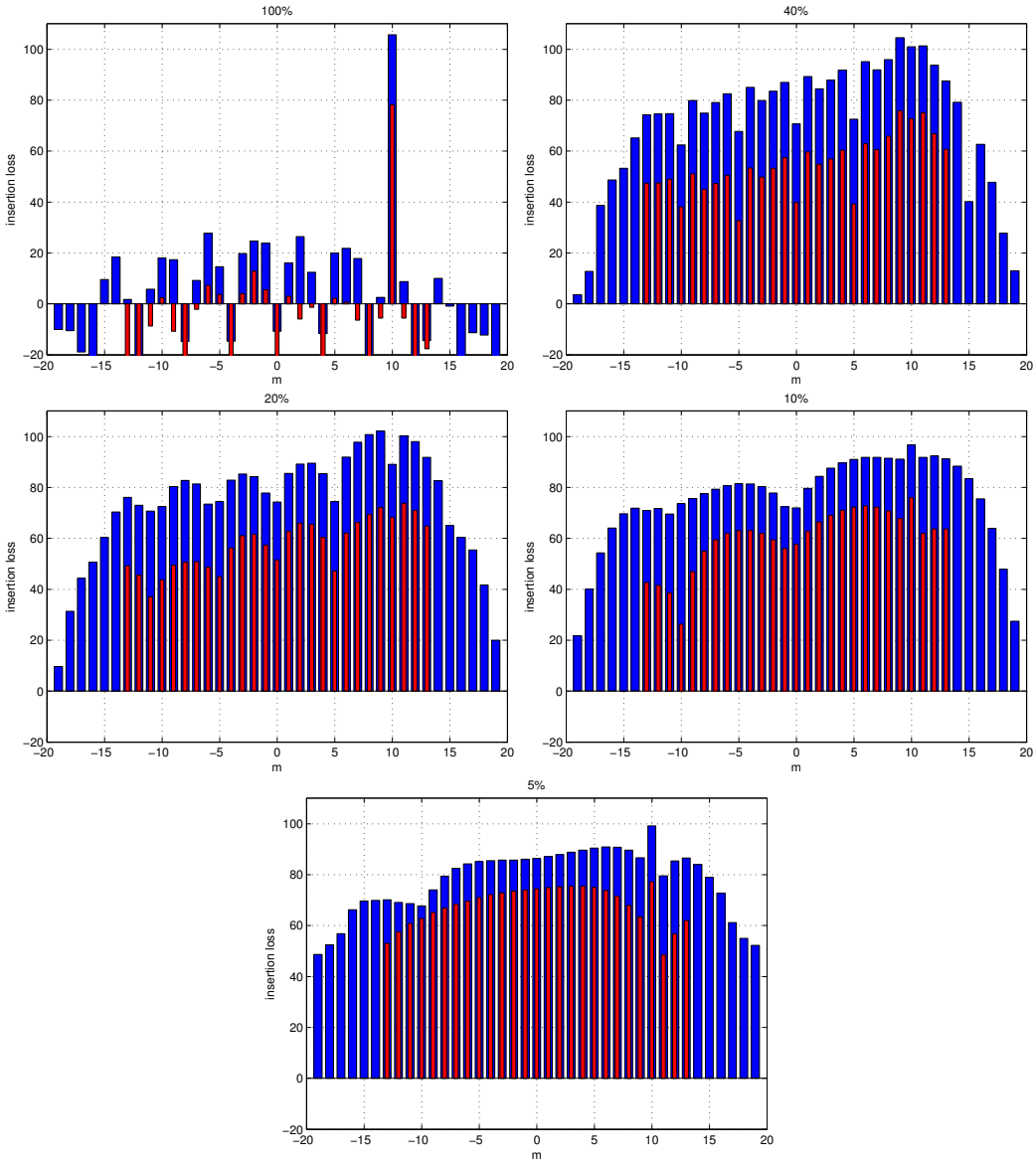

Figure 3. Modal power at the duct exhaust for different patch widths. The amplitude of the incident mode (10,1) is such that its modal power at the exhaust is 100dB. Blue: $n = 0$; Red: $n = 1$.

patches results in strongly skewed directivity in the far field. For patches of widths 40%, 30% and 10% increases of up to 4dB can be observed in the acoustic intensity radiated downward. Obviously this can have a dramatic impact on the perceived noise levels.

Another useful metric of the effect of hard patches is the total acoustic power radiated to the far field. Figure 4 shows the insertion loss for this quantity as a function of the patch width. Three different noise sources are considered: two tones with the plane wave and the mode (10,1) and a broadband multi-mode source (uncorrelated modes with equal energy distribution). The hard patches have little influence on the propagation of the plane wave with approximately 0.5dB increase in the acoustic power. This is due to the fact that with the plane wave acoustic energy is propagating parallel to the duct walls and the liner and the hard patch has only limited influence on the plane wave propagation. The propagation of broadband noise inside the duct is also only marginally modified by the hard patch. With broadband noise the sound field can be

considered as a diffuse field. In that case sound absorption is directly proportional to the area covered by the liner and the modal scattering introduced by the patch is of little importance. Therefore the main influence of the hard patch is to reduce the lined area and to slightly increase the transmitted acoustic power. By contrast modal scattering is largely responsible for the large increase observed for the mode (10,1). In the worst case, the acoustic power radiated to the far field is increased by 8.5dB by the patch of width 40%. Interestingly, the narrow patches of widths 5% and 10% create an increase of 3dB of the acoustic power.

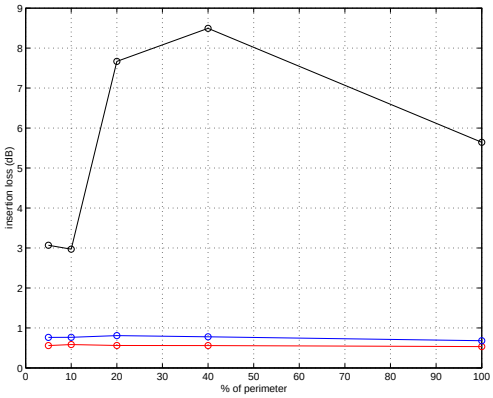

Figure 4. Insertion loss of the hard patch for the total power radiated in the far field with different incident sound fields. Red: plane wave; Black: mode (10,1); Blue: broadband noise.

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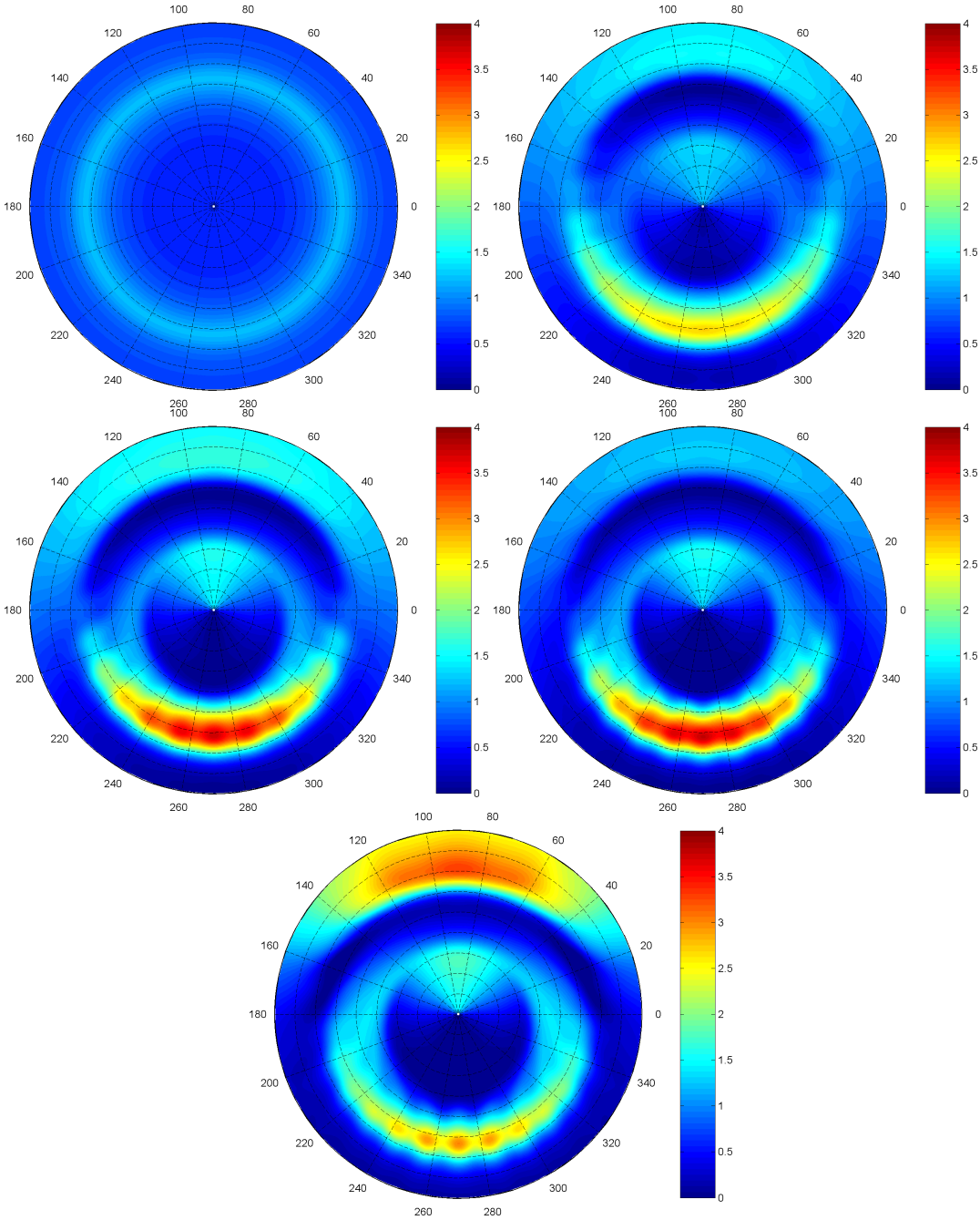

Figure 5. Insertion loss for the acoustic intensity in dB in the far field with broadband noise. The patch width is 100%, 40%, 20%, 10% and 5%. In these graphs the intensity is plotted on the hemisphere of the rear arc viewed from some position downstream on the axis. The patch is centered at the top of the outer wall (i.e. if α is the angular width of the patch then it is located between $90° - \alpha/2$ and $90° + \alpha/2$).