ICSV14
Cairns • Australia
9-12 July, 2007
14th International Congress on Sound & Vibration

VIBRATION ANALYSIS OF A BEAM CARRYING A MOVING MASS

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Abstract

This paper deals with the linear dynamic response of a cracked cantilever beam subjected to a moving mass. The velocity of the moving mass is assumed to be constant. The present analysis in its general form may well be applied to beams with various boundary conditions. Results from the numerical solutions of the differential equations of motion are shown graphically. Moreover, when considering the maximum deflection for the end point of the beam, the critical speeds of the moving mass have been evaluated. Experiments have been conducted to compare with the numerical results. It is observed that the experimental results are in good agreement with the numerical one.

1. INTRODUCTION

The general problem of transverse vibrations of beams resulting from the passage of moving loads (moving forces and moving masses) is of considerable practical interest in the dynamics of structures. It has a wide range of applications in many fields, from structural to mechanical to aerospace. This problem has been studied in the context of machinery operations and the behavior of bridges, runways, rails, roadways, pipelines, etc. In order to recognize when the structure is approaching an over stressed condition, it is necessary to understand the complexity of the dynamic interaction between the continuous system and the sub-system moving on it. A complete analysis must also take into account the presence of existing structural damage. Several investigations have been performed to study different aspects of such a problem.

At the beginning of twentieth century, Jeffcott [1] managed to calculate the vibration response of simple structures with a moving mass. The response characteristics of a beam subjected to a moving force were investigated by Florence [2], Stele [3], Kenney [4] and Smith [5]. In 1969, Stanisic and Hardin [6] determined the dynamic behavior of a simply supported beam carrying a moving mass, which is interesting enough, but their method is not easily applicable to different boundary conditions. A comprehensive treatment of the subject of the vibration of structures due to moving loads, which contains a large number of related cases, is that of Fryba [7].

Stanisic et al. [8] have developed a numerical-analytical method for determining the behaviour of beams with various boundary conditions and carrying a moving mass. Saigal [9] has developed expressions for beam structures with the help of Stanisic et al. theory [8], which has a higher degree of practical significance.

Later, Akin and Mofid [10] analysed such problems for finite beams with moving loads using a differential equation. Their theory is based on orthogonal functions, and the results indicate that the governing differential equation can be transferred into a series of coupled ordinary differential equations. The study has used the Continuous System Modeling Programming (CSMP) to solve the system of ordinary differential equation.

Behera et al. [11] have presented an analytical-computational method to study the dynamic behaviour of a cantilever beam with multiple cracks. They have compared between the numerical and experimental results. The results obtained from the experimental analysis show excellent agreement with the corresponding numerical results.

Mahmoud and Abou Zaid [12] have developed an iterative modal analysis approach to determine the effect of transverse cracks on the dynamic behavior of simply supported undamped Bernoulli-Euler beams subject to a moving mass. They have shown, the largest deflection in the beam for a given speed takes longer to build up and a discontinuity appears in the slope of the beam-deflected shape at the crack location. Behera et al. [13] have presented an analytical method to determine the response of a cracked beam with different boundary conditions, carrying a moving mass.

In this paper, the dynamic behavior of a two-crack cantilever beam with a moving mass is studied. The effect of acceleration in the direction of deflection, the effect of Coriolis force, and the effect of the path curvature are included in the problem formulation. The numerical results are compared with the experiment. A very good agreement is obtained between the numerical and experimental results.

2. EQUATION OF MOTION

A cantilever beam as shown in Fig.1, of length L, mass per unit length *m* and flexural rigidity *EI*, made from a uniform homogeneous and isotropic material is considered. The beam is subjected to a load F of mass M, moving with a constant velocity v. The force acting on the beam is given by

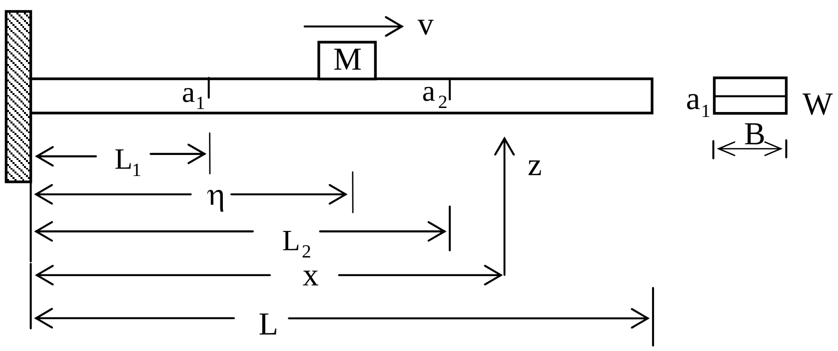

Figure 1. Geometry of cracked beam with a moving mass

$$F(t) = Mg - M\,\frac{D^2 y(\eta,t)}{Dt^2}$$

(1)

where $\eta = vt$,g is thegravitational acceleration and

$$\frac{D^2 y(\eta,t)}{Dt^2} = \frac{\partial^2 y(\eta,t)}{\partial t^2} + 2v\frac{\partial^2 y(\eta,t)}{\partial x \partial t} + v^2 \frac{\partial^2 y(\eta,t)}{\partial x^2}.$$

Starting from the equality sign, the terms on the right hand side of the above equation represent: the effect of acceleration in the direction of deflection $y(\eta,t)$, the effect of the Coriolis force, and the effect of the path curvature.
Thus, the equation of motion can be written as

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + m\frac{\partial^2 y(x,t)}{\partial t^2} = P(x,t), \tag{2}$$

with $0 \le t \le L/v$, and δ is the Dirac delta function [14] . The external force *P(x, t)* is taken as

$$\mathrm{P}(x,t) = \mathrm{F}(t)\delta(x - vt). \tag{3}$$

Using Eq. (1), Eq. (2) and Eq. (3), one can find

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y(x,t)}{\partial t^2} = \left[Mg - M\frac{D^2 y(\eta,t)}{Dt^2}\right]\delta(x-\eta). \tag{4}$$

A series solution of Eq. (2) can be taken in the form of

$$y(x,t) = \sum_{n=1}^{\infty} Y_n(x) T_n(t), \tag{5}$$

where $Y_n(x)$ is the known eigen functions of a freely vibrating beam and $T_n(t)$ is the modal amplitude to be found, and n is the number of contributed modes. To find $Y_n(x)$, the equation will be

$$Y_n^4(x) - \gamma_n^4 Y_n(x) = 0,$$
(6)

where $\gamma_n^4 = \rho A \omega_n^2 / EI$, A is the cross-section area, ρ is the material density, ω_n are the natural frequencies of the beam, and $T_n(t)$ are functions of time which have to be calculated.
Due to the localized crack effect, the double-cracked beam can be simulated as one with three segments. The general solution for the displacement on each part of the beam, based on the Euler-Bernoulli beam theory, from Eq. (6) is

$$Y_n(x) = a_1 \sin(\gamma_n x) + b_1 \cos(\gamma_n x) + c_1 \sinh(\gamma_n x) + d_1 \cosh(\gamma_n x) \text{ for } 0 \le x \le L_1; \tag{7a}$$

$$Y_n(x) = a_2 \sin(\gamma_n x) + b_2 \cos(\gamma_n x) + c_2 \sinh(\gamma_n x) + d_2 \cosh(\gamma_n x) \text{ for } L_1 < x \le L_2; \tag{7b}$$

$$Y_n(x) = a_3 \sin(\gamma_n x) + b_3 \cos(\gamma_n x) + c_3 \sinh(\gamma_n x) + d_3 \cosh(\gamma_n x) \text{ for } L_2 < x \le L, \tag{7c}$$

where a_i, b_i, c_i and d_i (i =1,2,3) are constant coefficients to be determined depending upon the boundary conditions[11,15].

$T_n(t)$ in Eq. (5) is the function of time and can be calculated by rewriting the right hand side of Eq. (4) as

$$\left[Mg - M\,\frac{D^2 y(\eta,t)}{Dt^2} \right] \delta(x-\eta) = \sum_{n=1}^{\infty} Y_n(x) S_n(t)\,. \tag{8}$$

Substituting Eq. (5) into Eq. (8), Eq. (8) converts to

$$\left[Mg - M\,\frac{D^2}{Dt^2}\left(\sum_{n=1}^{\infty} Y_n(\eta) T_n(t) \right) \right] \delta(x-\eta) = \sum_{n=1}^{\infty} Y_n(x) S_n(t) \tag{9}$$

Multiplying both sides of Eq. (9) by $Y_p(x)$, then integrating over the beam length, using the dirac delta integral property [14] and by using the orthogonal property of the function the equation of motion can be written in a coupled ordinary differential equation form [13] as

$$EI\sum_{n=1}^{\infty} \gamma_n^4 Y_n(x) T_n(t) + \rho A \sum_{n=1}^{\infty} Y_n(x) T_{n,tt}(t)$$

$$= \sum_{n=1}^{\infty} Y_n(x) \frac{M}{V_n} \left[g - \sum_{q=1}^{\infty} \frac{D^2}{Dt^2} Y_q(\eta) T_{q,tt}(t) \right] Y_n(\eta)\,. \tag{10}$$

Eq. (10) may be rearranged as

$$\sum_{n=1}^{\infty} Y_n(x) \left\{ EI\gamma_n^4 T_n(t) + \rho A T_{n,tt}(t) - \frac{M}{V_n} \left[g - \sum_{q=1}^{\infty} \frac{D^2}{Dt^2} Y_q(\eta) T_q(t) \right] Y_n(\eta) \right\} = 0\,. \tag{11}$$

As Eq. (11) must be satisfied, for any arbitrary value of x (i.e., each point of the beam), and this is possible only when the expression in the bracket is equal to zero for arbitrary *n*,

$$EI\gamma_n^4 T_n(t) + \rho A T_{n,tt}(t) - \left(\frac{M}{V_n} \right) \left[g - \sum_{q=1}^{\infty} \frac{D^2}{Dt^2} Y_q(\eta) T_q(t) \right] Y_n(\eta) = 0\,. \tag{12}$$

Equation (12) is a set of coupled ordinary differential equations and a numerical procedure Runge-Kutta method is used to solve it.

3. NUMERICAL ANALYSIS

The values $T_n(t)$ are obtained at the required position using equation (12). The Runge-Kutta numerical method is used to solve it. Then the displacement of the beam at any instant of time can be obtained by using Eq. (5). The first three modes of vibration are calculated and the total deflections of the beam are found out. An aluminum cantilever beam (0.8x0.038x0.006m) as shown in Fig.1 is considered for the beam response. The relative crack positions (zl1, zl2) are taken as 0.125 and 0.25 for two cracks. The relative crack depths(a_1/W , a_2/W) at the crack locations vary between 0.1667 and 0.334.The numerical results for displacement at the end point of the cantilever beam with and without crack are plotted in Figs.3 & 4. Fig. 5 shows the 3-D plot for the variation of displacement at the free end of cracked beam with variation of time for different velocities of the moving mass.

4. EXPERIMENTAL ANALYSIS

The experimental set up used for conducting the experiment is shown in Fig.2. A number of tests were carried out on an aluminium beam specimens (L = 0.8 m, B = 0.038 m, W = 0.006 m) with and without transverse cracks. The relative crack locations were taken such that zl1= 0.125 and zl2 = 0.25. The relative crack depths were chosen such that, a_1/W , a_2/W = 0.1667 , and 0.334 for the vibration analysis. Moving masses for the tests were selected as 1.0 and 1.5 kg. The velocity of the moving body was considered as10.28 km/h.The LVDT was positioned at the free end of the beam. The free end deflection of the beam for various positions of the moving mass was recorded. The outputs from the LVDT were taken to a computer using a data acquisition card. The results for different velocities of the moving mass are plotted in Figs.3 & 4. The corresponding numerical results are also shown in the same graph for comparison. While conducting the experiments, precautions were taken in positioning the LVDT and adjusting the variac accurately to have desired constant velocity of the moving masses.

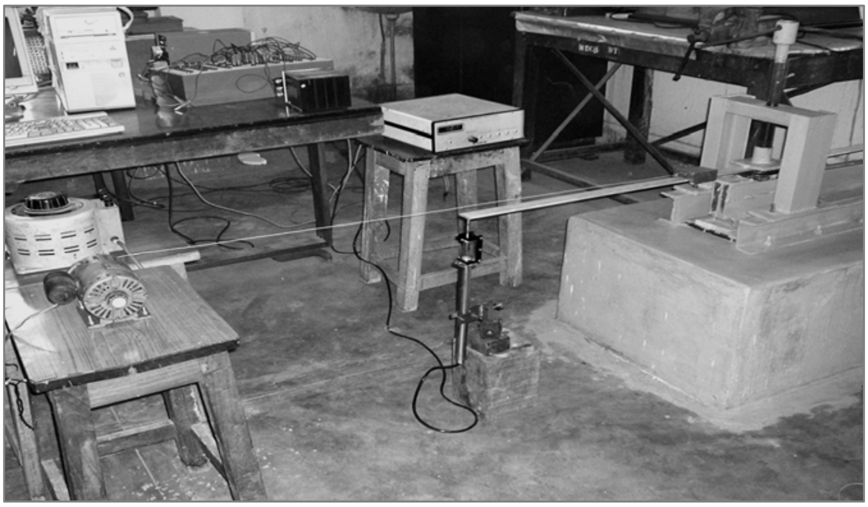

Figure 2. Experimental set up

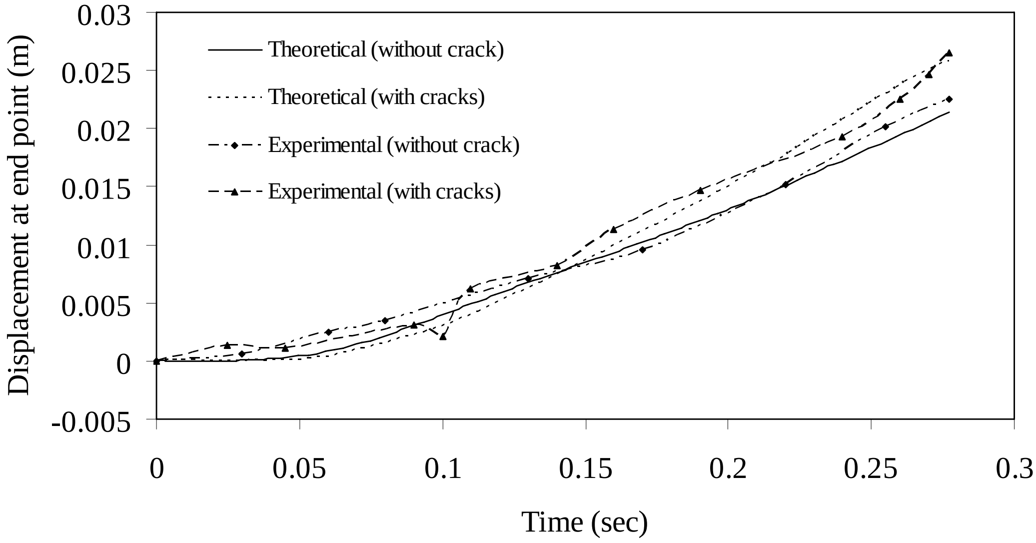

Figure 3. Displacement at end point (m) vs. time (sec), M=1.0 kg, v=10.28 km/h
$a_1 / W = 0.1667, a_2 / W = 0.1667$, zl1=0.125, zl2=0.25

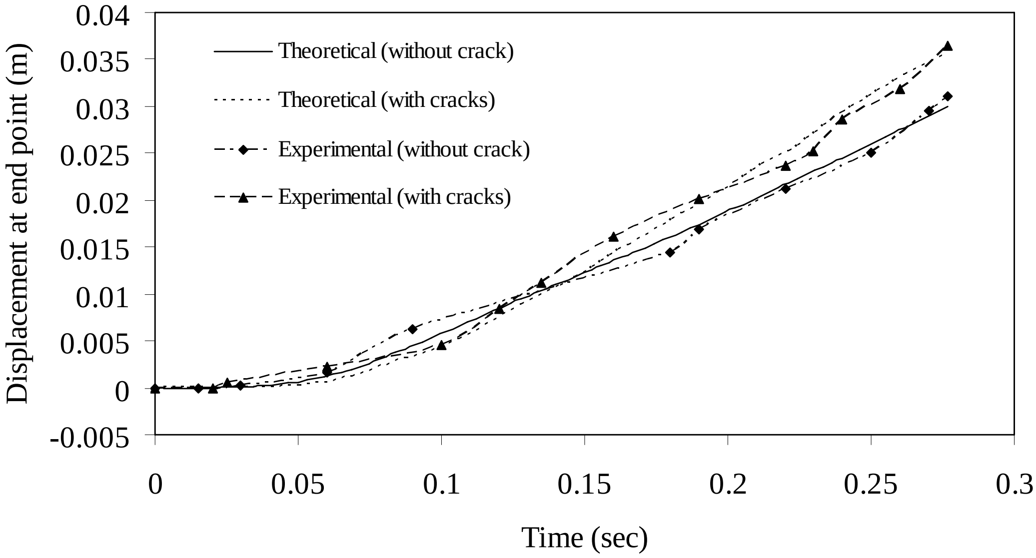

Figure 4. Displacement at end point (m) vs. time (sec), M=1.5 kg, v=10.28 km/h.
$a_1 / W = 0.1667, a_2 / W = 0.1667$, zl1= 0.125, zl2 = 0.25

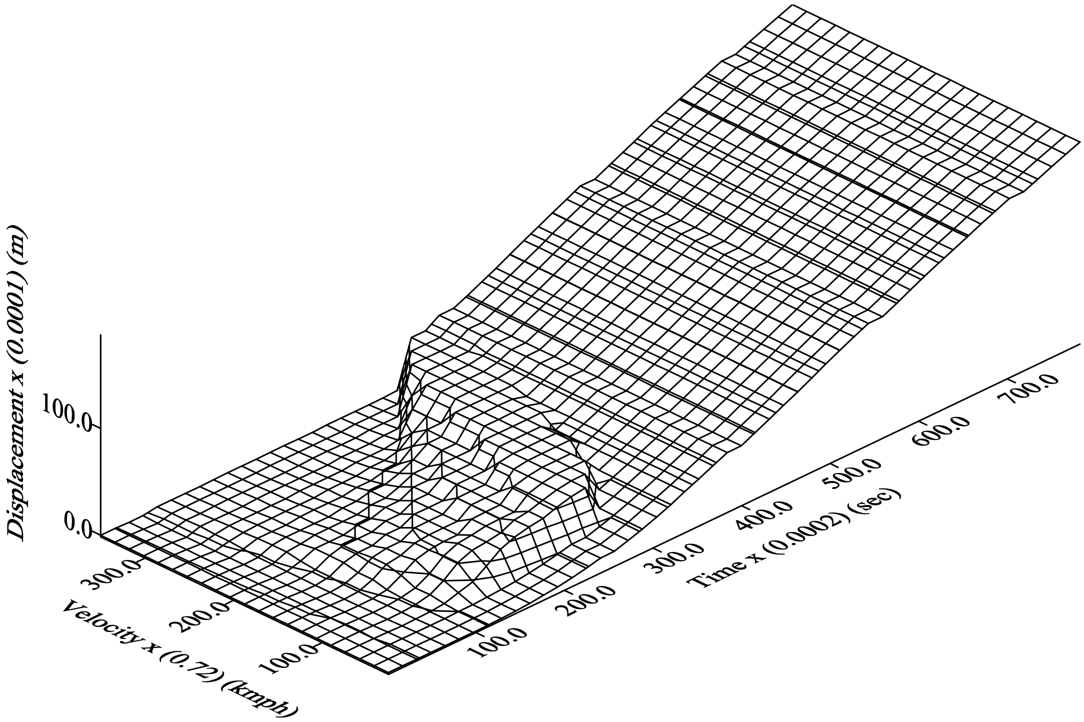

Figure 5. 3-D plot for the displacement at end point (m)-velocity (km/h)- time (sec), M = 1.0 kg, $a_1 / W = 0.334, a_2 / W = 0.334$, zl1= 0.125, zl2 = 0.25

5. DISCUSSIONS

From Figs.3 & 4 it is noticed that, as the mass of the moving body increases, the end point deflection of the cracked beam increases relative to that of the corresponding uncracked beam. As the speed of the moving mass increases the maximum dynamic deflection of the end point increases until it reaches a maximum. Above this speed, the maximum deflection decreases with increase in moving mass speed. The moving mass speed that corresponds to the maximum end-point deflection is called critical speed of the moving mass. The maximum dynamic deflections of the free end are shown in Fig.5.

6. CONCLUSIONS

The behaviour of a cracked cantilever beam, when carrying a moving mass with uniform speed, has been analyzed. In describing the dynamic effect of the moving mass load, the total differential must be considered since the mass is moving on a vibrating path. It has been shown that the inertial effect of the mass load causes the set of differential equations of motion to be coupled. Hence, ignoring this effect results in solving a set of uncoupled linear second order differential equations, which is the solution for the corresponding moving force, and not the mass, problem. In order to solve the governing differential equation, the Runge-Kutta numerical technique was employed. The solution series converged rapidly and usually required a small number of modal shapes for relatively accurate results. At the critical speed the deflection of the beam reaches to a maximum value. Beyond the critical speed the deflection of the beam decreases.

REFERENCES

[1] H.H. Jeffcott, "On the vibration of beams under the action of moving loads", *Phil. Mag* **7**(8), 66 (1929).

[2] A.L. Florence, "Travelling force on a Timoshenko beam", *Trans. ASME, Journal of Applied Mechanics* **32**, Ser.E (2): 351-358. (1965).

[3] C. R. Steele, "The finite beam with a moving load", *Trans. ASME, Journal of Applied Mechanics* **34**, Ser.E (1), 111-118 (1967).

[4] J.T. Kenney, "Steady-state vibration of a beam on elastic foundation for moving load", *Trans. ASME, Journal of Applied Mechanics* **21**(4), 359-364 (1954).

[5] C.E. Smith, "Motion of a stretched string carrying a moving mass particle", *Trans. ASME, Journal of Applied Mechanics* **31**, Ser.E (1), 29-37 (1964).

[6] M.M. Stanisic and J.C. Hardin, "On response of beams to an arbitrary number of moving masses", *Journal of Franklin Institute*, 287 (1969).

[7] L. Fryba, *Vibration of solids and structures under moving mass*. London: Thomas Telford, 1999.

[8] M.M. Stanisic, J.A. Euler and S.T. Montgomery, "On a theory concerning the dynamic behavior of structures carrying moving mass", *Ingenur Archieve* **43**, 295-305 (1974).

[9] S. Saigal, "Dynamic behavior of beam structure carrying moving masses",*Trans. ASME, Journal of Applied .Mechanics* **53**(1) 222-224 (1986).

[10] J.E. Akin and M. Mofid, "Numerical solution for response of beam with moving mass. *ASCE Journal of Structural Engineering* **115**(1), 21-131(1989).

[11] R.K. Behera, D.R.K. Parhi and S.K. Sahu, "Dynamic characteristics of cantilever beam with transverse cracks", *International Journal of Acoustics and Vibration*, **11**(3), 131-144 (2006).

[12] M.A. Mahmoud and M.A Abou Zaid, "Dynamic response of a beam with a crack subjected to a moving mass", *Journal of Sound and Vibration*, **256**(4), 591-603, 2002.

[13] R.K. Behera, D.R.K. Parhi and S.K. Sahu, "Dynamic response of a beam with a crack subjected to a moving mass", *International Journal of Acoustics and Vibration*, **10**(4), 131-144 (2005).

[14] F.S. Tse, I.E. Morse and R.T. Hinkle, "Mechanical Vibrations", CBS Publishers, Second Edition, 1983.

[15] A.D. Dimarogonas and S. Haddad, *Vibration for Engineers*, Prentice Hall, Englewood Cliffs, New Jersey 07632, 1992.